Deterministic Annealing for Hybrid Beamforming Design in Multi-Cell MU-MIMO Systems

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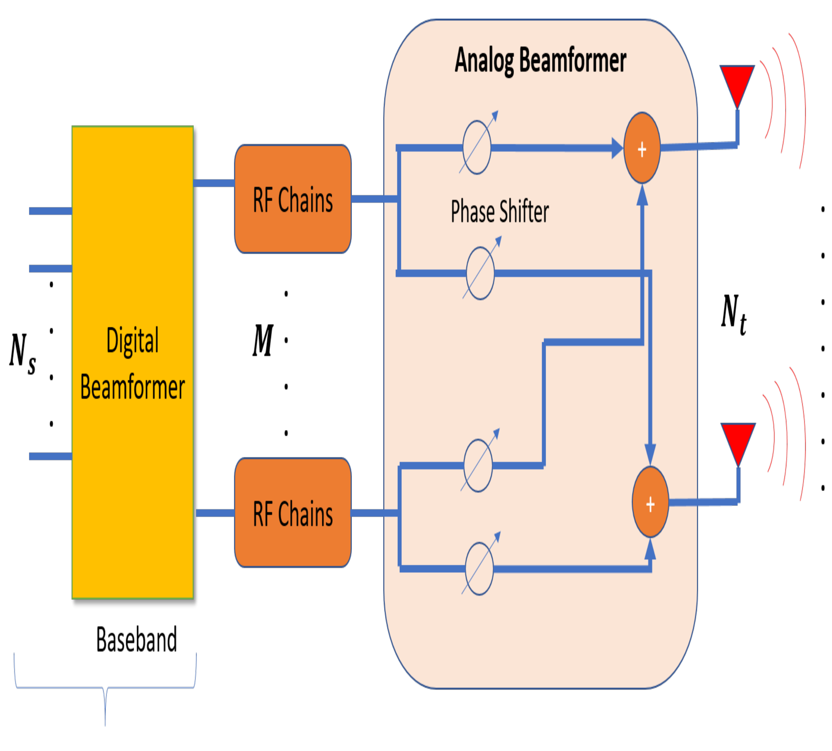
Motivation & State of the Art (SotA)

Why Hybrid Beamforming (HBF) ?

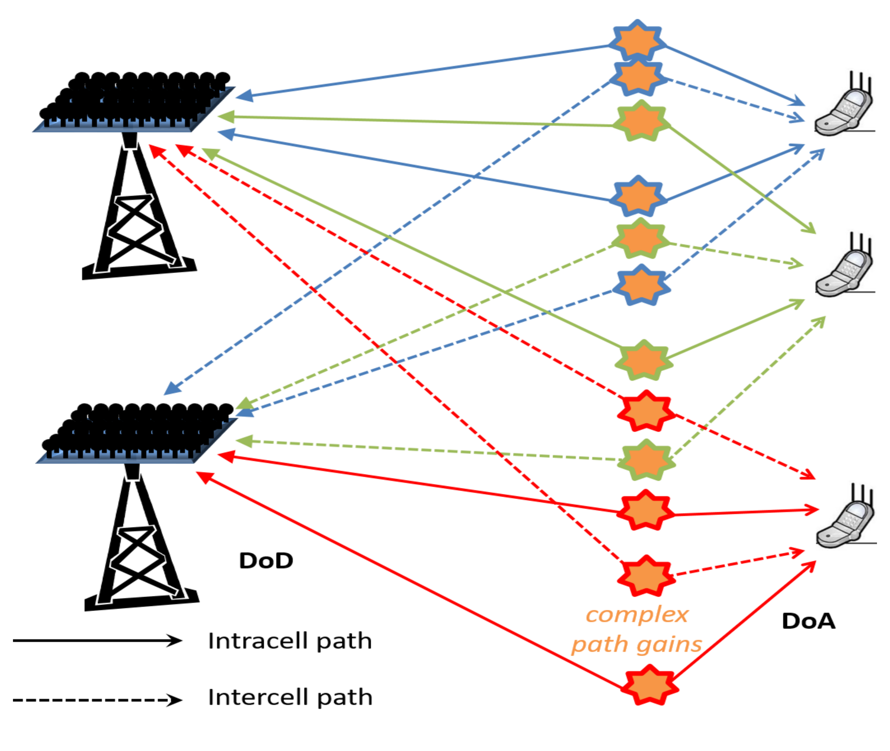
- Massive MIMO is largely beneficial for a millimeter wave communication system for 5G cellular.
- Conventional MIMO uses one RF chain for each antenna.
- Not feasible for a Massive MIMO system.
- **Promising Solution:** Using hybrid transceivers, proposed as a low complexity/high throughput enabling technology for 5G cellular communication.

SotA & Our SPAWC Contribution	SU/MU	Digital	Analog	Cell
Zhang:TSP05	SU	MRT	All Digital	Single
Ayach:TWC14,Alkh:JSTSP14,Rusu:ICC15	SU	LS	OMP	Single
SohrabiYu:JSTSP16	MU	ZF	Power Opt.	Single
Bogale:TWC16	MU	QR Dec.	QR	Single
ZhuHuang:JSAC17	MU	MMSE	ICI	Multi
Our Contribution (Global Opt. Design)	MU	WSR	WSR(DA)	Multi

System Architecture



Fully Connected HBF Architecture



Multi-Cell MU System

Multi-Cell Multi-User MIMO System Model

$$\mathbf{y}_k = \underbrace{\mathbf{H}_{k,b_k}\mathbf{V}^{b_k}\mathbf{G}_k\mathbf{s}_k}_{\text{signal}} + \underbrace{\sum_{i\neq k, b_i=b_k}\mathbf{H}_{k,b_k}\mathbf{V}^{b_k}\mathbf{G}_i\mathbf{s}_i}_{\text{intracell interf.}} + \underbrace{\sum_{c\neq b_k}\sum_{i:b_i=c}\mathbf{H}_{k,c}\mathbf{V}^{b_i}\mathbf{G}_i\mathbf{s}_i}_{\text{intercell interf.}} + \mathbf{v}_k$$

Problem Formulation: Weighted Sum Rate (WSR) Maximization

$$[\mathbf{V}\ \ \mathbf{G}] = \arg\max_{\mathbf{V},\mathbf{G}} WSR(\mathbf{G},\mathbf{V}) = \arg\max_{\mathbf{V},\mathbf{G}} \sum_{k=1}^{K} u_k \ln\det(\mathbf{R}_{\overline{k}}^{-1}\mathbf{R}_k),$$

$$\text{tr}(\sum_{i:b_i=c}^{K}\mathbf{Q}_i) \leq P_c, \text{ where, } \mathbf{Q}_i = \mathbf{V}^c\mathbf{G}_i\mathbf{G}_i^H\mathbf{V}^{c\,H}, \mathbf{G}_i = \mathbf{G}'_i\mathbf{P}_i^{1/2},$$

$|\mathbf{V}^c_{m,n}| = 1 \Rightarrow \mathbf{V}^c_{m,n} = e^{j\,\theta^c_{m,n}}$, $\mathbf{R}_k$ is Signal + Interference + Noise Power.

Alternating Minorizer Approach

- $WSR(\mathbf{G},\mathbf{V}) = u_k \ln\det(\mathbf{R}_{\overline{k}}^{-1}\mathbf{R}_k) + WSR_{\overline{k}}$,
- Difference of Convex functions programming [KimGiannakis:TIT11] introduces first order Taylor series expansion in $\mathbf{Q}_k$ around $\widehat{\mathbf{Q}}$.
- Constitutes a lower bound, hence a minorizer.
- Augmenting the WSR cost function with the Tx power constraints, we get the Lagrangian which gets maximized alternatingly [Stoica:SPMag04] between digital and analog BF,

$$\mathcal{L}(\mathbf{V},\mathbf{G},\mathbf{\Lambda}) = \sum_{c=1}^{C}\lambda_c P^c + \sum_{k=1}^{K}(u_k \ln\det(\mathbf{I}+\mathbf{Q}_k\widehat{\mathbf{B}}_k) - \text{tr}\{\mathbf{Q}_k(\widehat{\mathbf{A}}_k + \lambda_{b_k}\mathbf{I})\})$$

- Min-max problem: $\min_{\Lambda_c}\max_{\mathbf{G}',\mathbf{P},\mathbf{V}} \mathcal{L}(\mathbf{G}',\mathbf{P},\mathbf{V}^c,\mathbf{\Lambda}_c)$, leads to **generalized eigen vector solution**,

$$\mathbf{G}'_k = \mathbf{V}_{1:d_k}\left(\mathbf{V}^{b_k\,H}\widehat{\mathbf{B}}_k\mathbf{V}^{b_k}, \mathbf{V}^{b_k\,H}\left(\widehat{\mathbf{A}}_k + \lambda_{b_k}\mathbf{I}\right)\mathbf{V}^{b_k}\right),$$

Interference Leakage Aware Water-Filling (ILA-WF) for $\mathbf{P}_k$:

$\mathbf{P}_k = \left(u_k\mathbf{T}_k(\lambda_{b_k})^{-1} - \mathbf{S}_k^{-1}\right)^+$, $\mathbf{S}_k = \mathbf{G}'^H_k\mathbf{V}^{b_k H}\widehat{\mathbf{B}}_k\mathbf{V}^{b_k}\mathbf{G}'_k$,

$\mathbf{T}_k(\lambda_{b_k}) = \mathbf{G}'^H_k\mathbf{V}^{b_k H}(\widehat{\mathbf{A}}_k + \lambda_{b_k}\mathbf{I})\mathbf{V}^{b_k}\mathbf{G}'_k$, Bisection for λ_c,

Unconstrained $\mathbf{V}^c = unvec(\mathbf{V}_{max}(\mathbf{B}_c,\mathbf{A}_c))$ with

$$\mathbf{B}_c = \sum_{k:b_k=c}\left((\mathbf{G}_k\mathbf{G}_k^H\mathbf{W}_k)^T \otimes \widehat{\mathbf{B}}_k\right), \mathbf{A}_c = \sum_{k:b_k=c}\left((\mathbf{G}_k\mathbf{G}_k^H)^T \otimes (\widehat{\mathbf{A}}_k + \lambda_c\mathbf{I})\right).$$

Phasor Optimization: Analog BF

- Given $\mathbf{G}'_k$ and $\mathbf{P}_k$, $\forall k$, the analog beamformers $\mathbf{V}^c$ can be found by performing alternating optimization elementwise.
- $WSR(\theta^c_{m,n}) = e^{j\theta^c_{m,n}}a^c_{m,n} + e^{-j\theta^c_{m,n}}b^c_{m,n} +$ "terms not containing $\theta^c_{m,n}$", $a^c_{m,n}$ is a function of the channel, $\mathbf{G}$ and other auxiliary parameters.
- The alternating minimization gives the phasor value as $\theta^c_{m,n} = \pi - \frac{1}{2}\angle\frac{a^c_{m,n}}{b^c_{m,n}}$.

Hybrid BF Design via Alternating Minorizer

Algorithm 1 - Given: $P_c, \mathbf{H}_{k,c}, u_k\ \forall k, c$.

Initialization: $(\mathbf{V}^c)^{(0)} = e^{j\angle\mathbf{V}_{1:M^c}(\sum_{k:b_k=c}\mathbf{\Theta}^c_k, \sum_{i:b_i\neq c}\mathbf{\Theta}^c_i)}$, $\mathbf{G}^{(0)}_k$ are ZF precoders

Iteration (j) :

1. Compute $\widehat{\mathbf{B}}_k, \widehat{\mathbf{A}}_k$, update $\mathbf{G}'^{(j)}_k$, $\forall k$. Then ILA-WF for $\mathbf{P}^{(j)}_k$, $\lambda^{(j)}_c$, $\forall k, c$.
2. Update $(\mathbf{V}^c_{p,q})^{(j)}$, $\forall c$, $\forall (p,q)$ phasor constrained or unconstrained.
3. Check for convergence of the WSR: if not go to step 1).

Algorithm Convergence:

- The ingredients required are minorization, alternating or cyclic optimization (also called block coordinate descent) [Stoica:SPMag04], Lagrange dual function, saddle-point interpretation and KKT conditions [BoydVandenberghe:CambridgeUPress04].

Hybrid Beamformer Capabilities

Theorem:

For multi-cell MU-MIMO system with $M \geq N_p$ (total no of multi-paths) and phasor antenna responses, to achieve opt. all-digital precoding performance, the analog BF can be chosen as the Tx side concatenated antenna array responses.

Deterministic Annealing for Analog BF ($M < N_p$)

Let $\mathbf{V}^c = |\mathbf{V}^c| \odot \boldsymbol{\theta}^c\,(\mathbf{V}^c_{i,j} = |\mathbf{V}^c_{i,j}|e^{j\theta^c_{i,j}})$. Let the unconstrained $\mathbf{V}^c$ design (joint $\mathbf{V}^c$ and $\mathbf{G}_k$) using Algorithm 1 converge first.

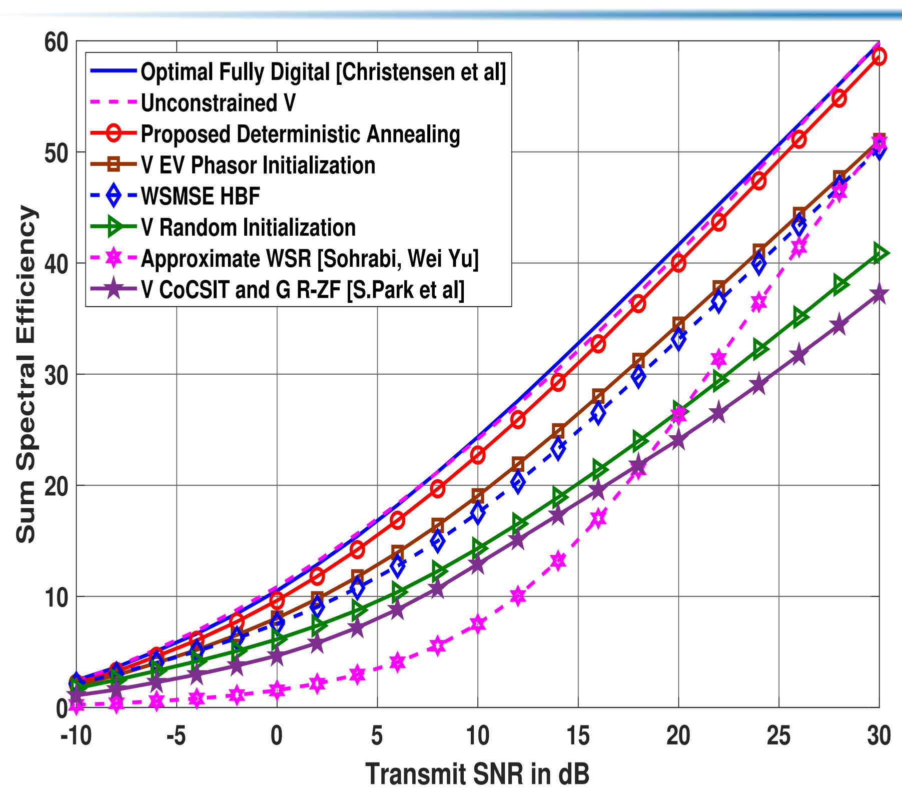
1. Scale $\forall(i,j) : |V^c_{i,j}| \leftarrow |V^c_{i,j}|^b$.
2. Reoptimize all $\theta^c_{i,j}$ and all digital BFs using Algorithm 1.
3. Update stream powers and Lagrange multipliers.
4. Go to 1) for a number of iterations.
5. Finally redo 2)-3) a last time with all $|\mathbf{V}^c_{i,j}| = 1$ in 1).

Homotopy Method:

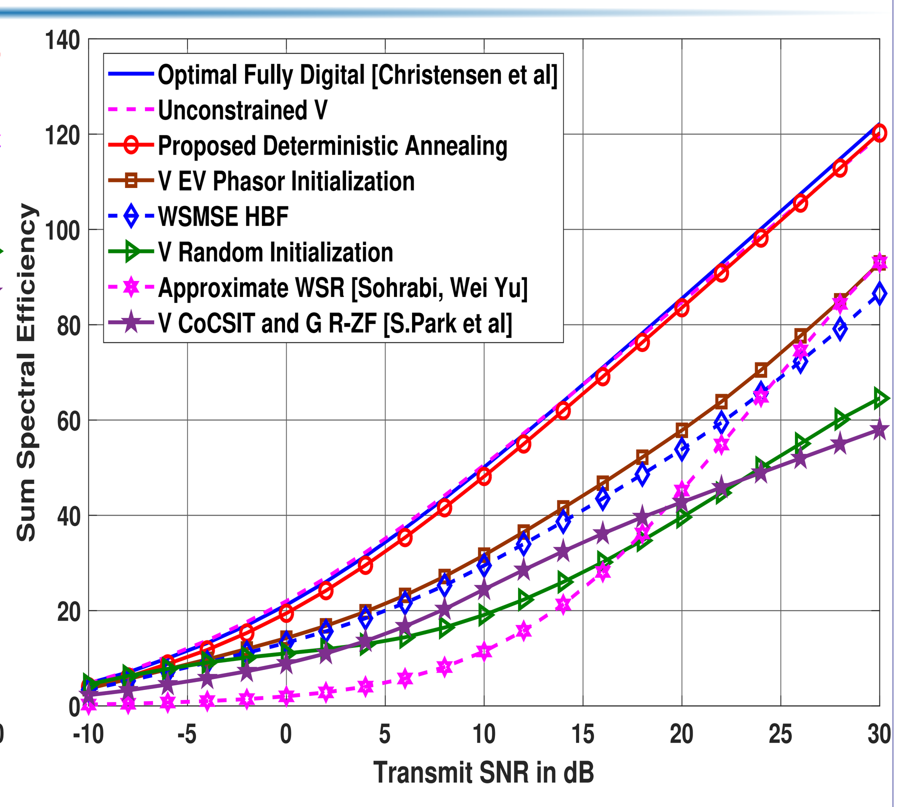
$$(\mathbf{\Lambda}^{(n)}, \boldsymbol{\theta}^{(n)}, \mathbf{G}'^{(n)}, \mathbf{P}^{(n)}) = \arg\min_{\mathbf{\Lambda}}\max_{\boldsymbol{\theta},\mathbf{G}',\mathbf{P}} \mathcal{L}((|\mathbf{V}|)^{t_n}, \boldsymbol{\theta}, \mathbf{G}', \mathbf{P}, \mathbf{\Lambda}),\ t_n = b^n, b < 1,$$

initialized by $(\mathbf{\Lambda}^{(n-1)}, \boldsymbol{\theta}^{(n-1)}, \mathbf{G}'^{(n-1)}, \mathbf{P}^{(n-1)})$.

Spectral Efficiency Results



$N_t = 32, M = 16, K = 8, C = 1, L = 4.$



$N_t = 64, M = 16, K = 16, C = 1, L = 2.$

Conclusion and Future Work

- Optimization of the WSR using alternating minorization for HBF design, with provable convergence guarantees.
- Deterministic annealing based solution further narrows the gap to the fully digital solution.
- Hybrid beamforming for partially connected structures, under realistic per-antenna or per-RF power constraints, partial CSIT designs.
- MmWave Channel estimation in hybrid beamforming massive MIMO.

References

[1] F. Negro et al, "Deterministic annealing design and analysis of the noisy MIMO interference channel", *ITA*, 2011.