Enhancing Favorable Propagation in Cell-Free Massive MIMO Through Spatial User Grouping

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1 Introduction

We investigate the impact of the network configuration on the level of favorable propagation for a cell-free (CF) Massive MIMO network. Leveraging users' spatial diversity, we formulate a user grouping and scheduling optimization problem. The formulated optimization problem is NP-hard. We design an efficient randomization algorithm based on semidefinite relaxation method to efficiently find a sub-optimal solution.

2 System Model

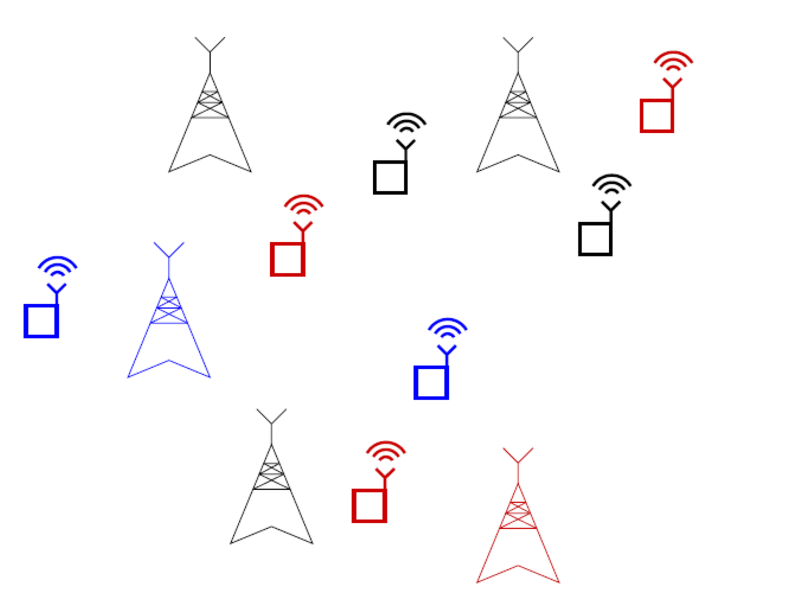

Figure 2: Cell-free Massive MIMO

- M single antenna APs serve simultaneously K single omni-directional antenna users ($K << M$)
- The m-th AP performs minimum mean-square error (MMSE) channel estimation

$$\hat{g}_{mk} = \frac{\sqrt{\rho_p}\beta_{mk}}{\rho_p\beta_{mk}+1}\left(\sqrt{\rho_p}g_{mk} + \boldsymbol{n}_{m,p}\mathbf{q}_k^\dagger\right),\ k = 1,\cdots,\tau$$

where,

- g_{mk}: channel coefficient between the k-th user and the m-th AP.
- $\mathbf{q}_k$ training sequence of the kth user.
- β_{mk}: large scale fading coefficients.
- ρ_p: transmit power during training phase.
- $\boldsymbol{n}_{m,p}$: AWGN vector at the m-th AP.
- τ: uplink training duration with $\tau < T_c$ (coherence interval).

3 Which users can be active simultaneously?

Favorable propagation: mutual orthogonality between users' vector wireless channel

$$\boldsymbol{g}_k^\dagger\boldsymbol{g}_j = \begin{cases} 0 & \text{if } k \neq j, \\ \|\boldsymbol{g}_k\|^2 \neq 0 & \text{otherwise,} \end{cases}$$

Asymptotically,

$$\frac{\boldsymbol{g}_k^\dagger\boldsymbol{g}_j}{M} \longrightarrow 0,\ M \longrightarrow \infty \quad \text{for} \quad k \neq j,$$

which is equivalent to

$$\frac{\sum_{m=1}^{M}\sqrt{\beta_{m,k}}\sqrt{\beta_{m,j}}h_{m,k}^{*}h_{m,j}}{M} \longrightarrow 0 \quad \text{for} \quad k \neq j,$$

where

- $h_{mk} \sim \mathcal{CN}(0,1)$: small-scale fading coefficients.

Alternative: consider the complementary CDF of the inner product of two given users' channel

$$P_\theta = \Pr\left\{\frac{\boldsymbol{g}_k^\dagger\boldsymbol{g}_j}{M} \geq \theta\right\}$$

Objective: making P_θ, $\forall\theta \geq 0$ very small to achieve NEAR ORTHOGONALITY between users' channel vectors, and therefore FAVORABLE PROPAGATION .

Invoking Chebychev's inequality, P_θ can be lower-bounded by

$$P_\theta = \Pr\left\{\frac{\boldsymbol{g}_k^\dagger\boldsymbol{g}_j}{M} \geq \theta\right\} \leq \frac{1}{1+\frac{M^2\theta^2}{\sum_{m=1}^{M}\beta_{m,k}\beta_{m,j}}},$$

Challenge: design a scheme to minimize P_θ.

4 Graphical Modeling and proposed solution

4.1 Scheduling design

1. Step 1 : Construct a spatial correlation graph $\mathcal{G}(\mathcal{V},\mathcal{E})$ that captures the level of favorable propagation for a set of users which are active simultaneously.
   - $\mathcal{V}$: set of vertices stands for the users in the coverage area.
   - Each edge $e_{k,j} \in \mathcal{E}$ is associated with a weight $\omega_{k,j} \triangleq \sum_{m=1}^{M}\beta_{mk}\beta_{mj}$, directly related to the spatial correlation between two users' channel.
2. Step 2 : Group active users such that the spatial correlation between their channels is minimized.

4.2 Problem formulation and algorithm design

Define the following variable

$$x_{k,c} = \begin{cases} 1 & \text{if user } k \text{ is allocated to the } c\text{-th group} \\ 0 & \text{otherwise} \end{cases}$$

The user grouping problem is formulated as

$$\begin{aligned} \max_{x_{k,c}\in\{0,1\},\forall c,\forall k} \quad & \sum_{c=1}^{C}\sum_{k\in\mathcal{V}}\sum_{j\in\mathcal{V},j\neq k} w_{k,j}\left(1-x_{k,c}\right)x_{j,c} \\ \text{s.t.} \quad & \sum_{c=1}^{C} x_{k,c} \leq \alpha, \forall k \in \mathcal{V}, \\ & \sum_{k\in\mathcal{V}} x_{k,c} \leq \tau, \forall c = 1,\ldots,C, \end{aligned} \tag{1}$$

where,

- C: total number of groups.
- α: maximum number of groups to which a user can belong at the same time.

Lemma 1: Computational tractability

Problem (1) is NP-hard in general.

GOAL: Design a low-complexity algorithm to sub-optimally solve problem (1).

Define following variables and changes of variables

$$\mathbf{x_c} \triangleq \left(x_{1,c}\cdots,x_{K,c}\right)^\top,\ \mathbf{y_c} \triangleq 2\mathbf{x_c} - \mathbf{1}_K$$

$$\mathbf{W} \triangleq \begin{pmatrix} 0 & w_{2,1} & \cdots & w_{K,1} \\ w_{1,2} & 0 & \cdots & w_{K,2} \\ \vdots & \vdots & \ddots & \vdots \\ w_{1,K} & w_{2,K} & \cdots & 0 \end{pmatrix}$$

where,

- $\mathbf{1}_K$: entry one column vector.

Combining SEMIDEFINITE RELAXATION method with the SCHUR COMPLEMENT, problem (1) can be relaxed as

$$\begin{aligned} \max_{\mathrm{Y}_c\succeq 0,\forall c} \quad & \frac{1}{4}\sum_{c=1}^{C}\left(\varsigma - \mathrm{tr}\left(\mathbf{W}\mathrm{Y}_c\right)\right) \\ \text{s.t.} \quad & \sum_{c=1}^{C}\mathbf{y}_c \leq \bar{\alpha} \\ & \mathrm{tr}\left(\mathrm{diag}\left(\mathbf{y}_c\right)\right) \leq \bar{\tau},\ \forall c \\ & \mathrm{diag}\left(\mathrm{Y}_c\right) = \mathbf{1}_K \\ & \begin{pmatrix} \mathrm{Y}_c & \mathbf{y}_c \\ \mathbf{y}_c^\top & 1 \end{pmatrix} \succeq 0,\ \forall c \end{aligned} \tag{2}$$

Problem (2) is a standard convex optimization problem and can be efficiently solved using CVX.

We develop a randomized procedure, in the vein of Gaussian randomization, to convert the optimal solution of (2) into a feasible solution to problem (1).

Algorithm 1 A randomized algorithm to solve problem (1)

1: **input** an optimal solution $\mathrm{Y}_c^\star$, $\forall c$ to problem (2).
2: Generate $\boldsymbol{\xi}_c \sim \mathcal{N}(\mathbf{0},\mathrm{Y}_c^\star)$, $\forall c$;
3: Set $\bar{\boldsymbol{\xi}}_c = \boldsymbol{\xi}_c/\,\mathrm{tr}\left(\mathrm{diag}\left(\boldsymbol{\xi}_c\right)\right)$, $\forall c$;
4: Generate L vector samples $\bar{\mathbf{y}}_c^l$, $l = 1,\cdots,L$ feasible for problem (1) such that each entry $\bar{y}_{k,c}^l$, $k = 1,\cdots,C$ is drawn from the following distribution:

$$\bar{y}_{k,c}^l = \begin{cases} 1 & \text{with probability } (1+\bar{\xi}_{k,c})/2 \\ -1 & \text{with probability } (1-\bar{\xi}_{k,c})/2 \end{cases}$$

5: Compute $l^\star = \arg\max_{l=1,\cdots,L}\frac{1}{4}\sum_{c=1}^{C}\left(\varsigma - (\bar{\mathbf{y}}_c^l)^\top\mathbf{W}\bar{\mathbf{y}}_c^l\right)$;
6: **output** the solution $\bar{\mathbf{y}}_c = \bar{\mathbf{y}}_c^{l^\star}$, $\forall c$.

4.3 Bandwidth allocation problem

The bandwidth allocation problem is formulated as

$$\begin{aligned} \max_{0\leq\gamma_c\leq 1,\forall c} \quad & \sum_{c,k\in\Gamma(c)}\gamma_c\bar{R}_{k,c} \\ \text{s.t.} \quad & \overline{R}_k \leq \sum_{c=1}^{C}\gamma_c\bar{R}_{k,c}, \forall k \\ & \sum_{c=1}^{C}\gamma_c \leq 1 \end{aligned} \tag{3}$$

The rate of the kth user in group c is given by

$$\bar{R}_{k,c} = \mathcal{B}\frac{T_c - |\Gamma(c)|}{T_c}\log_2\left(1 + \frac{\rho_d\left(\sum_{m=1}^{M}\nu_{mk}\right)^2}{\rho_d\sum_{m=1}^{M}\sum_{\substack{k'\in\Gamma(c)\\k'\neq k}}^{\tau}\nu_{mk'}\beta_{mk} + 1}\right)$$

where,

- $\mathcal{B}$: bandwidth of the system.
- $\nu_{mk} \triangleq \frac{\rho_p\beta_{mk}^2}{1+\rho_p\beta_{mk}}$: variance of $\hat{g}_{mk}$.
- ρ_d: downlink transmit power.
- $\Gamma(c)$: set of users that belong to group c.

Problem (3): a convex linear optimization problem. Optimal solution: interior-point method.

5 Numerical Results

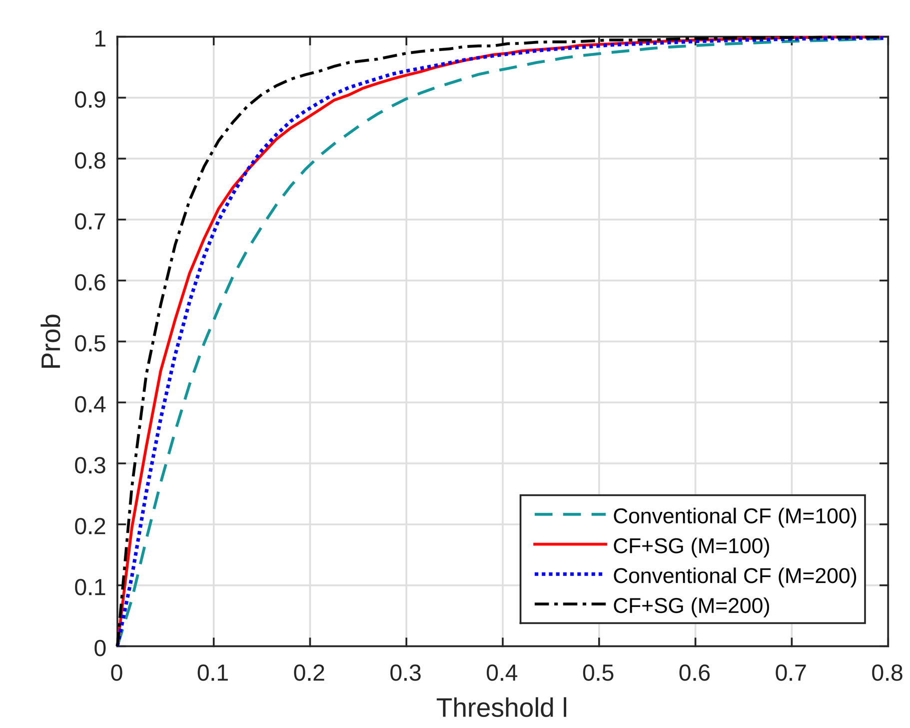

Figure 3: Comparison of CDFs of normalized large-scale fading correlation for $K = 20, \alpha = 6, C = 4$

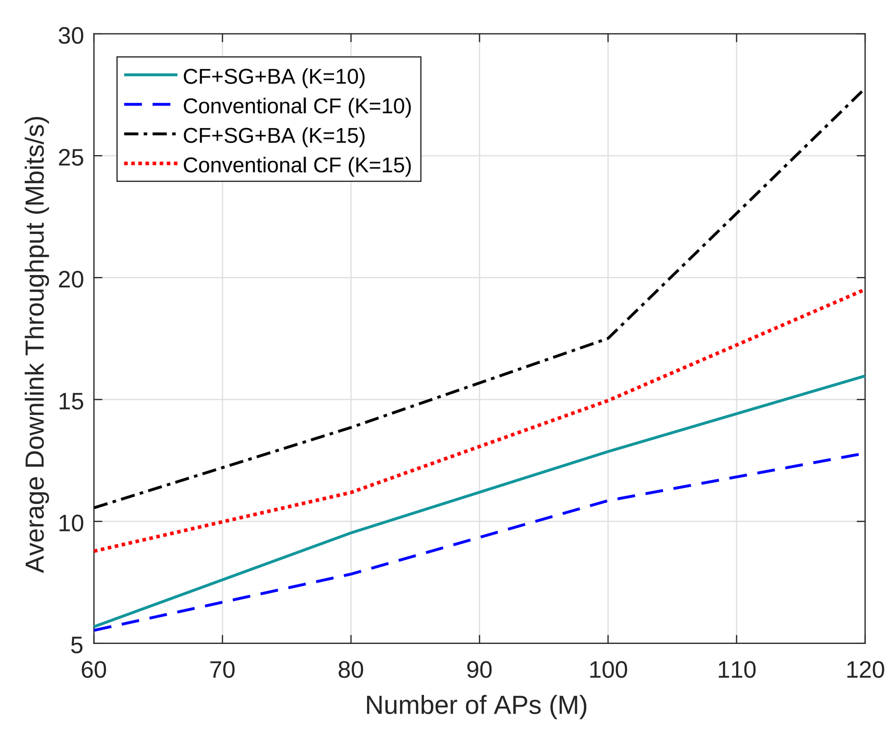

Figure 4: Average Downlink Throughput versus the number of APs and different K values ($\tau = K$)

6 Acknowledgements

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