SEISMIC SIGNAL COMPRESSION THROUGH DELAY COMPENSATED AND ENTROPY CONSTRAINED DICTIONARY LEARNING

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Introduction

- A seismic survey, which collects a huge amount of seismic data from the field, is a primary strategy utilized for the oil and gas exploration.
- Data compression is highly desirable to reduce the costs of the storage and transmission, especially for wireless seismic signal acquisition.
- All previous works demonstrate the potential of learning dictionary for the successful representation of seismic data.

Motivation

- Compensating for the delays between seismic traces makes them more similar to each other.
- Although, sparse coefficients is intuitively favorable for compression, they are not necessarily the most compressible ones. As such, reducing the entropy of the quantized coefficients is more essential.
- **The goal of this paper is to develop a dictionary learning method to represent the appropriately time-shifted seismic segments such that the sparse coefficients follow a more favorable distribution for compression.**

Dictionary Learning and Sparse Coding

- Dictionary learning aims at finding a dictionary for a data set such that each data sample in the data set can be efficiently estimated by sparse linear combination of atoms in the learned dictionary.
- Given the input data set $\mathbf{Y}=[\mathbf{y}_1,\ldots,\mathbf{y}_K]\in\mathbb{R}^{m\times K}$, traditional sparse dictionary learning techniques solve for a normalized dictionary $\mathbf{D}=[\mathbf{d}_1,\ldots,\mathbf{d}_n]\in\mathbb{R}^{m\times n}$ and a sparse coefficients matrix $\mathbf{W}=[\mathbf{w}_1,\ldots,\mathbf{w}_K]\in\mathbb{R}^{n\times K}$ such that the overall error $\|\mathbf{Y}-\mathbf{DW}\|_F^2$ is minimized within a given sparsity level L. This could be formulated as

$$\arg\min_{\mathbf{D},\mathbf{W}} \|\mathbf{Y}-\mathbf{DF}\|_F^2, \quad \text{s.t. } \|\mathbf{w}_i\|_0\le L, \quad i=1,\ldots,K$$

Problem Formulation

- It includes the following two sub-problems:

A.1 Delay compensated dictionary learning: For given maximum sparsity level, L, the goal is learning the time-shifts and the dictionary and coefficients such that

$$[\hat{T}_i,\bar{\mathbf{D}},\mathbf{w}_i]=\arg\min_{T_i,\mathbf{D},\mathbf{w}_i}\sum_{i=1}^{K}\|\phi(\mathbf{y}_i,T_i)-\mathbf{D}\mathbf{w}_i\|_2^2, \text{ s. t. } \|\mathbf{w}_i\|_0\le L,$$

$\phi(\mathbf{y}_i,T_i)$ is the circular shift operator that delays $\mathbf{y}_i$ by T_i samples.

A.2 Entropy constrained dictionary learning: For $\hat{T}_i$'s given from **A.1**, the dictionary and the coefficients are learned to minimize the compression rate. The corresponding optimization problem can be written as

$$[\hat{\mathbf{D}},\hat{\mathbf{w}}_i,\hat{h}(\mathbf{w}_i)]=\arg\min_{\mathbf{D},\mathbf{w}_i,h(\mathbf{w}_i)}\sum_{i=1}^{K}(\|\hat{\mathbf{y}}_i-\mathbf{D}\mathbf{w}_i\|_2^2+\lambda h(\mathbf{w}_i)), \text{ s. t. } \|\mathbf{w}_i\|_0\le L,$$

$h(\mathbf{w}_i)$ is the estimated compression rate (entropy) of the coefficients.

1 Delay Compensated Dictionary Learning

1-a) Delay compensated sparse coding

For fixed $\bar{\mathbf{D}}$, a dictionary set $\{\bar{\mathbf{D}}\}_{T_b}=\phi(\bar{\mathbf{D}},\hat{T}_b)$ is constructed. Then delay compensated sparse coding involves finding an appropriate delay T_b and sparse coefficients $\mathbf{w}_i$, which can be formulated as:

$$\arg\min_{T_b,\mathbf{w}_i}\sum_{i=(b-1)N+1}^{bN}\|\mathbf{y}_i-\{\bar{\mathbf{D}}\}_{T_b}\mathbf{w}_i\|_2^2, \text{ s.t. } \|\mathbf{w}_i\|_0\le L,$$

It can be efficiently solved via ORMP.

1-b) Delay compensated sparse coding

Given delay compensated data $[\hat{\mathbf{y}}_1,\ldots,\hat{\mathbf{y}}_K]$ and sparse coefficients $[\mathbf{w}_1^{(T_1)},\ldots,\mathbf{w}_K^{(T_k)}]$, the dictionary $\bar{\mathbf{D}}$ can be updated using existing methods such as MOD.

2 Entropy Constrained Dictionary Learning

2-a) Entropy constrained sparse coding

The solution of the entropy-constrained optimization problem can be written as $\mathbf{w}^{\gamma}=\Delta\,[w_1^r+\gamma_1,\ldots,w_n^r+\gamma_n]$ Where $\boldsymbol{\gamma}=[\gamma_1,\ldots,\gamma_n]$ is the vector of unknown parameters to be found. Then the optimization could be written as:

$$\arg\min_{\gamma}\|\hat{\mathbf{y}}_i-\textstyle\sum_{q\ne j}\mathbf{d}_q(z_q^r+\hat{\gamma}_k)\Delta-\mathbf{d}_j(z_j^r+\gamma)\Delta\|_2^2+\lambda h(z_j^r+\gamma),$$

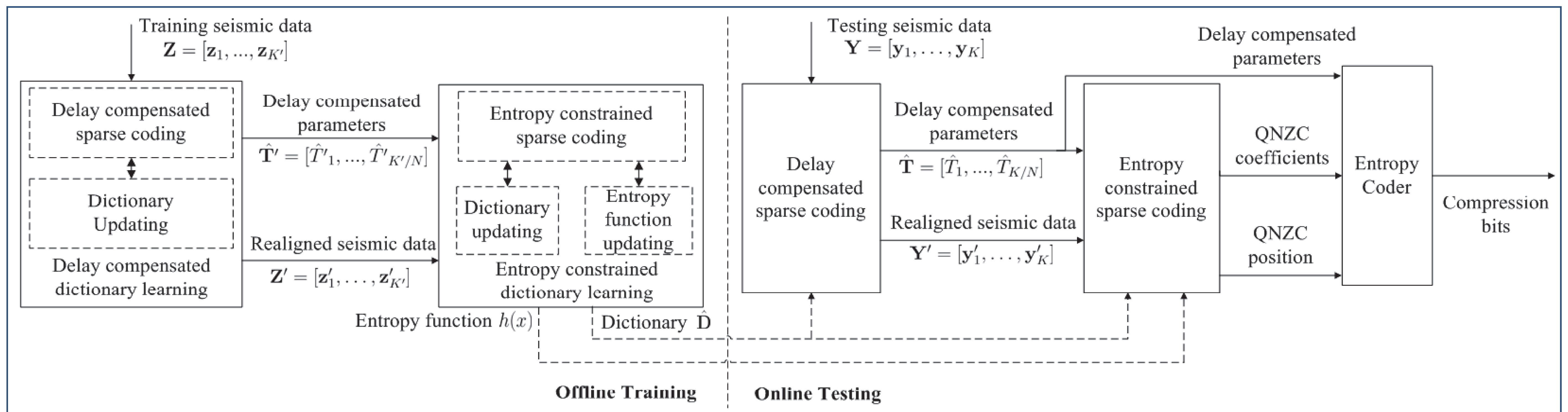
2-b) Entropy function updating

The entropy function $h(x)$ could be updated as: $h(x)=-p(x)\log_2 p(x)$ where $p(x)$ is the probability of nonzero coefficient x in all quantized NZC.

2-c) Dictionary updating

In our method, we chose to use MOD for dictionary updating.

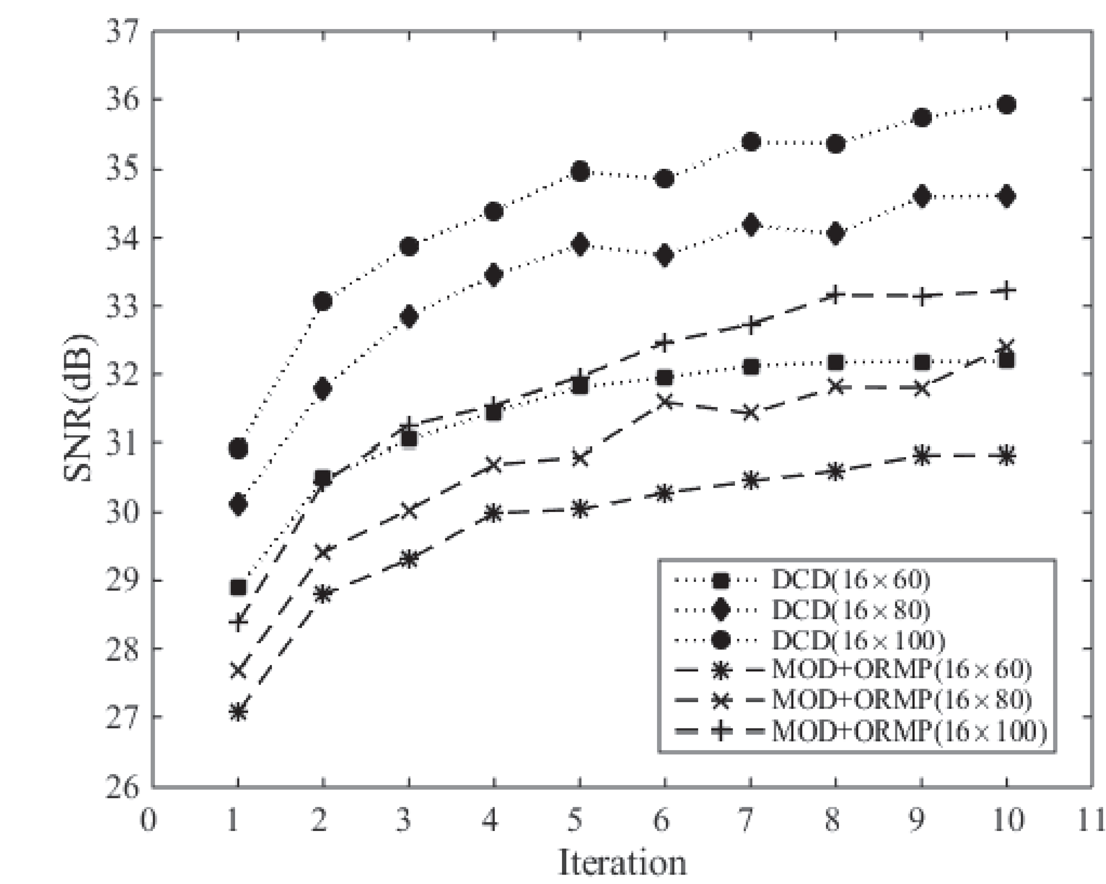
Implementation



- It includes two steps: offline training and online testing.
- Offline training is used to learn some prior information, such as the dictionary, codebook and the entropy function.
- Online testing includes 1) delay-compensation, 2) entropy-constrained sparse coding which uses the delay- compensated data from previous step for entropy constrained sparse coding.

Experimental Results

- The efficiency of Delay Compensated Dictionary learning (DCD) is verified. The traditional dictionary learning method using MOD + ORMP is compared to our proposed scheme.



- Next, we want to verify the efficiency of compression using Entropy Constrained Dictionary learning with Delay Compensated inputs (ECDDC).
- The proposed method outperforms other schemes by optimizing for the compression rate and compensating for the delays between different segments of traces which increases the reconstruction quality.