Estimating message transmission time over heterogeneous chain of disrupted links

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June 2018

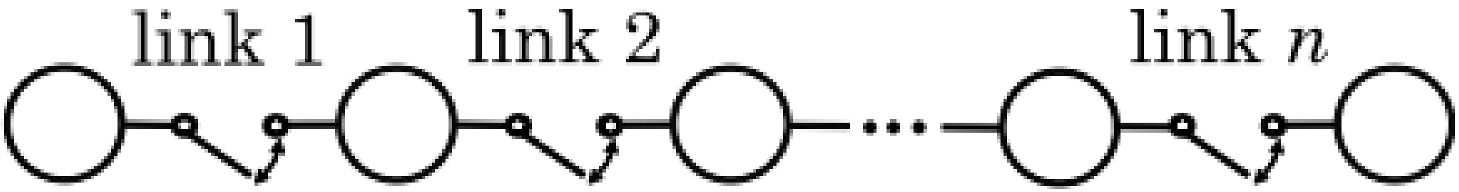
Outline

- What is the problem?
- What is our model?
- Approximation formulas
- Simulation results
- What next?

The problem

- A message has to sent over a chain of disrupted links
- One of the links is more disrupted than the others: **a bottleneck**
- → message should be fragmented (because of disruptions)
- → can we estimate the total transmission time based on characteristics of individual links ?

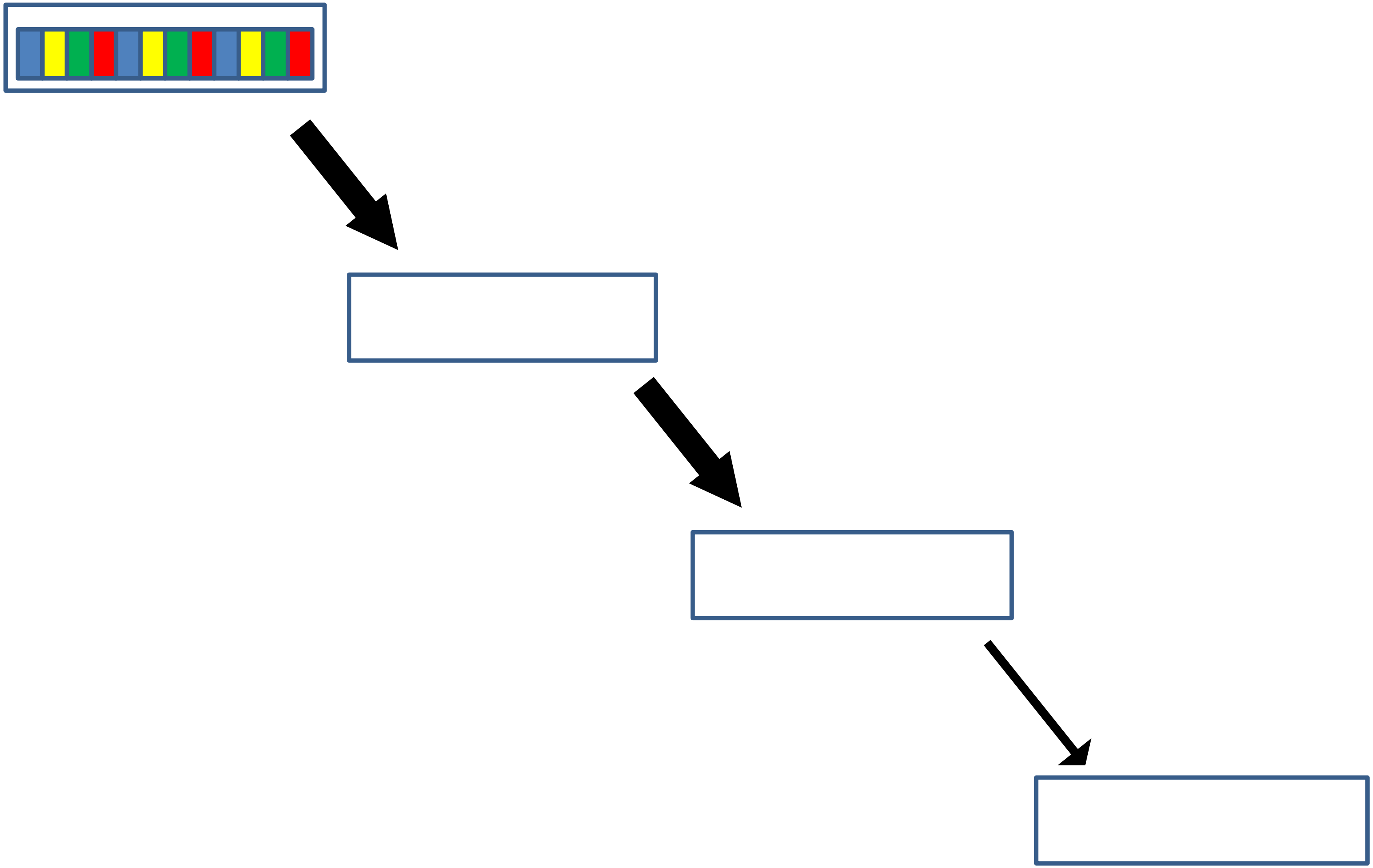
The model

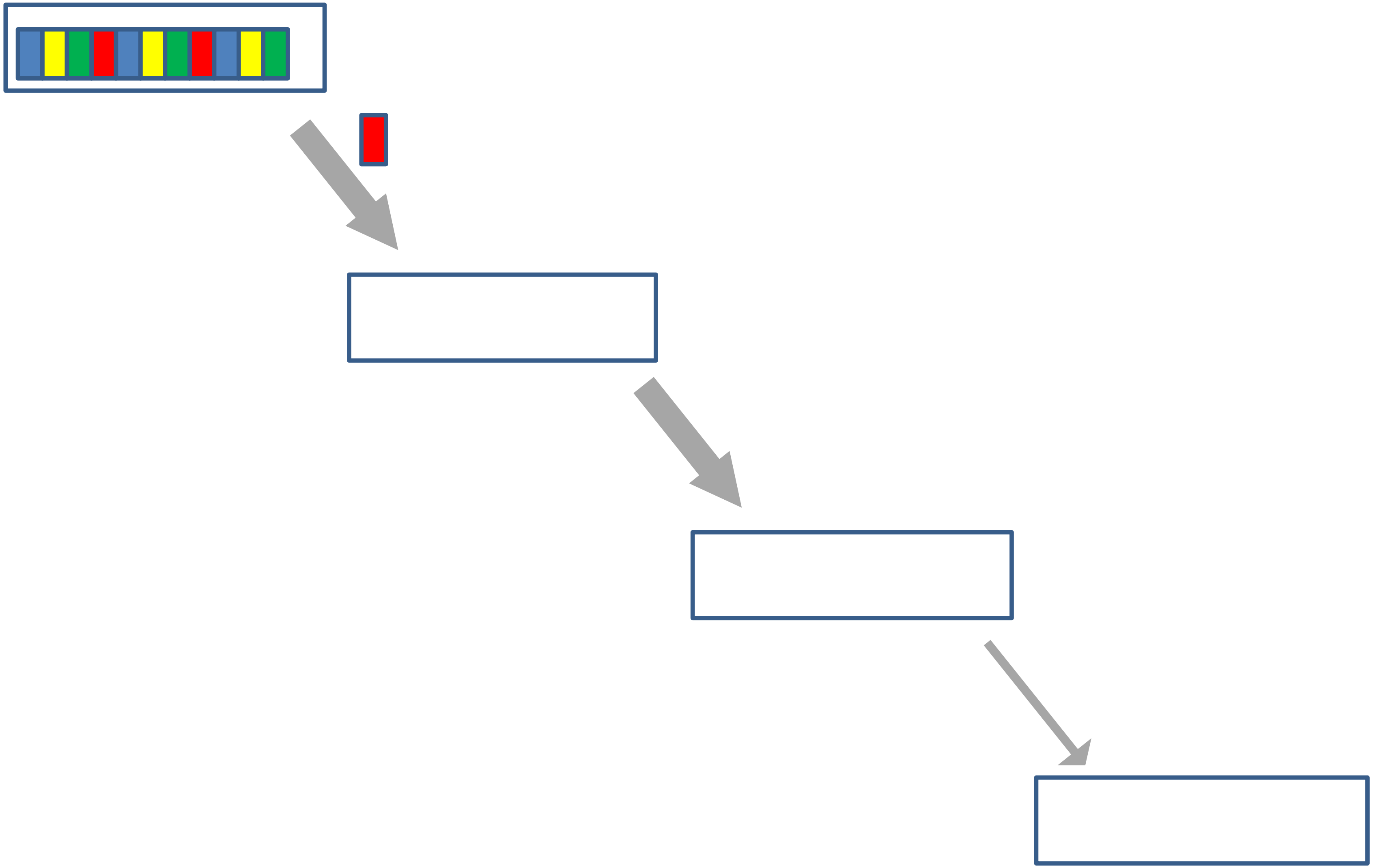


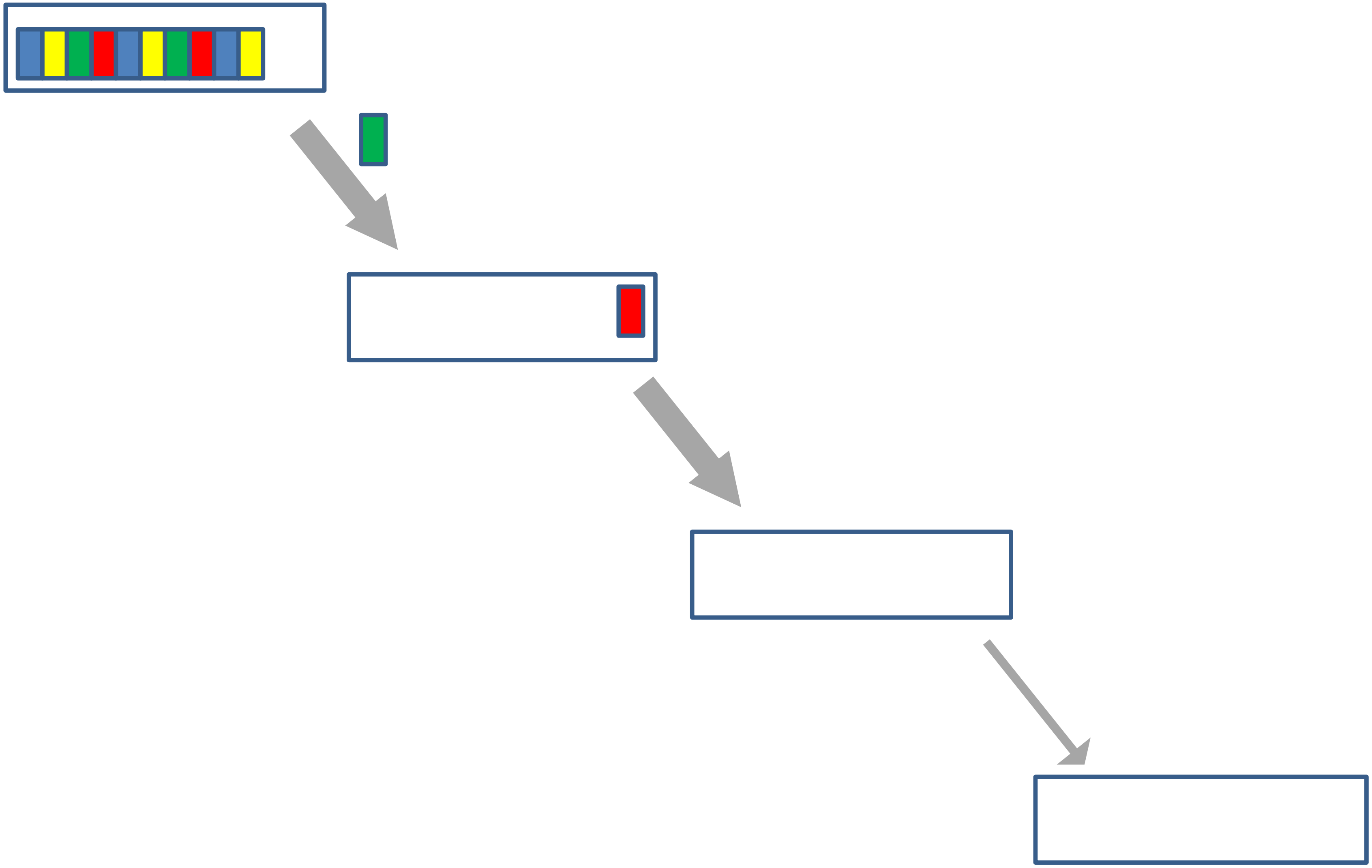
- The links alternate between **ON** state and **OFF** state
- Length of states are independent **random** variables
- For each link, **ON** (resp., OFF) state lengths follow same distributions
- Message is divided into **blocks** of size f.

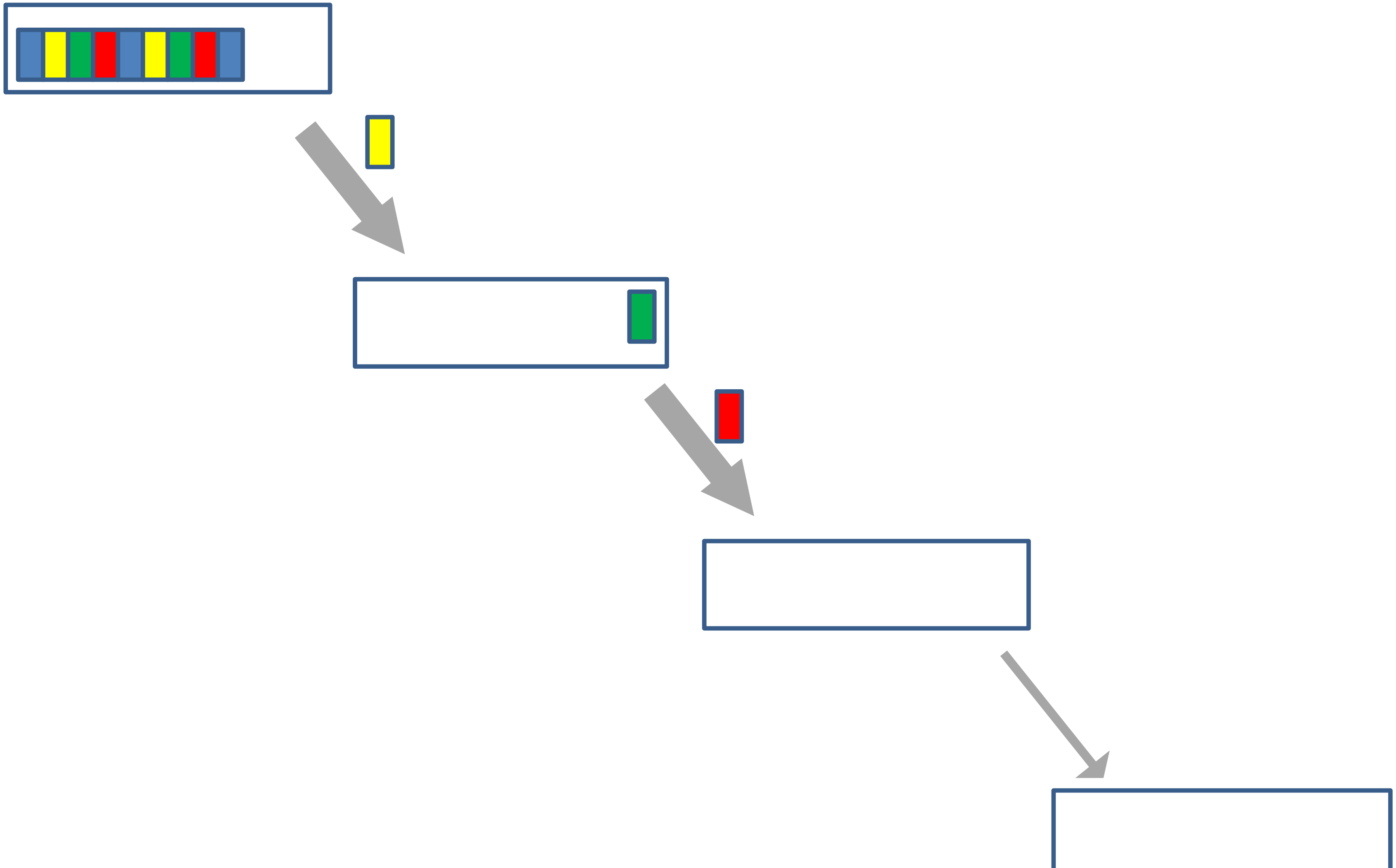
Example

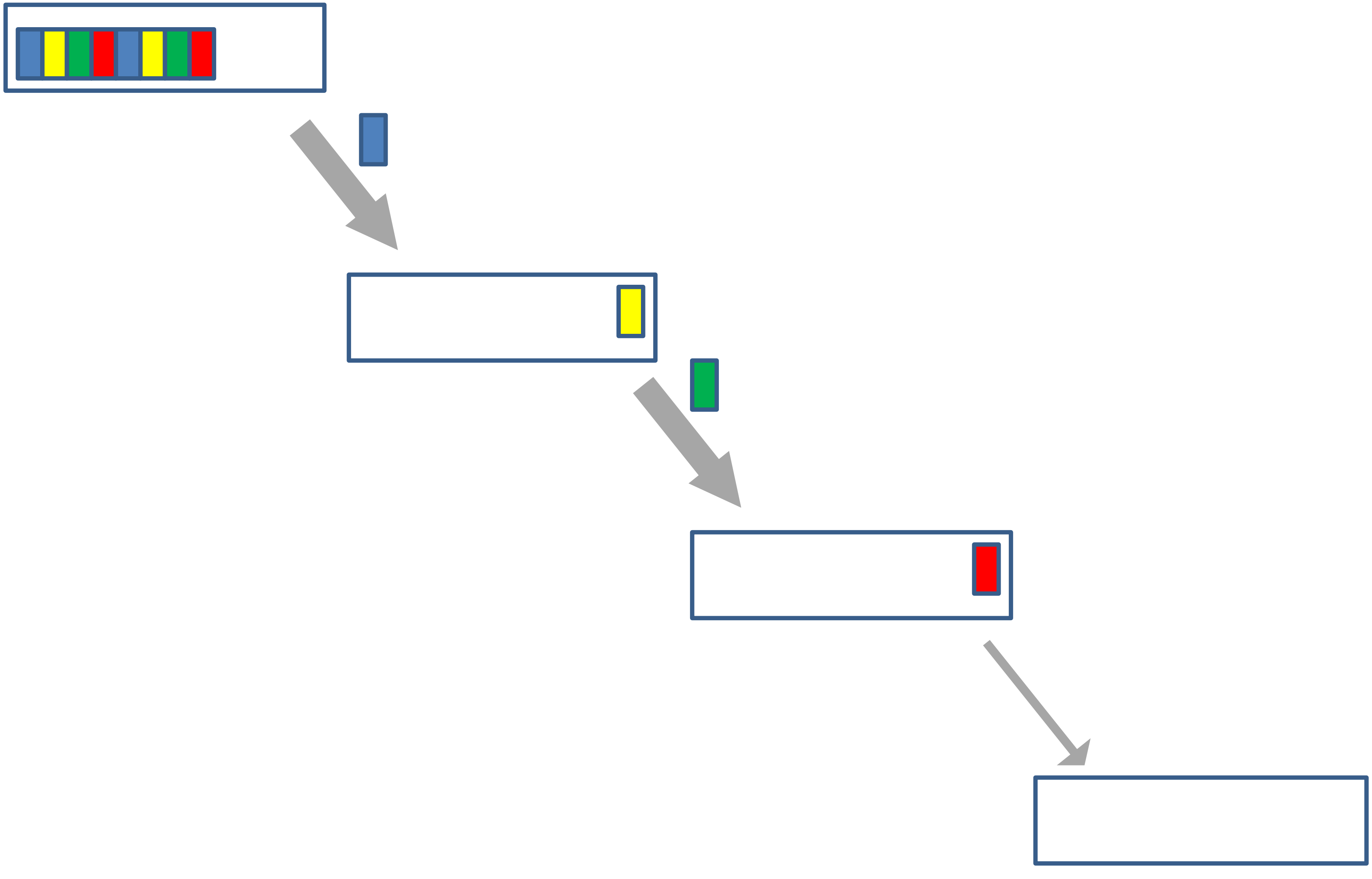
- 3 links
- Last one is the bottleneck
- Message divided into 12 blocks

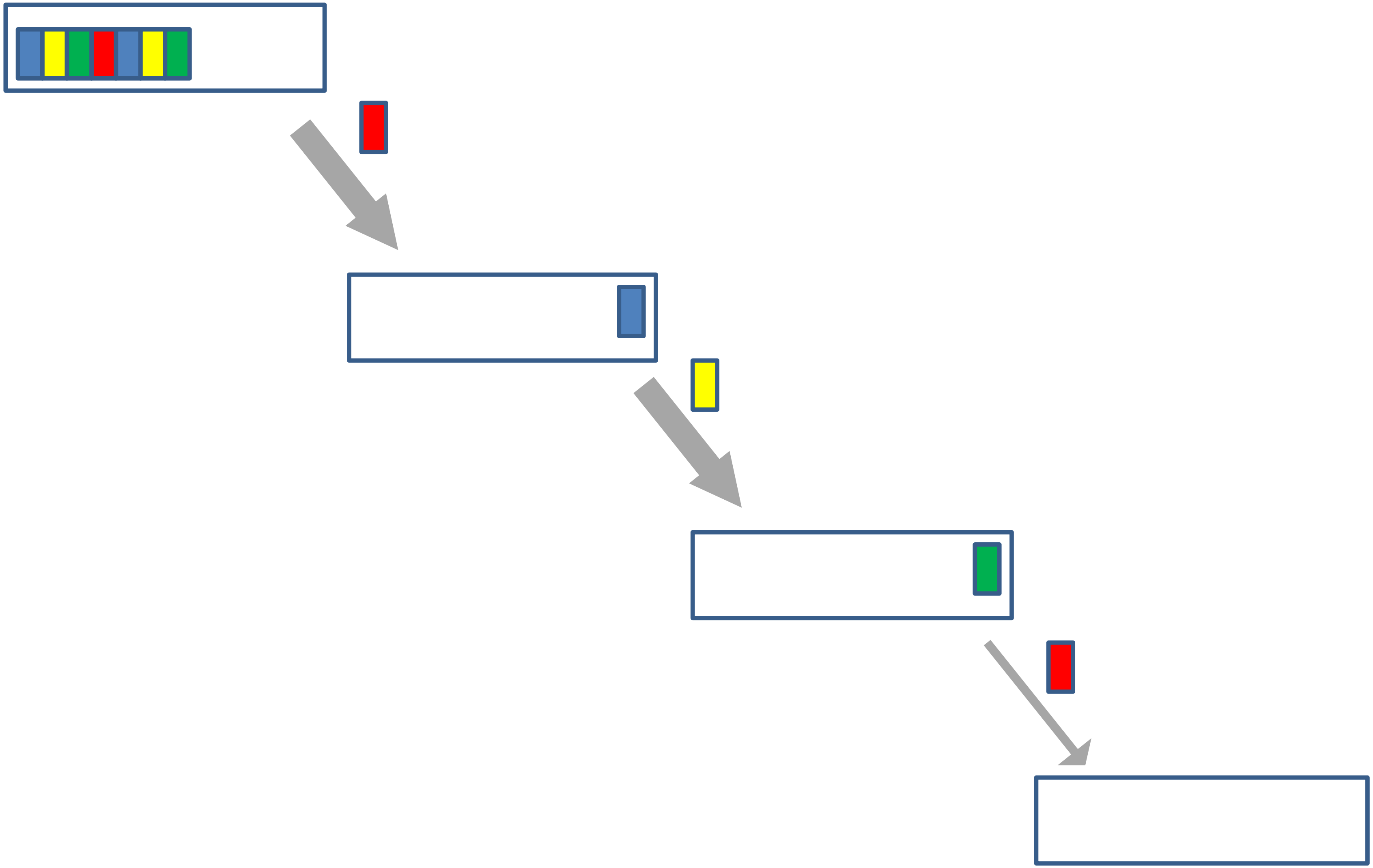


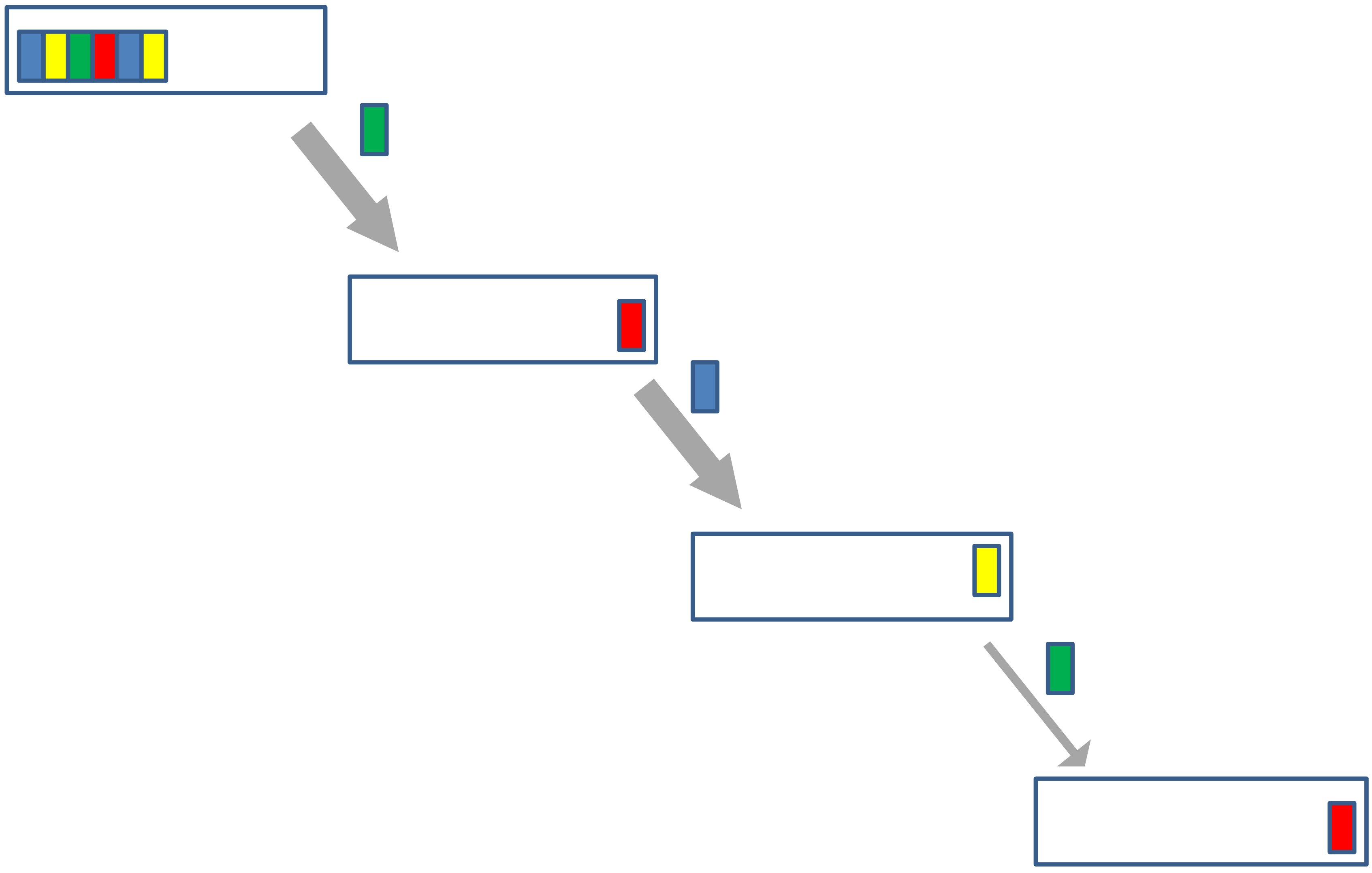


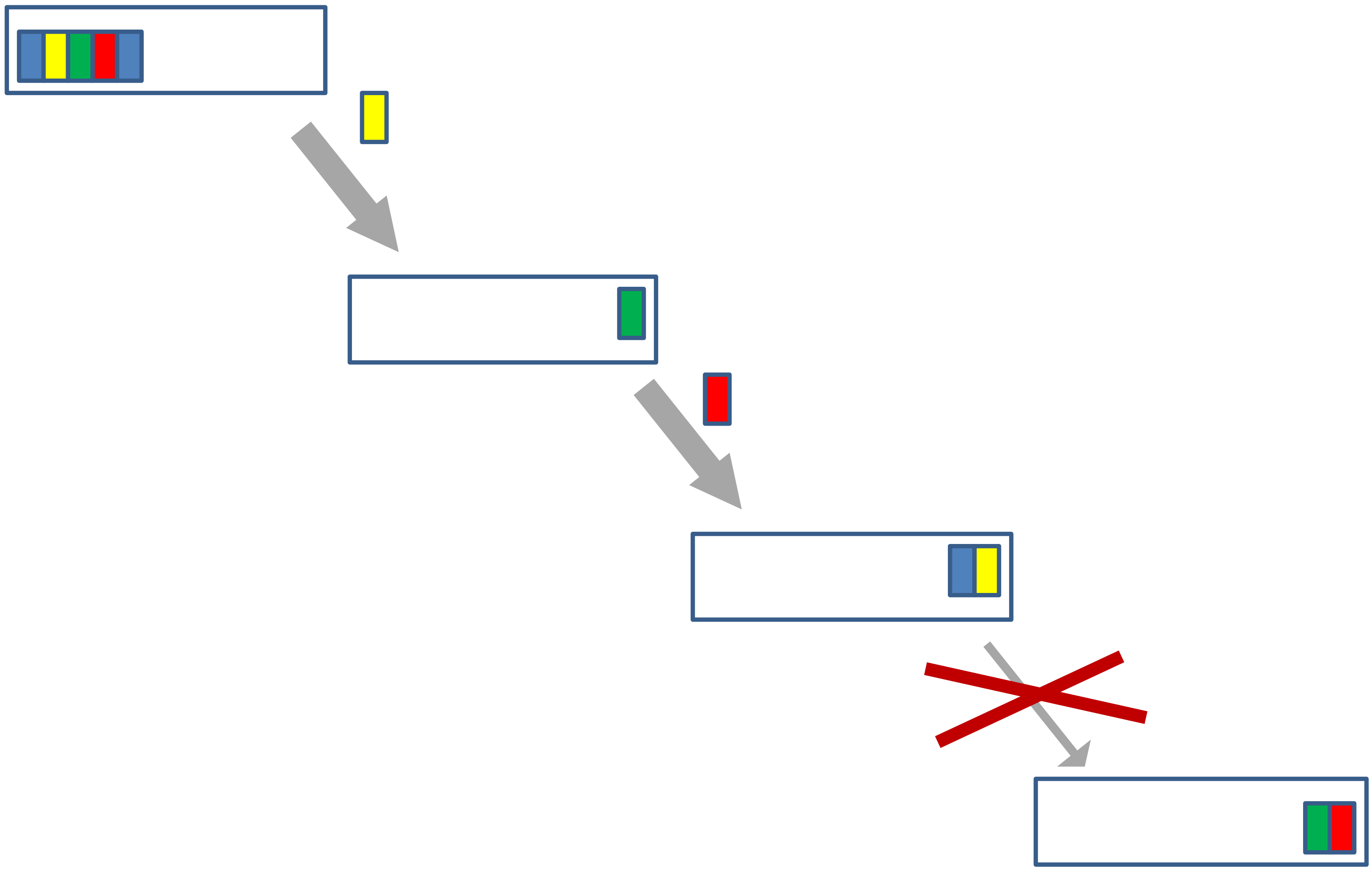


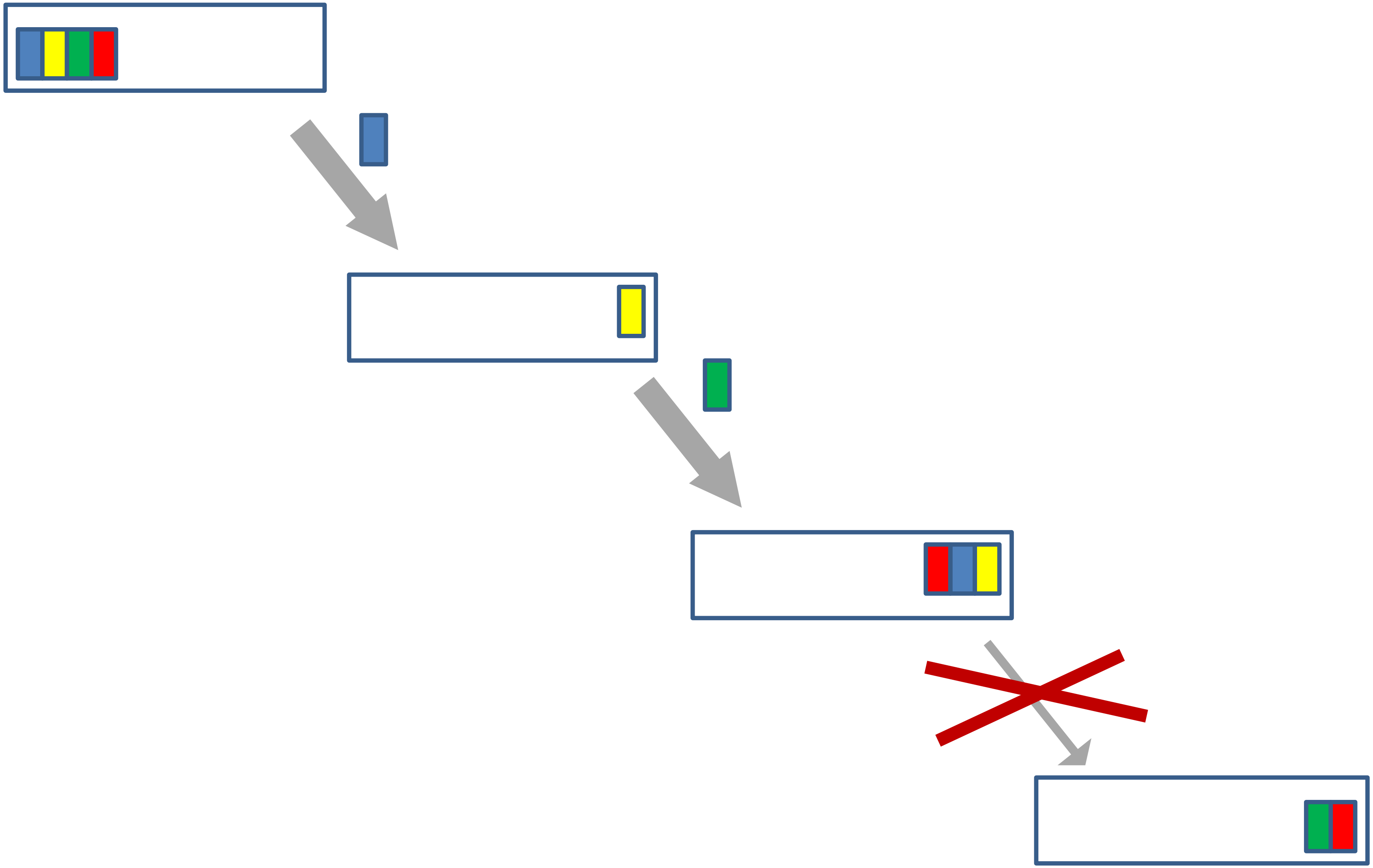


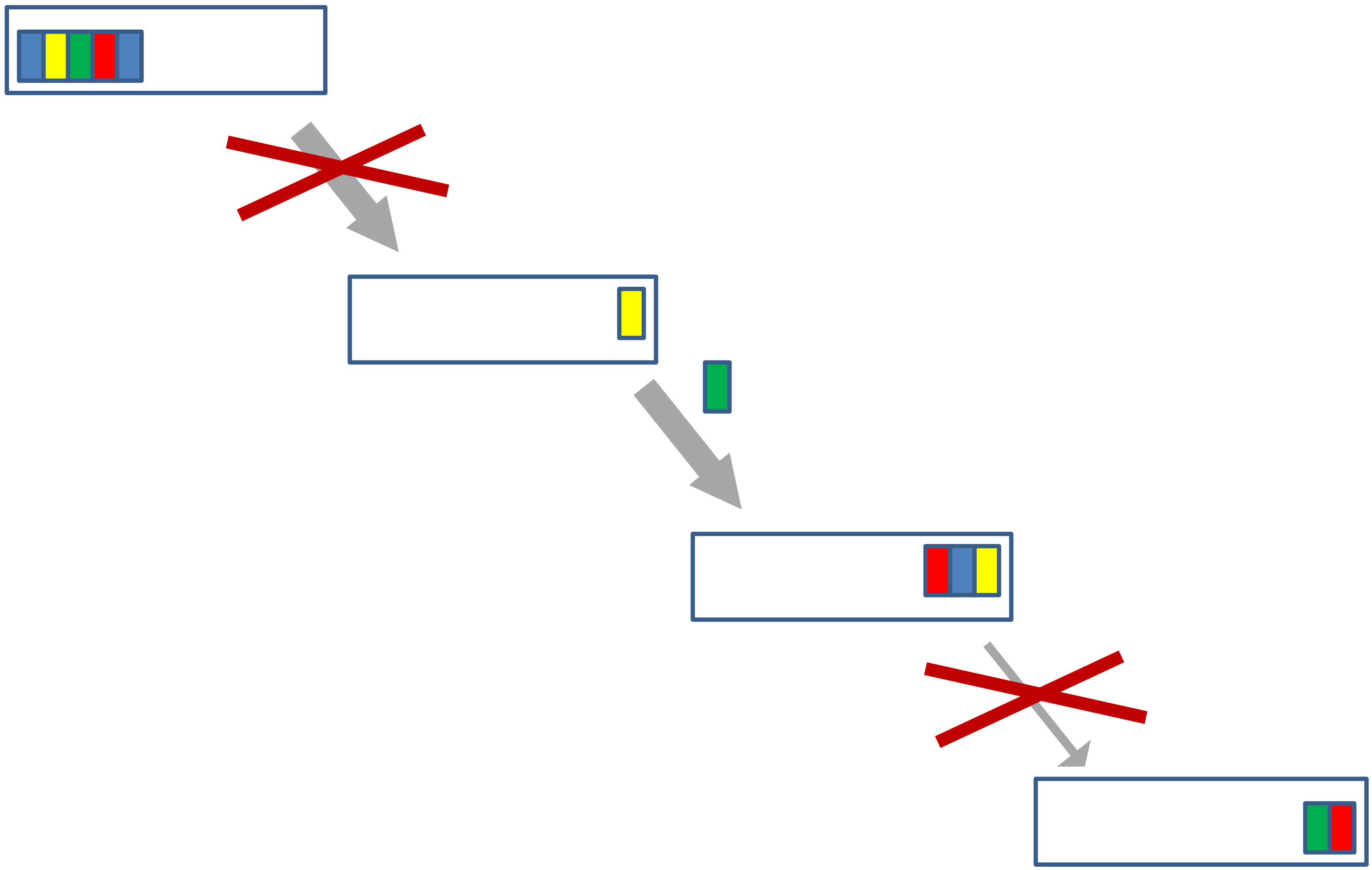


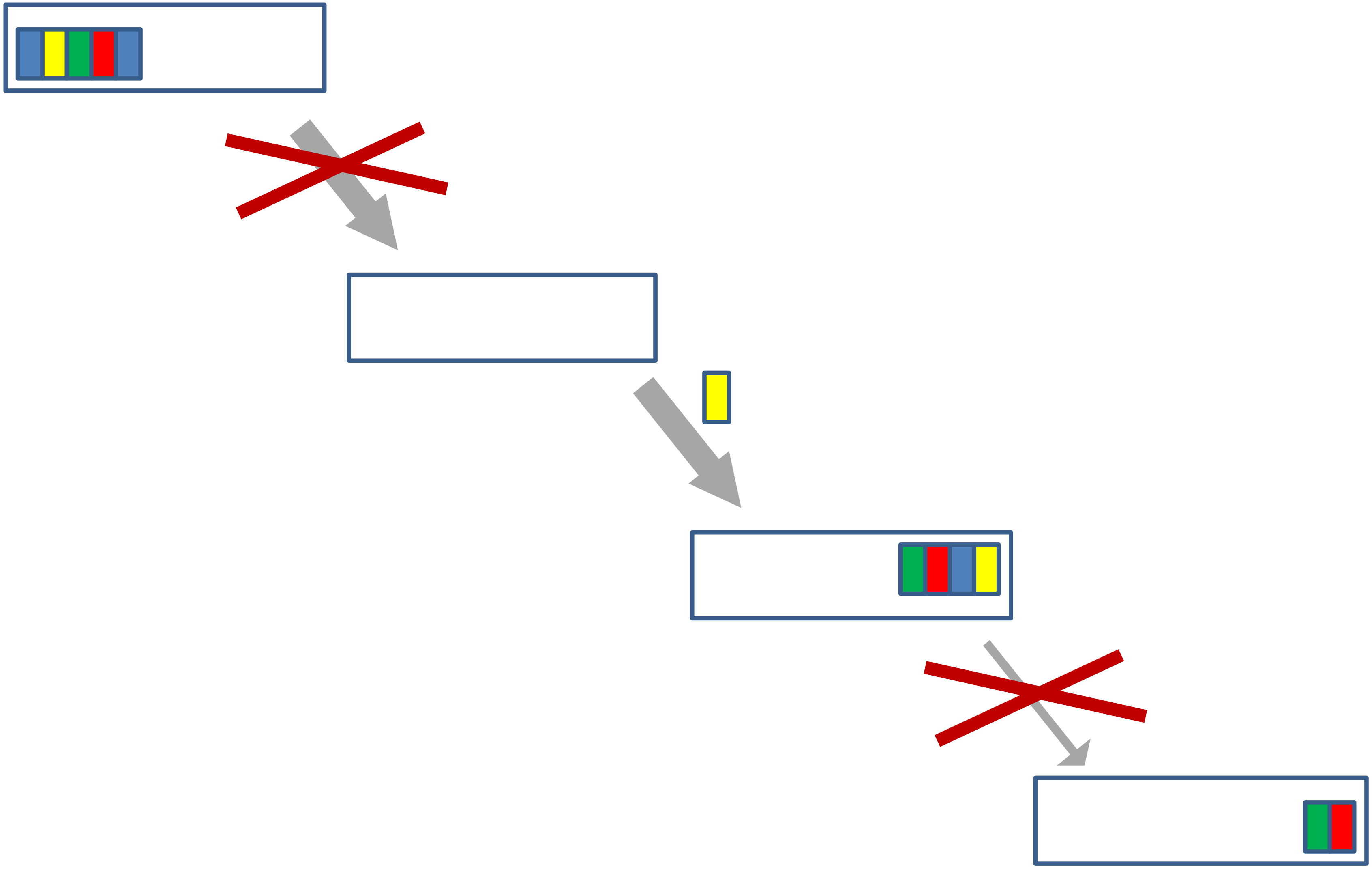


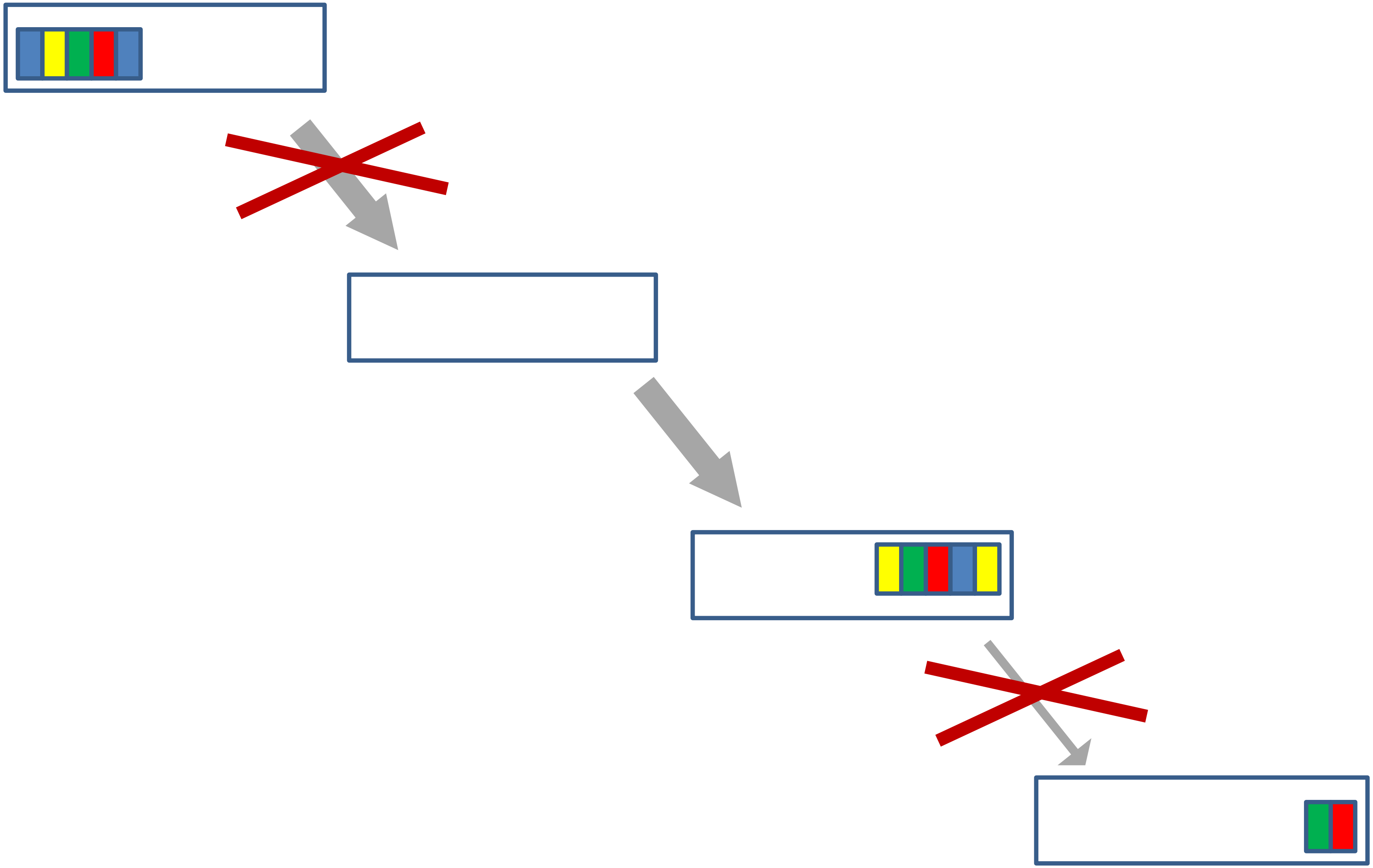


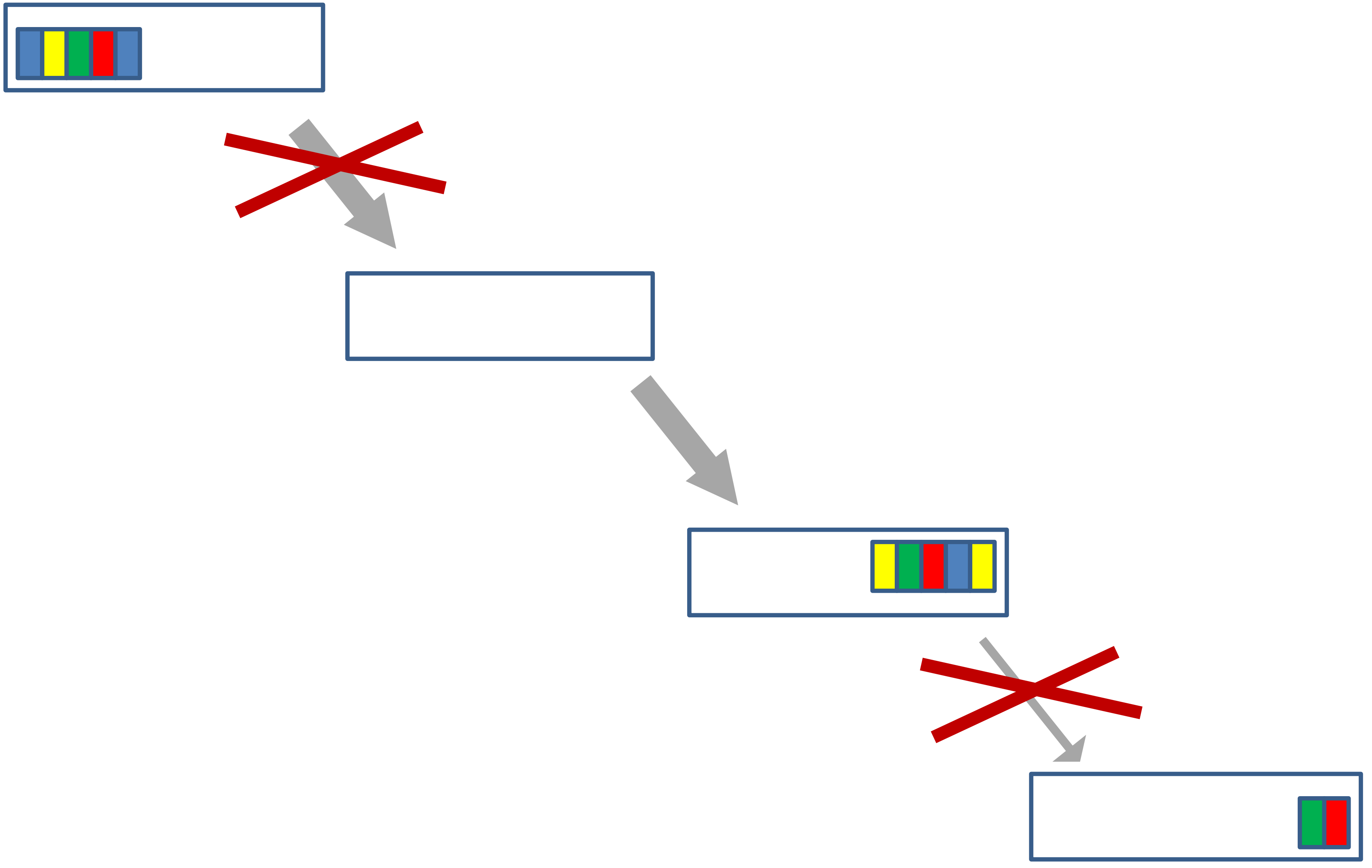


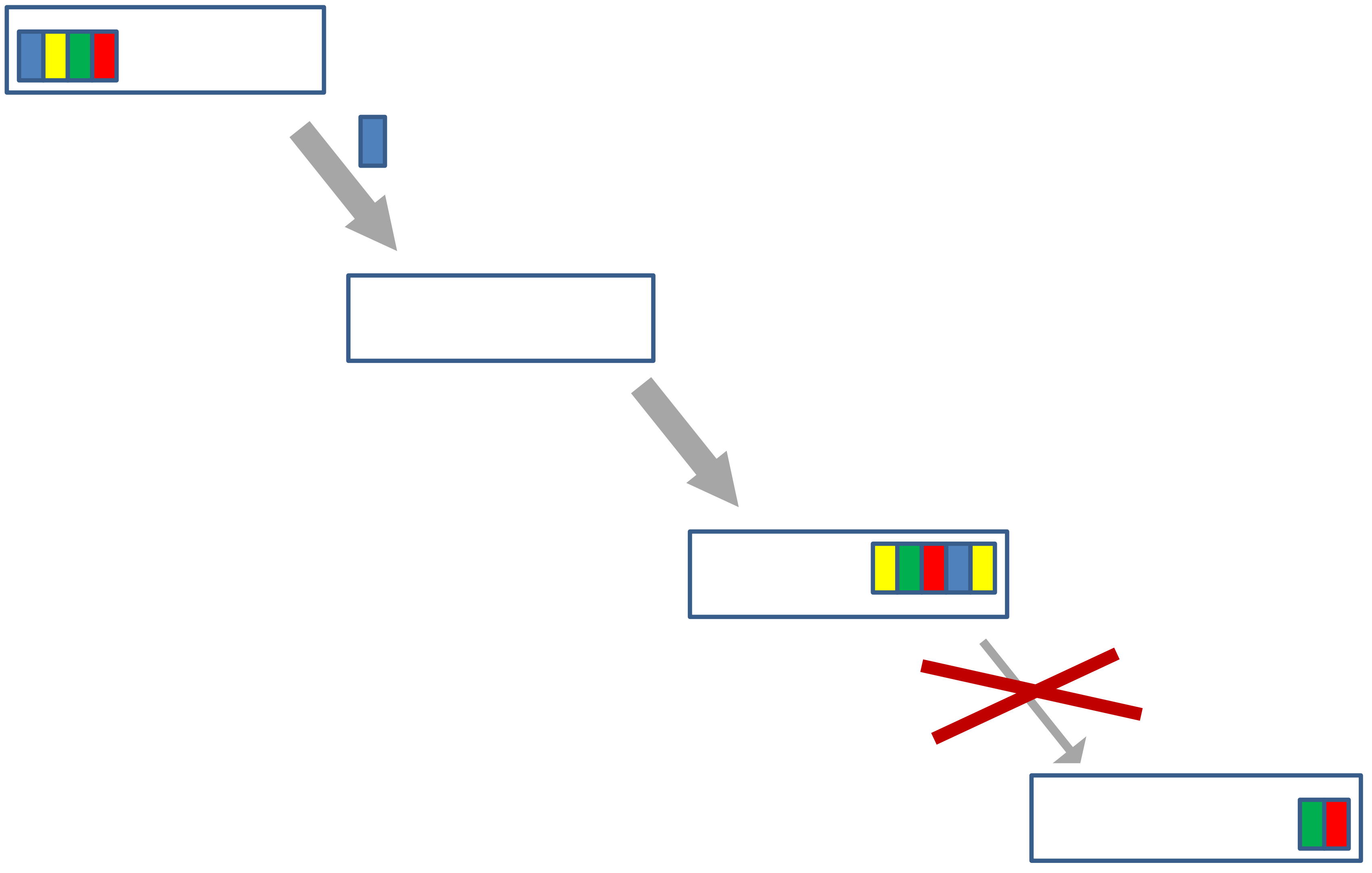


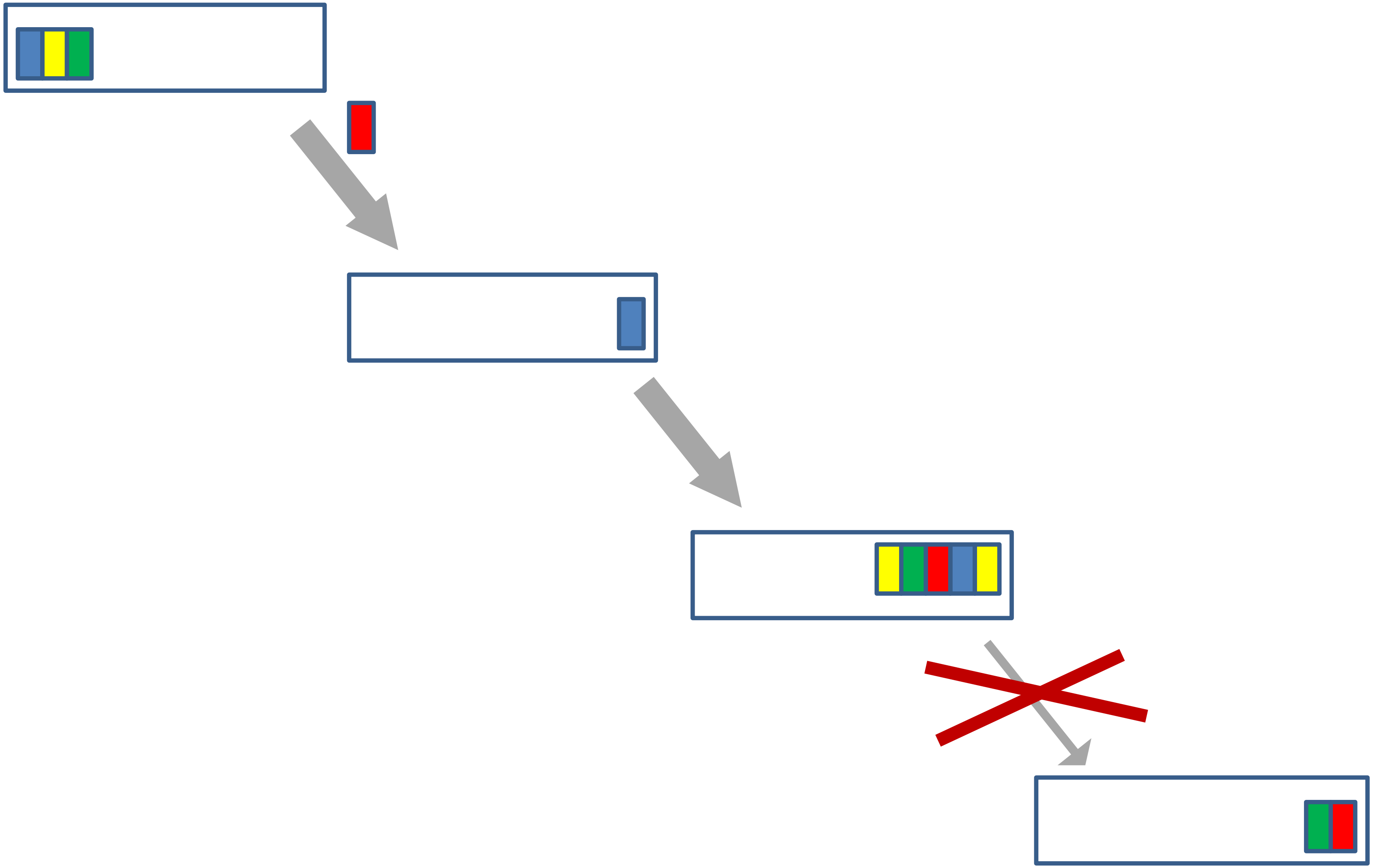


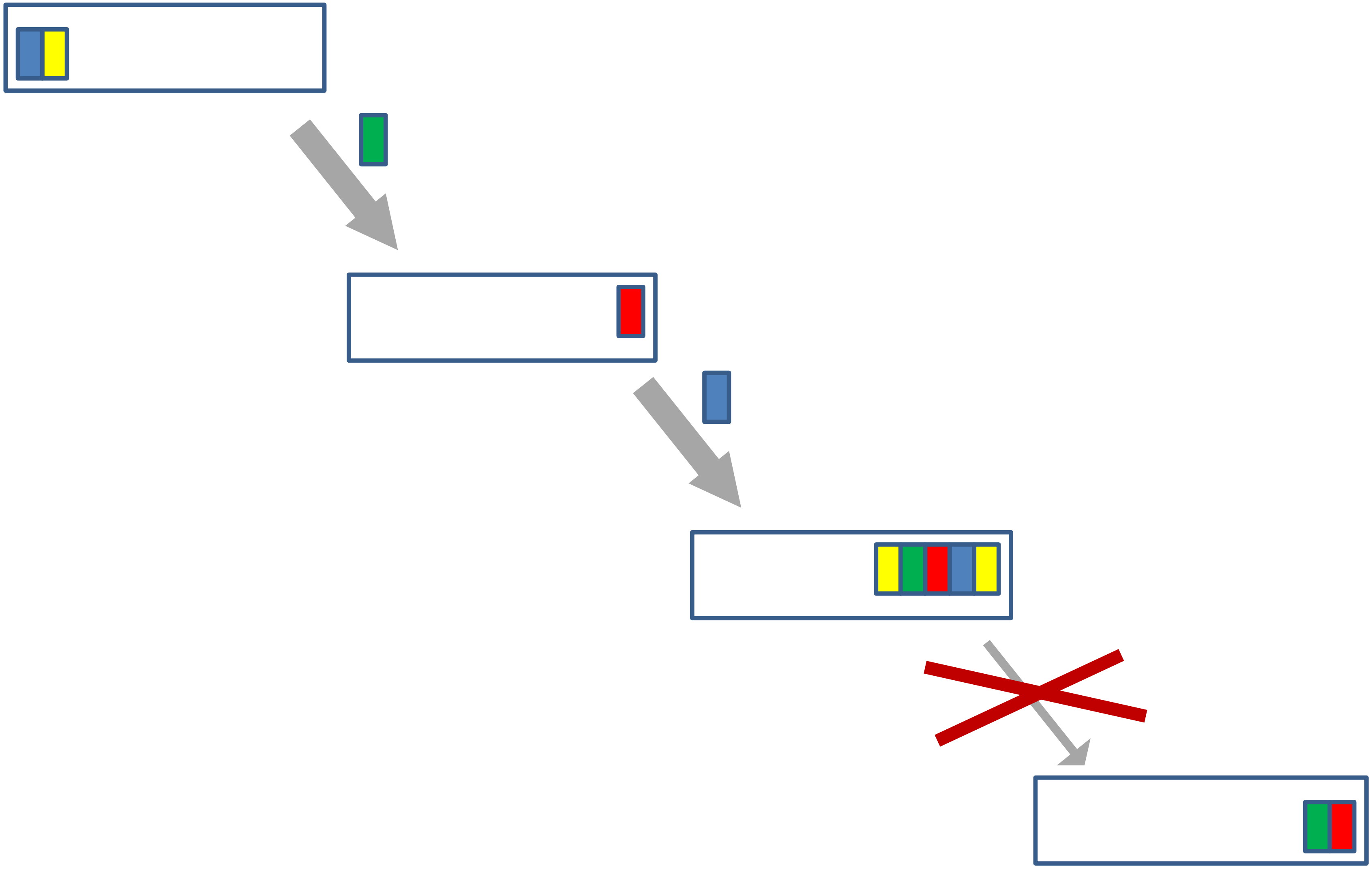


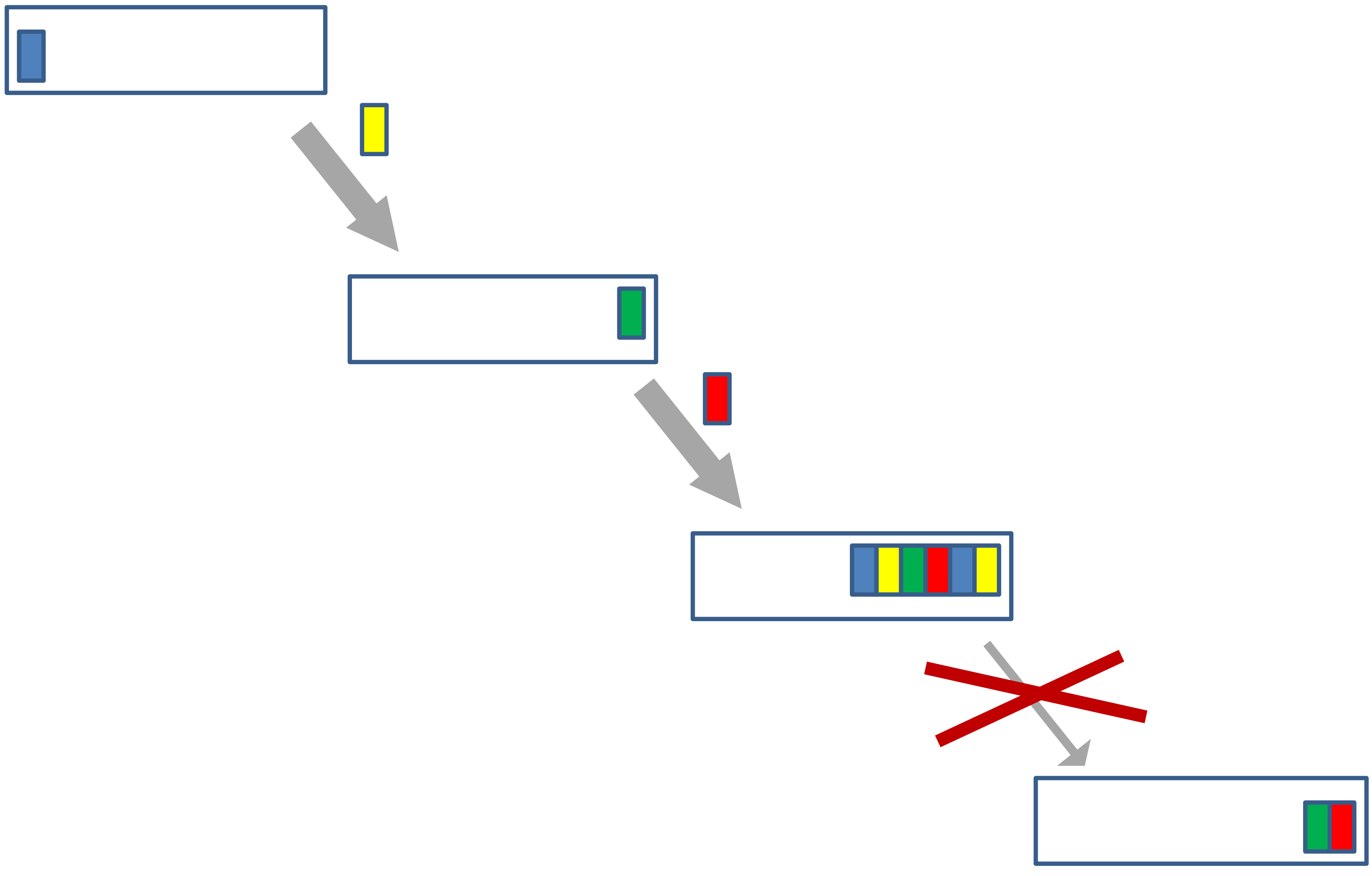


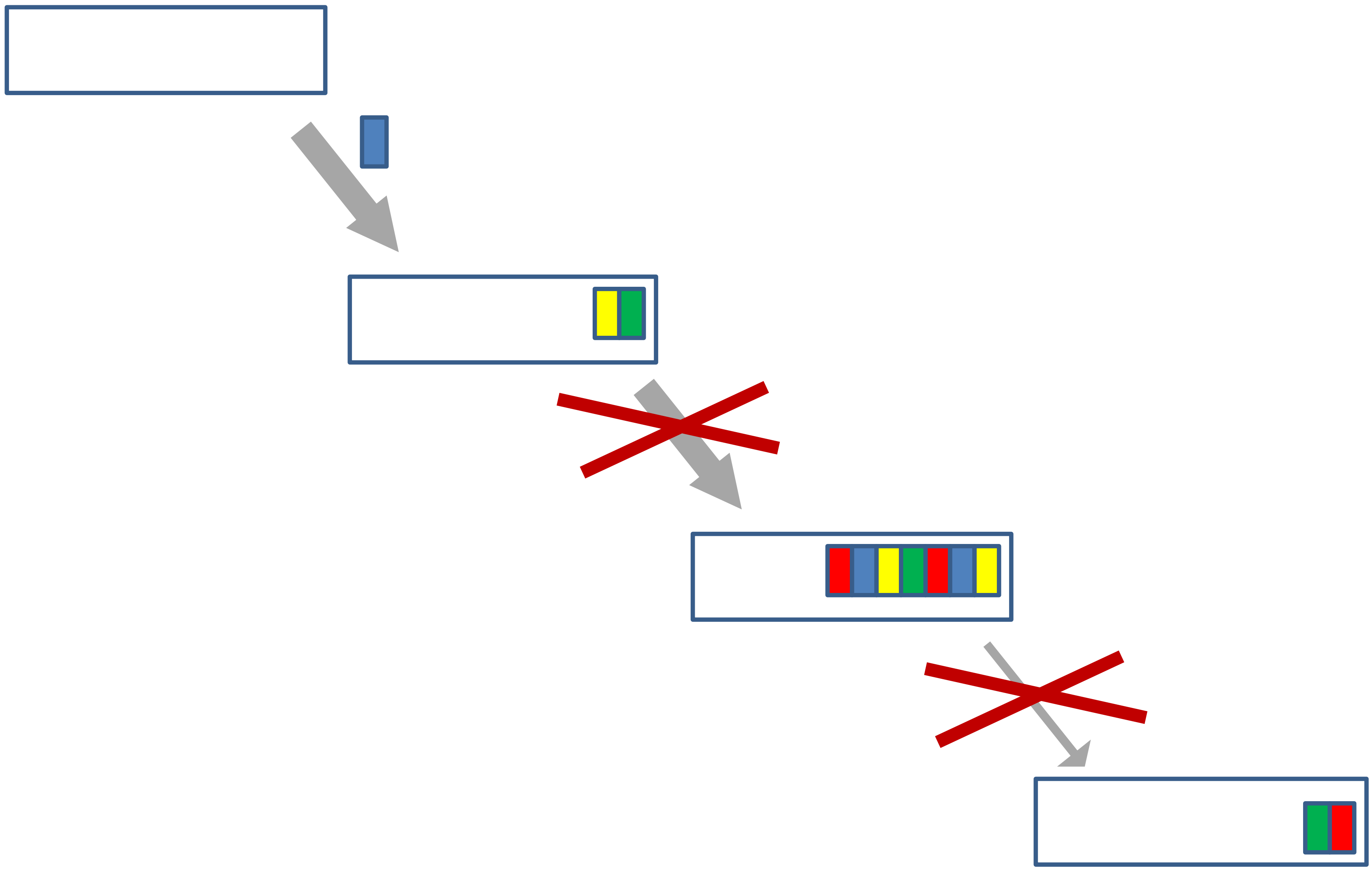


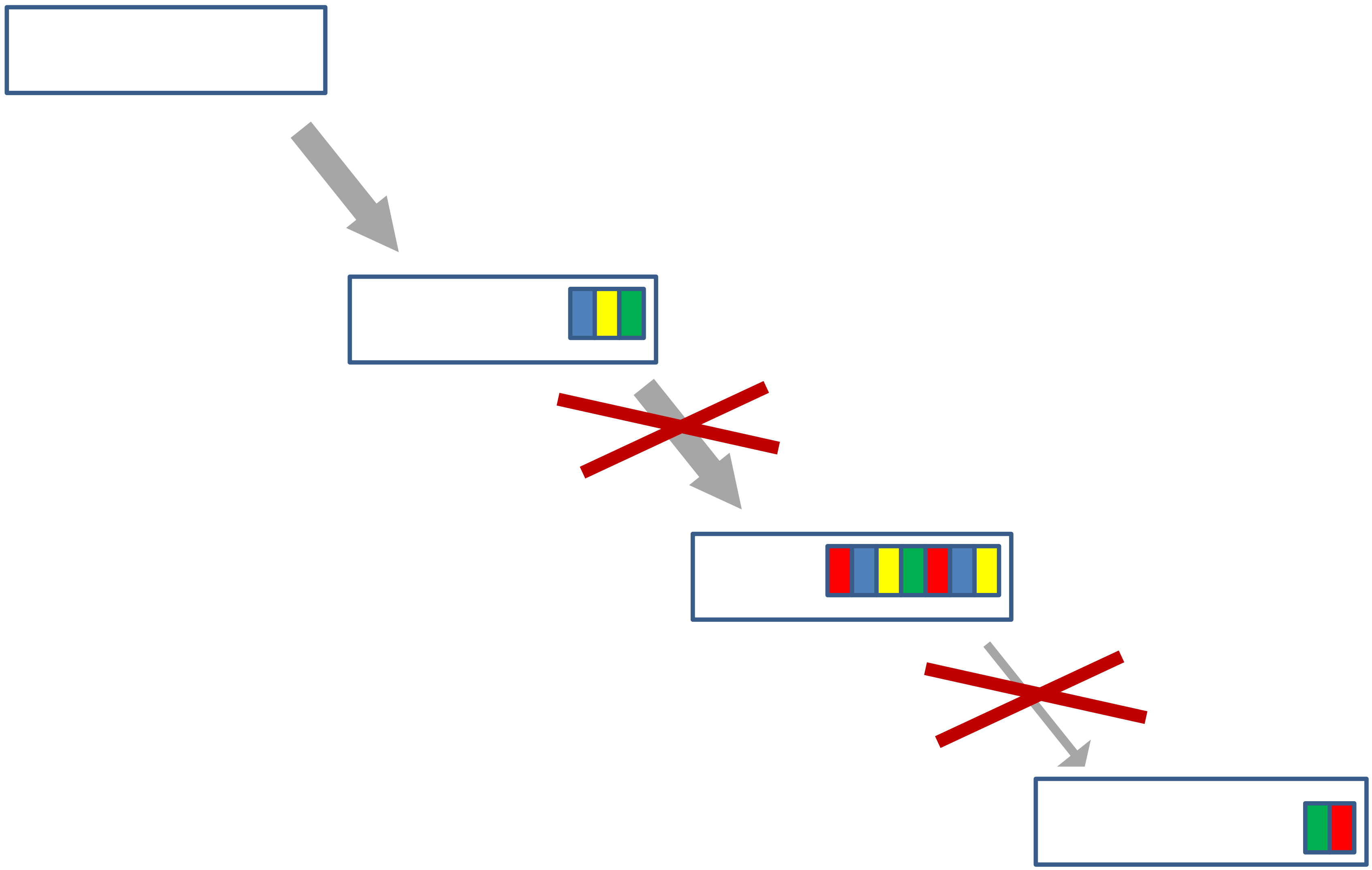


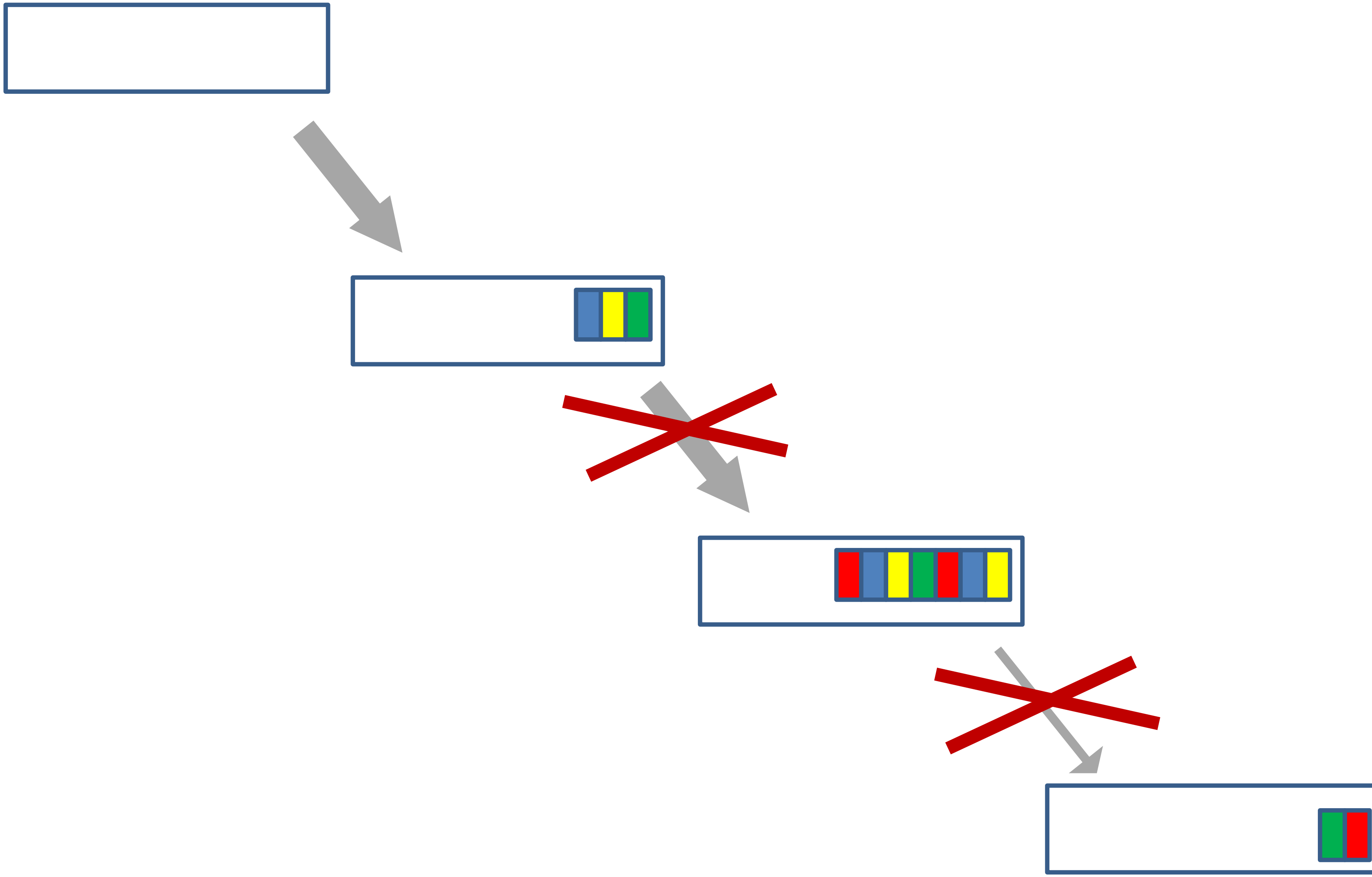


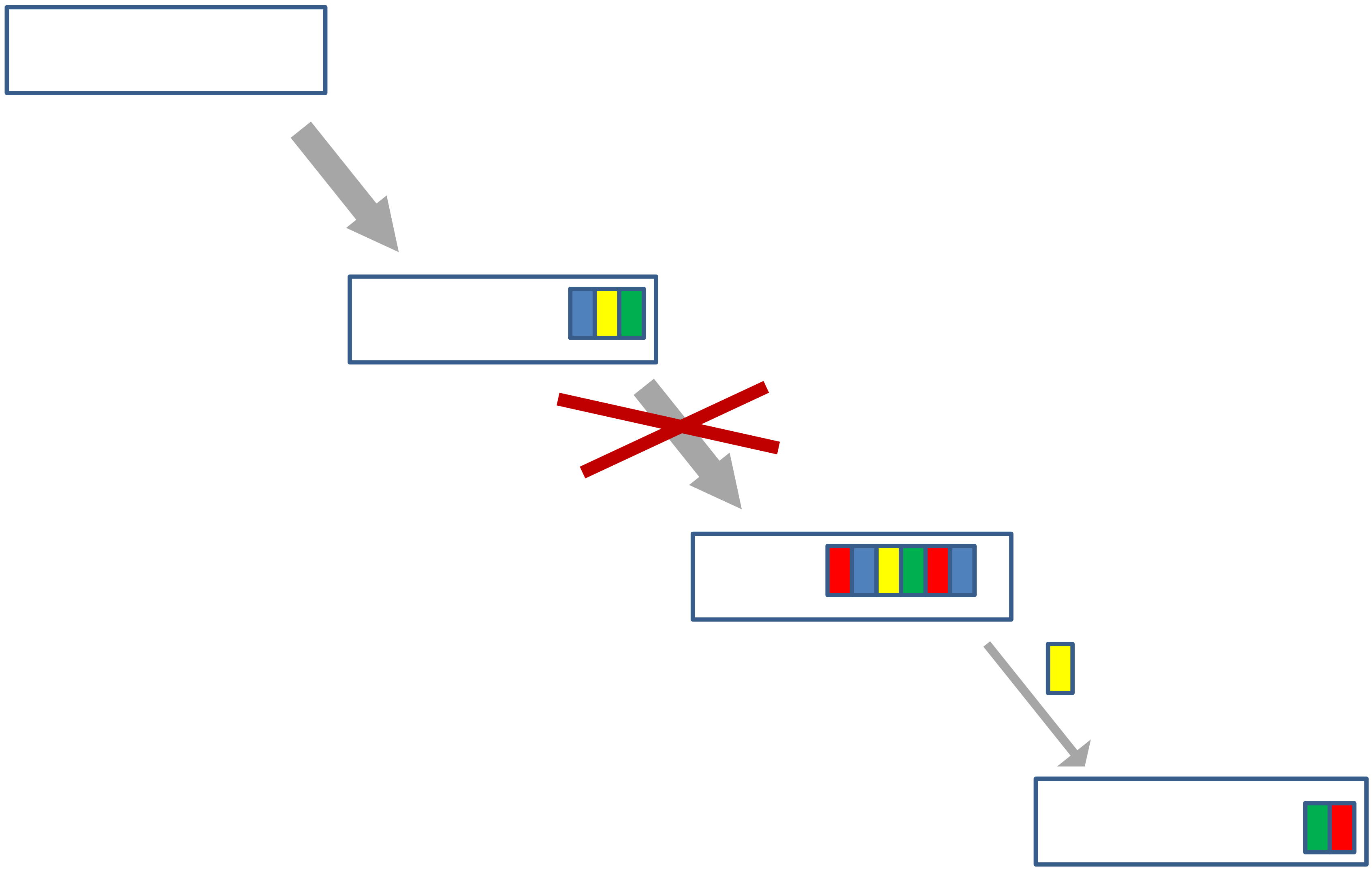


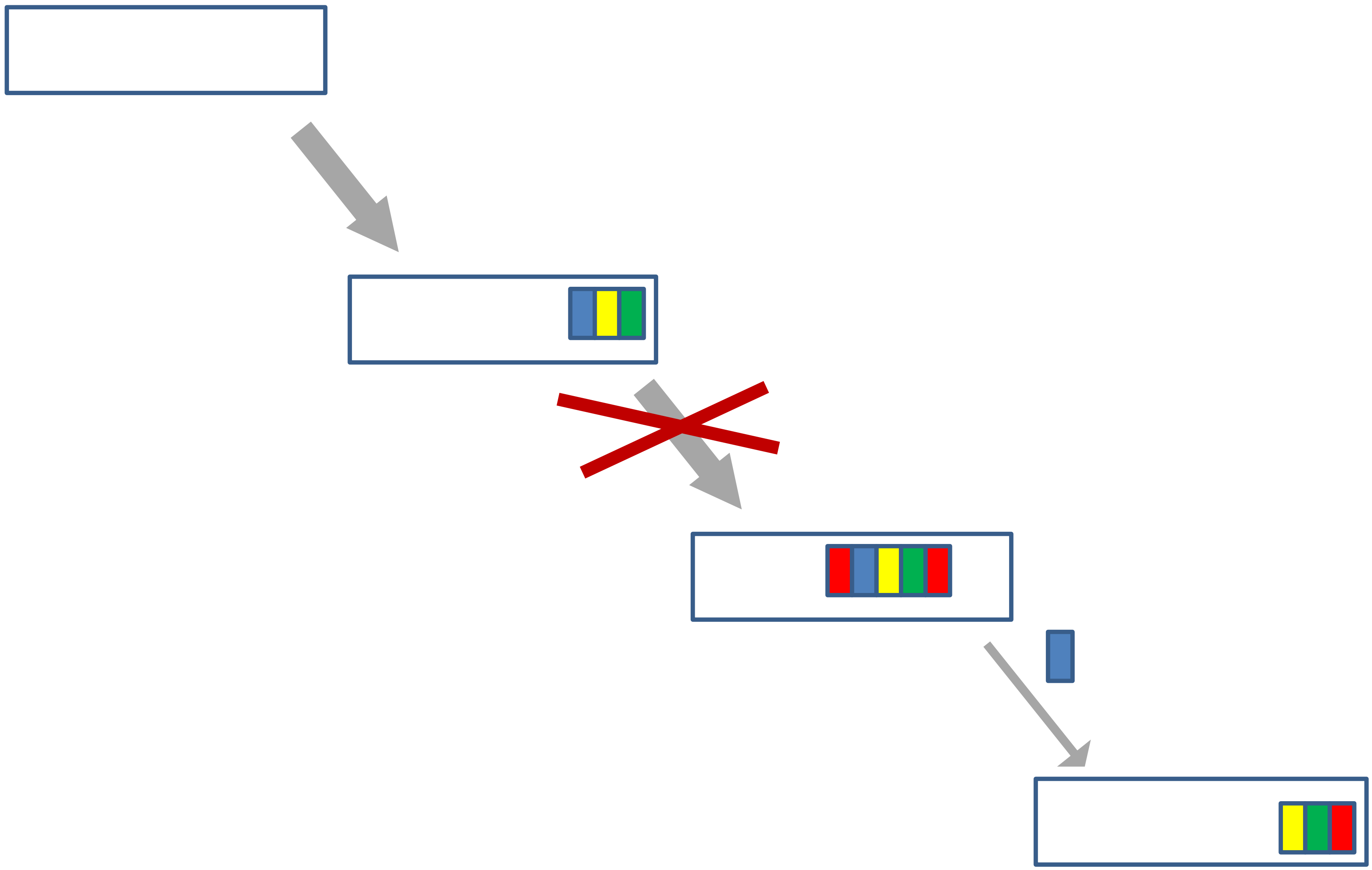


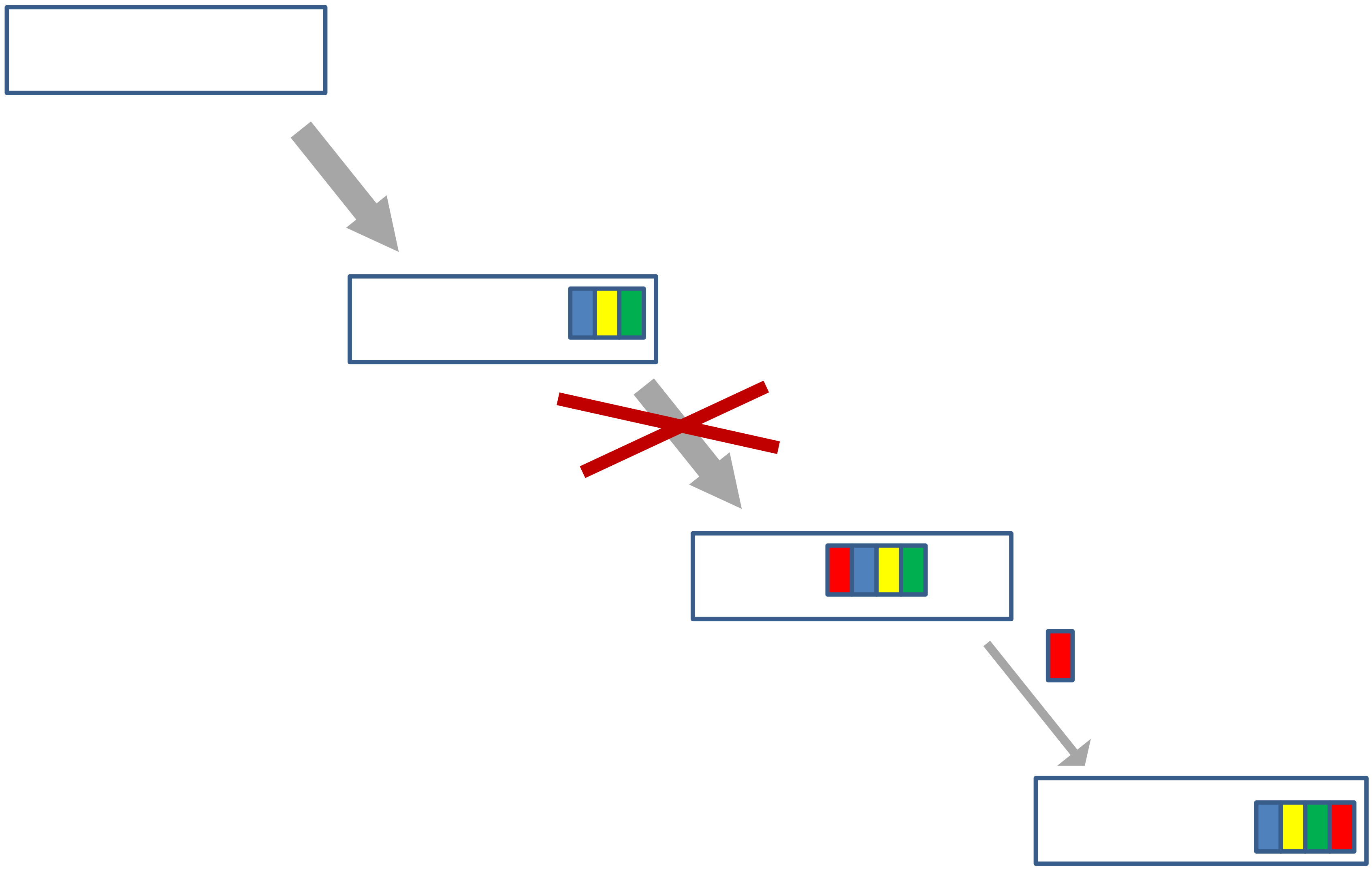


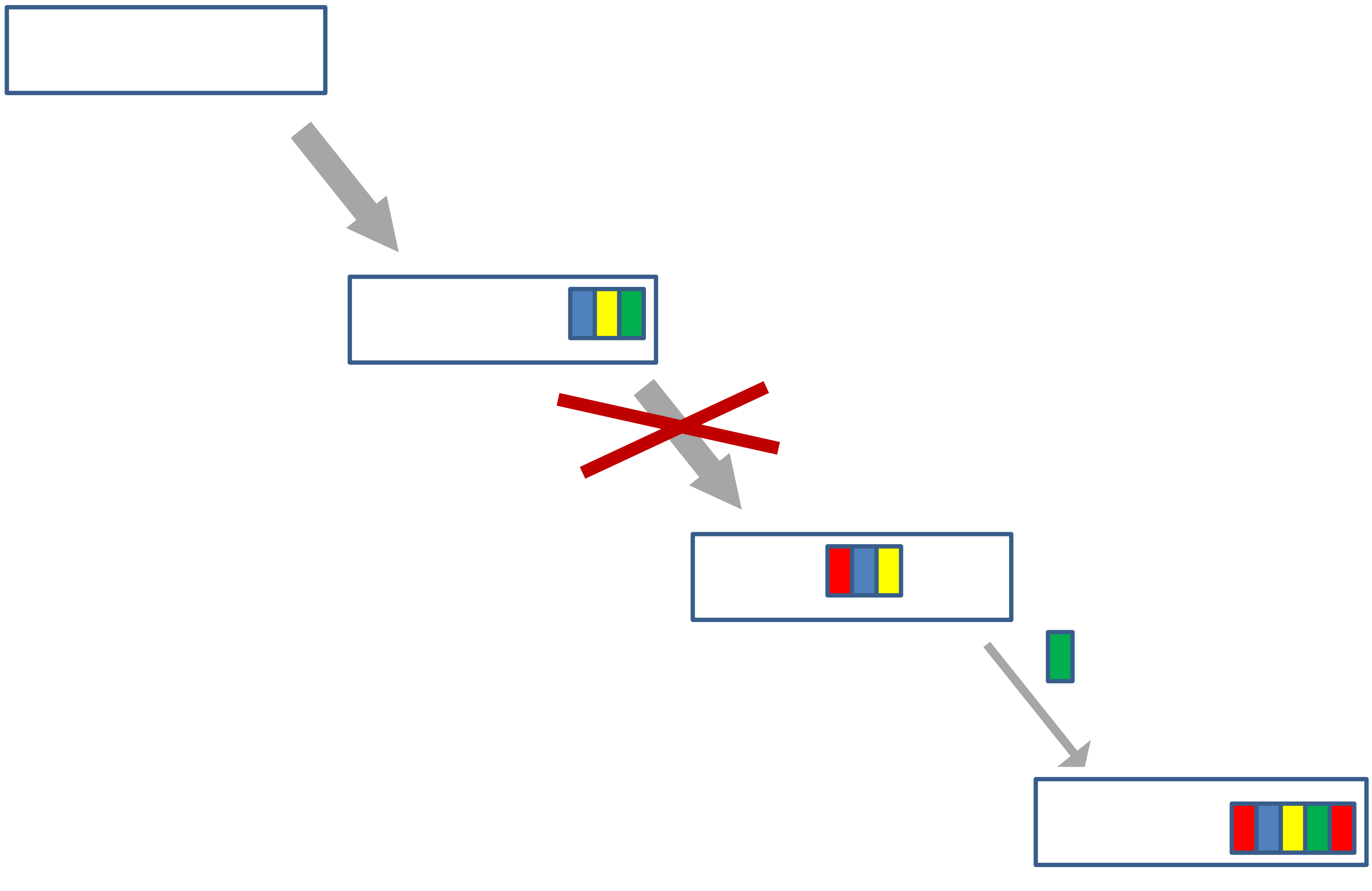


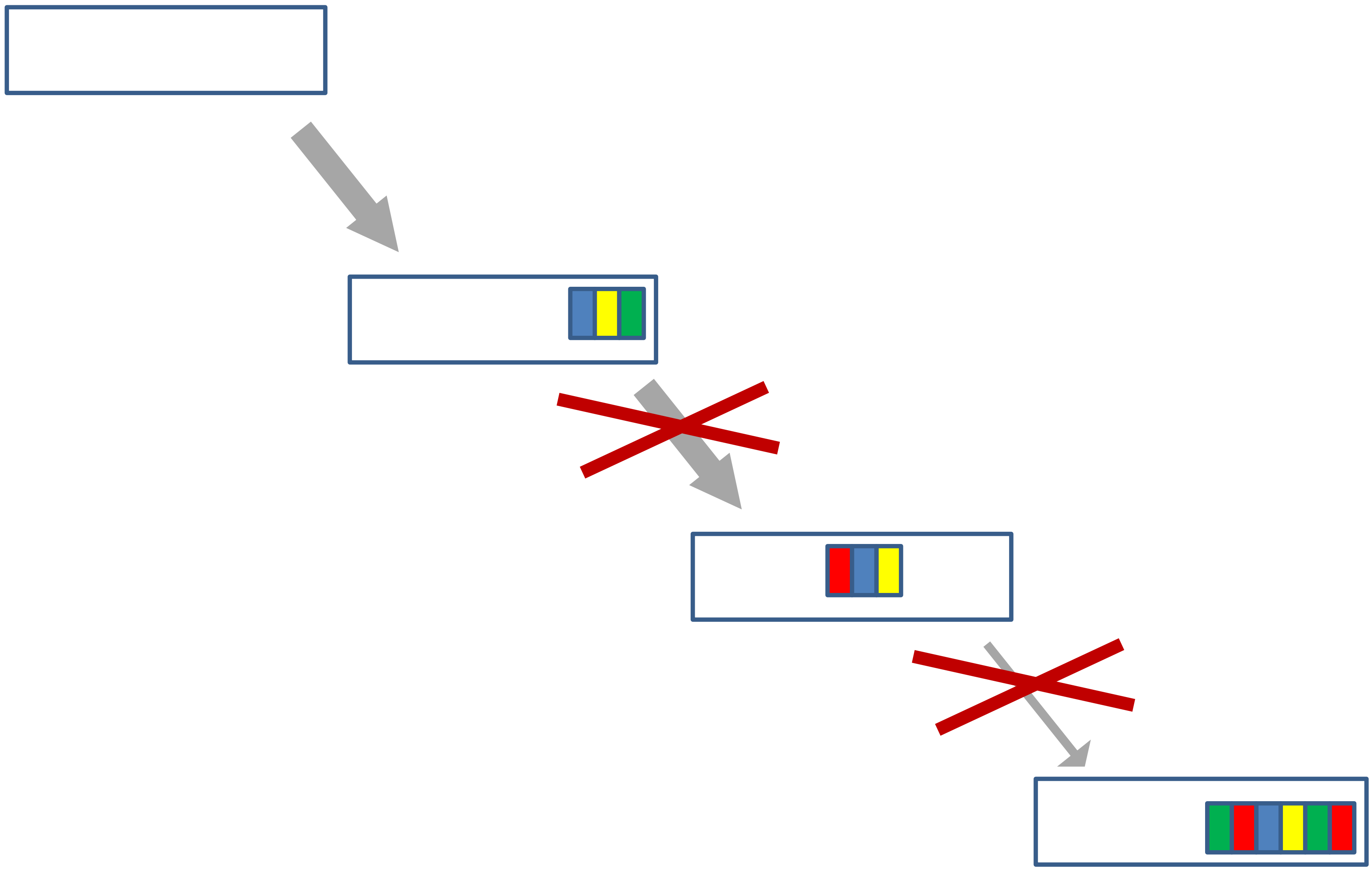


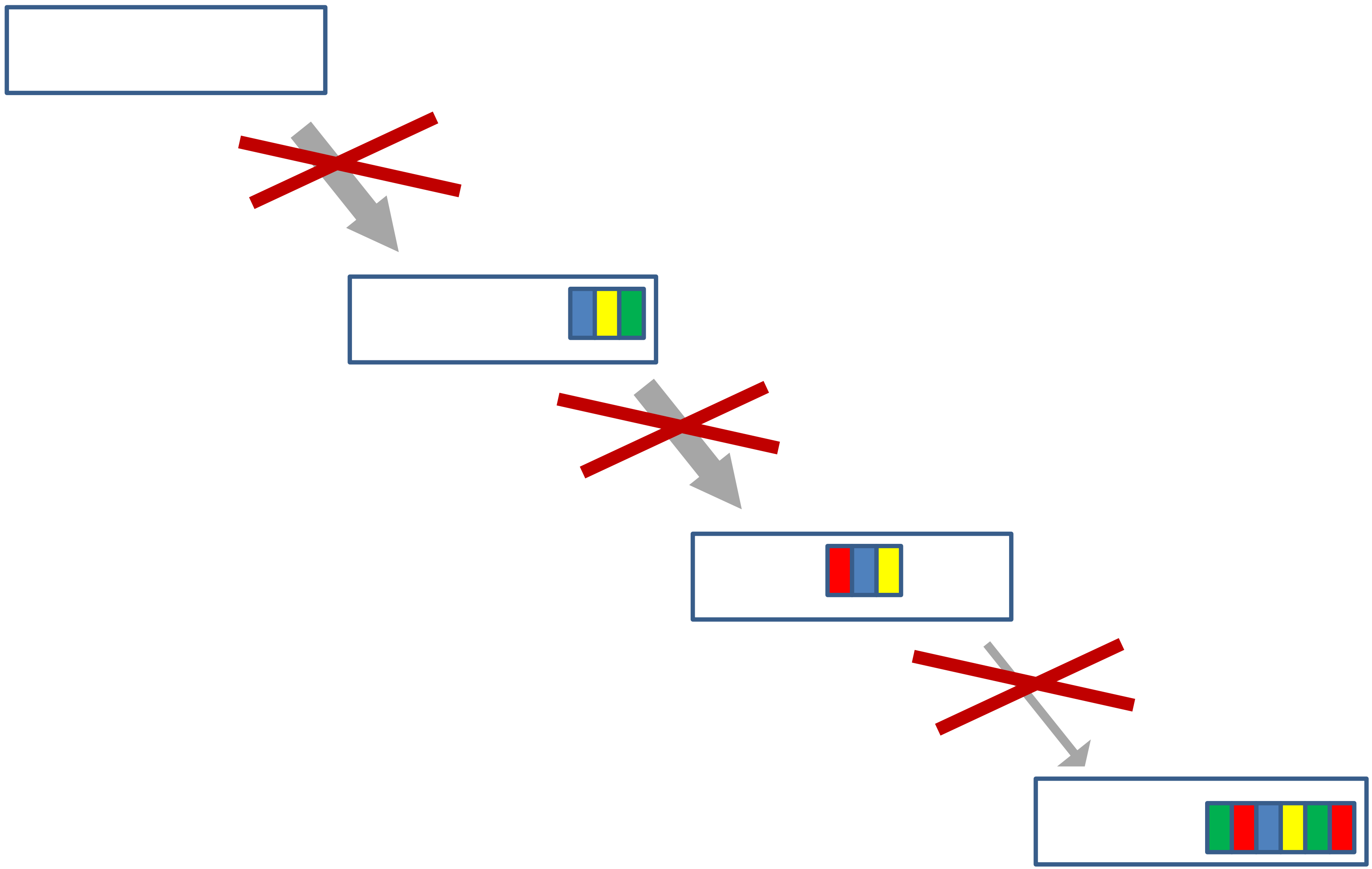


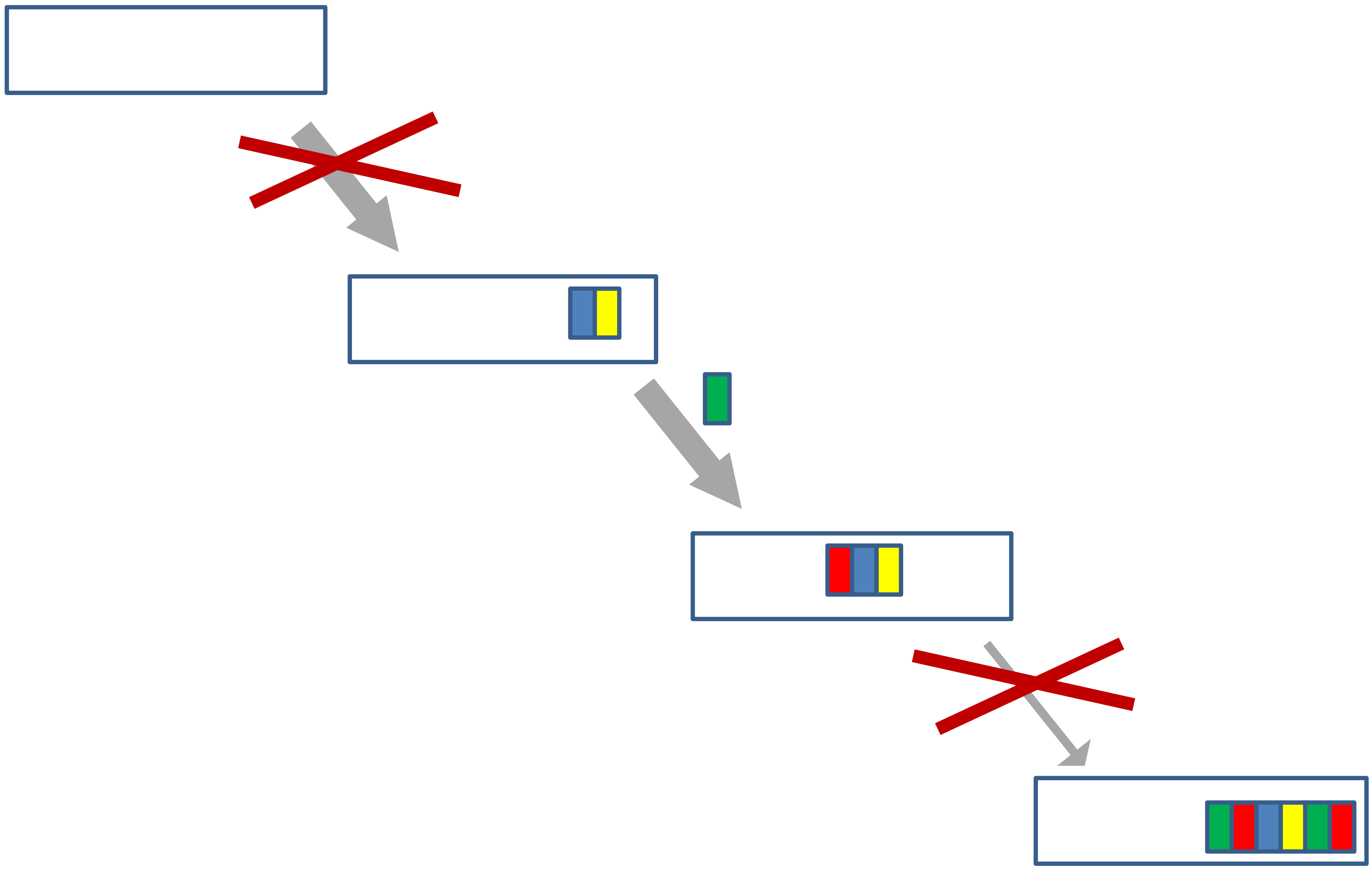


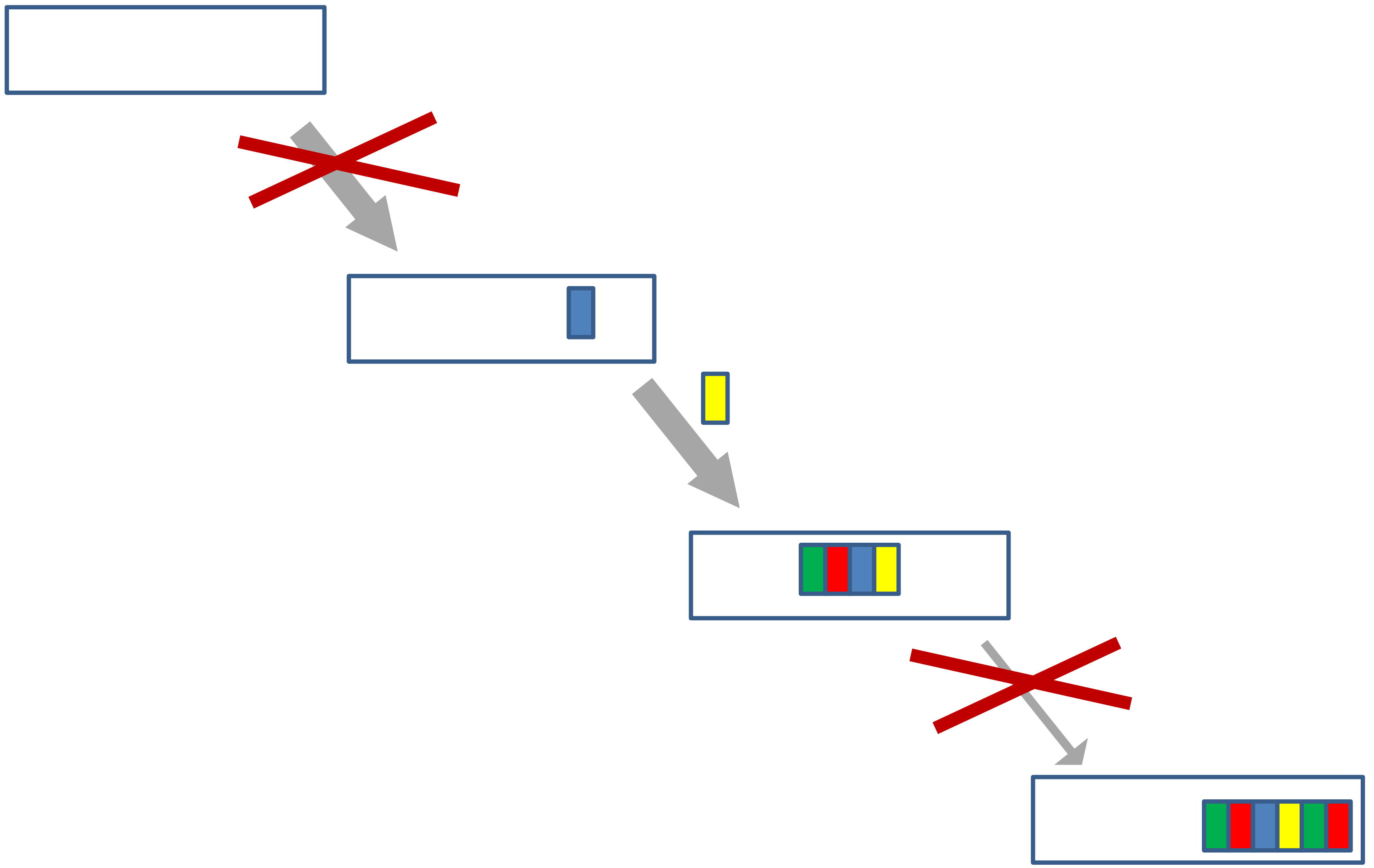


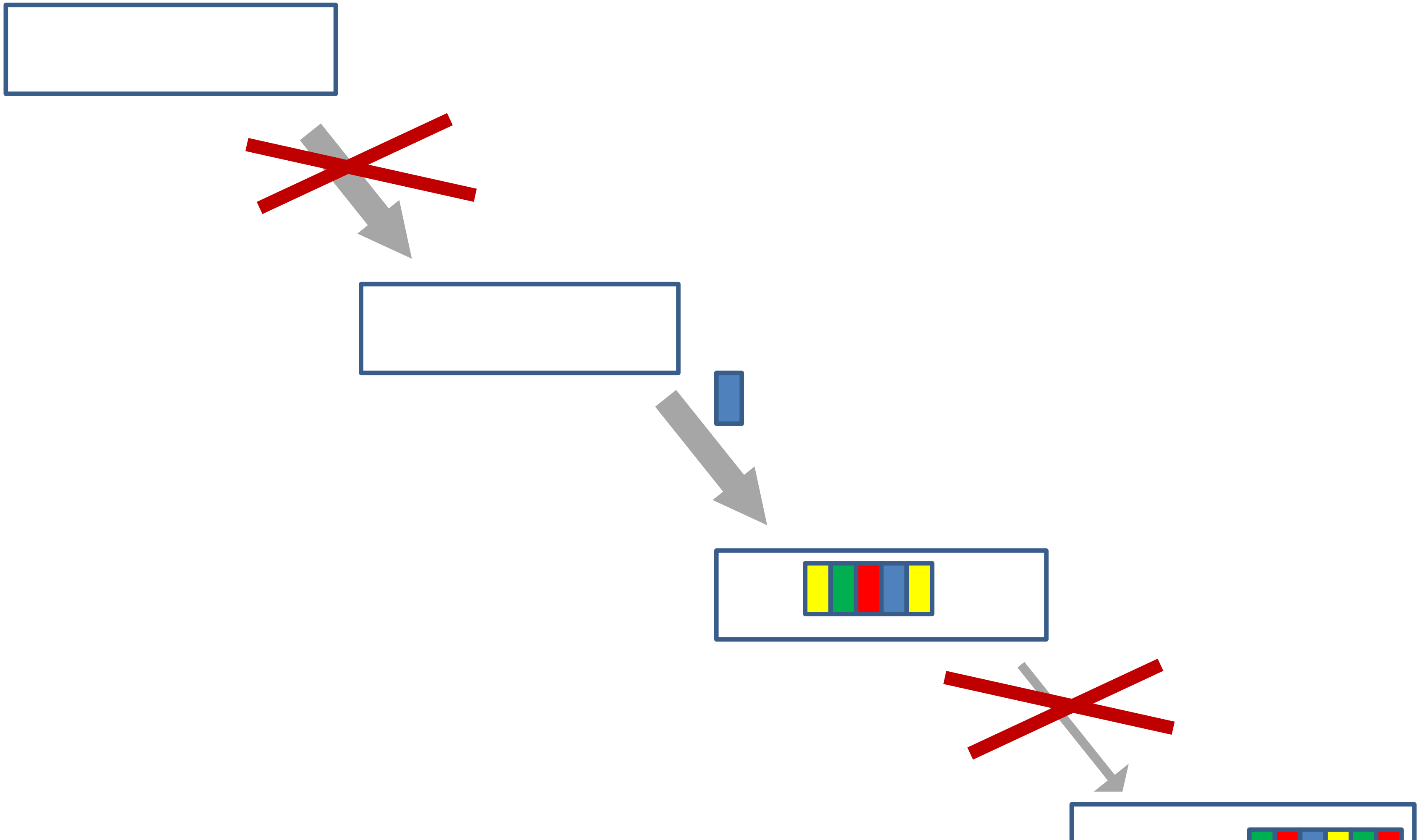


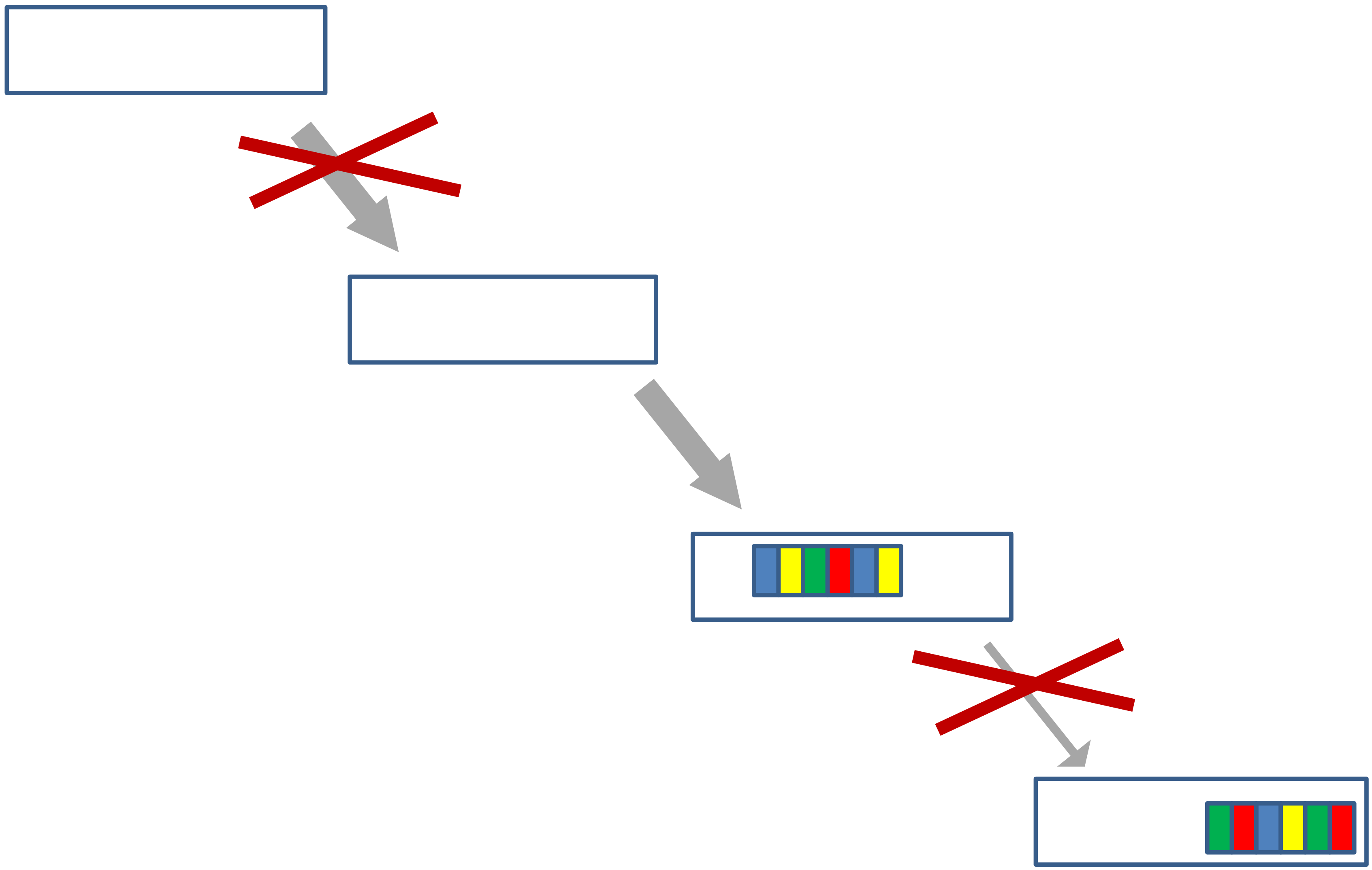


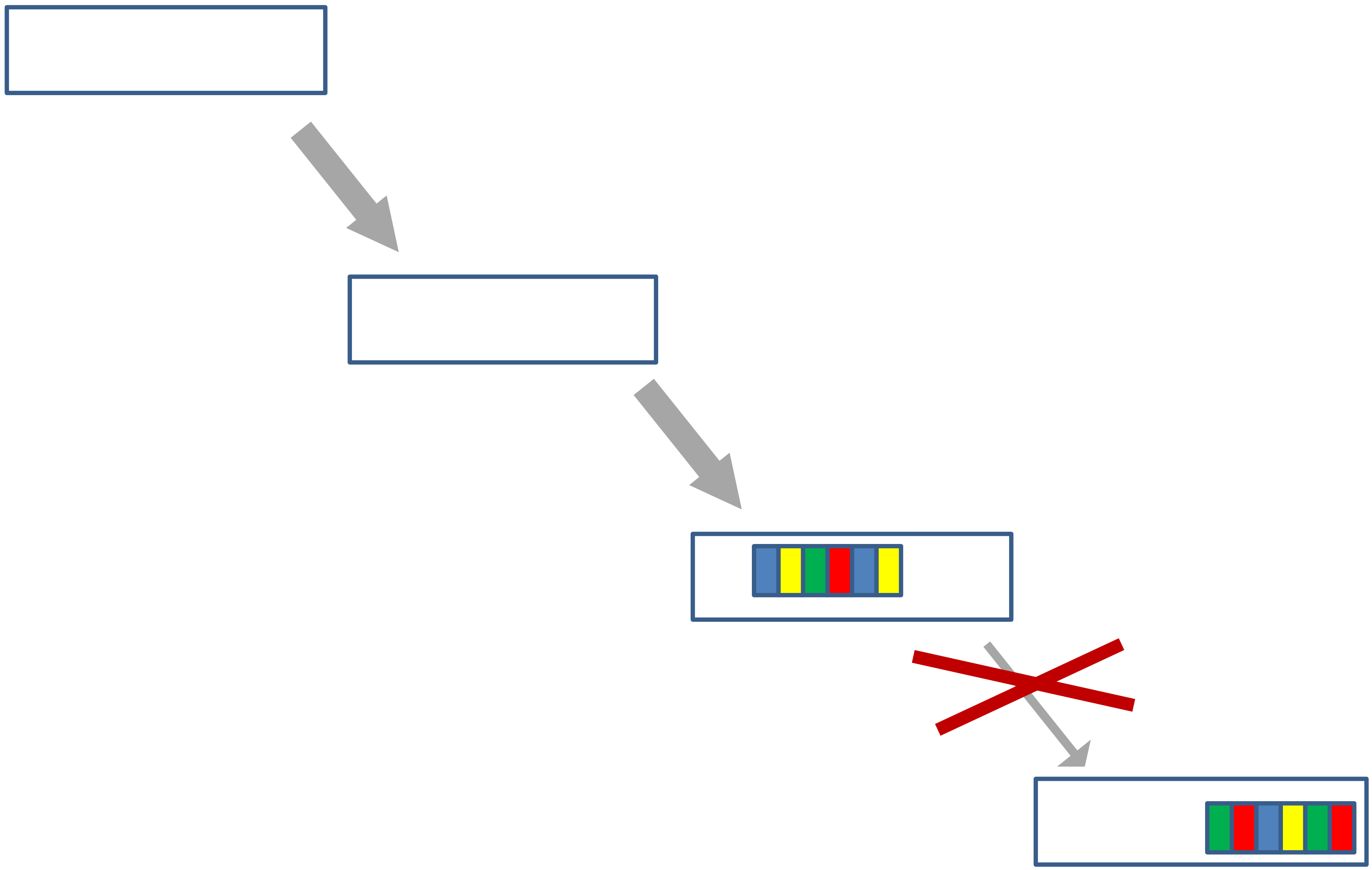


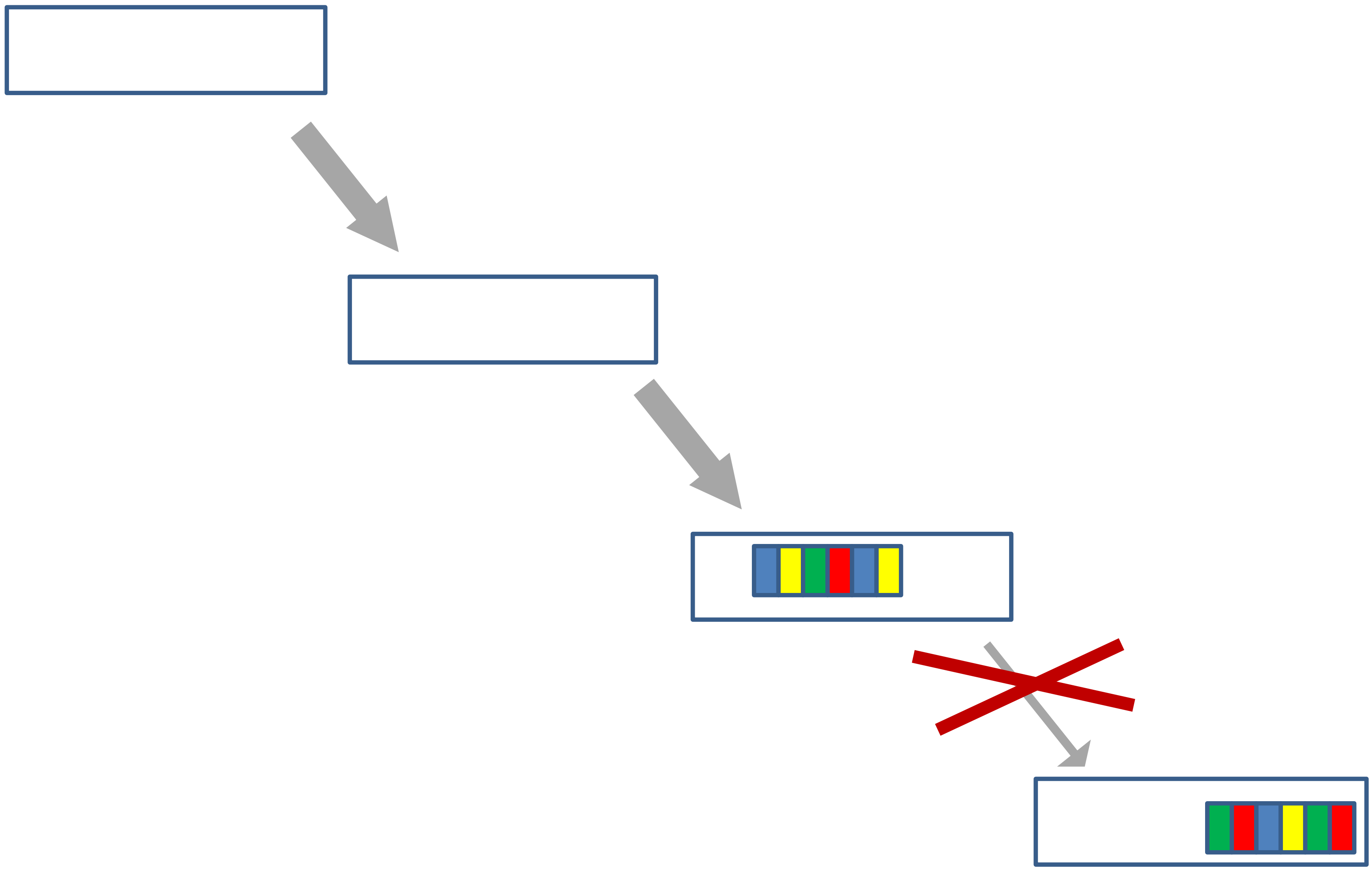


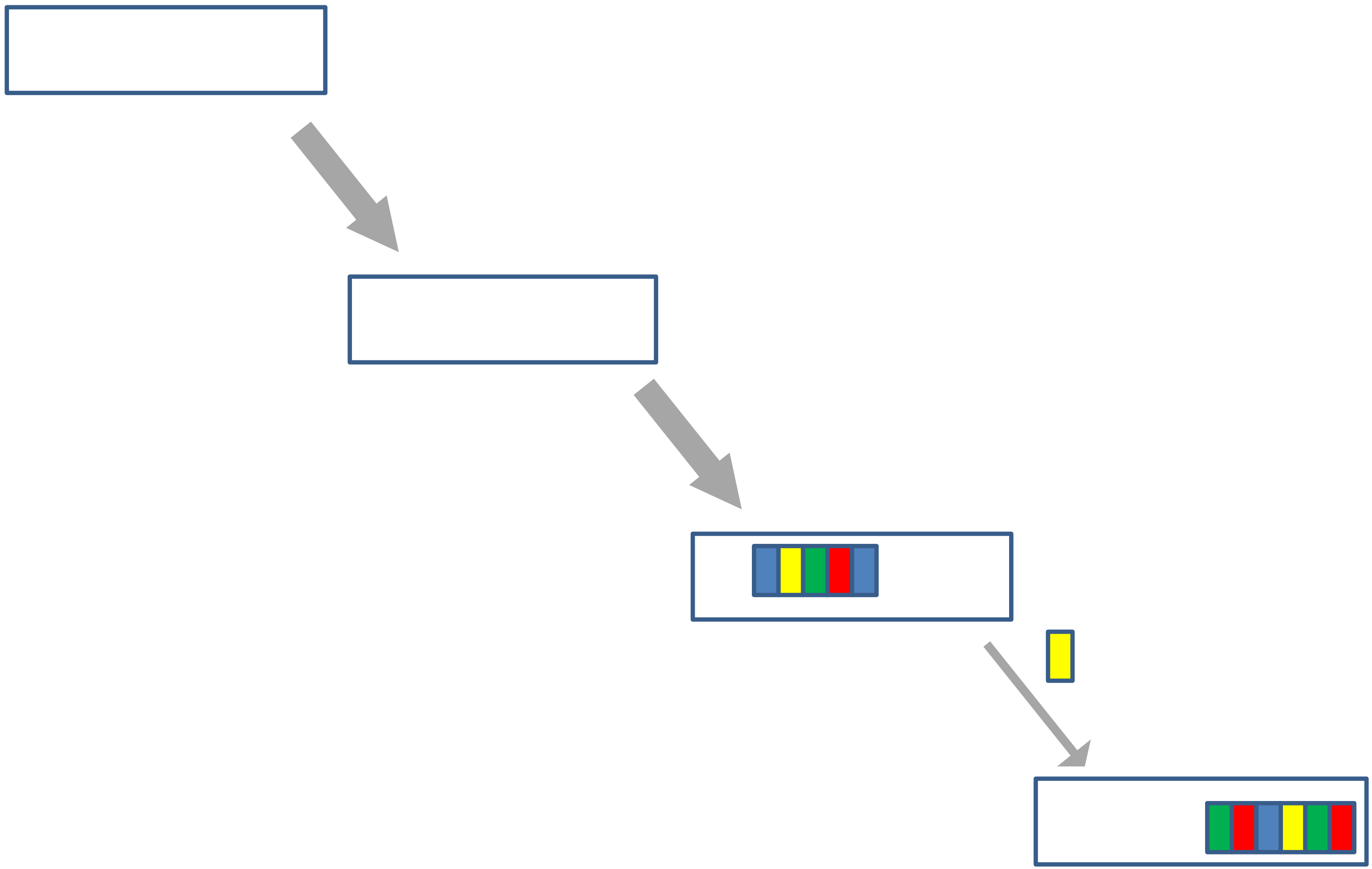


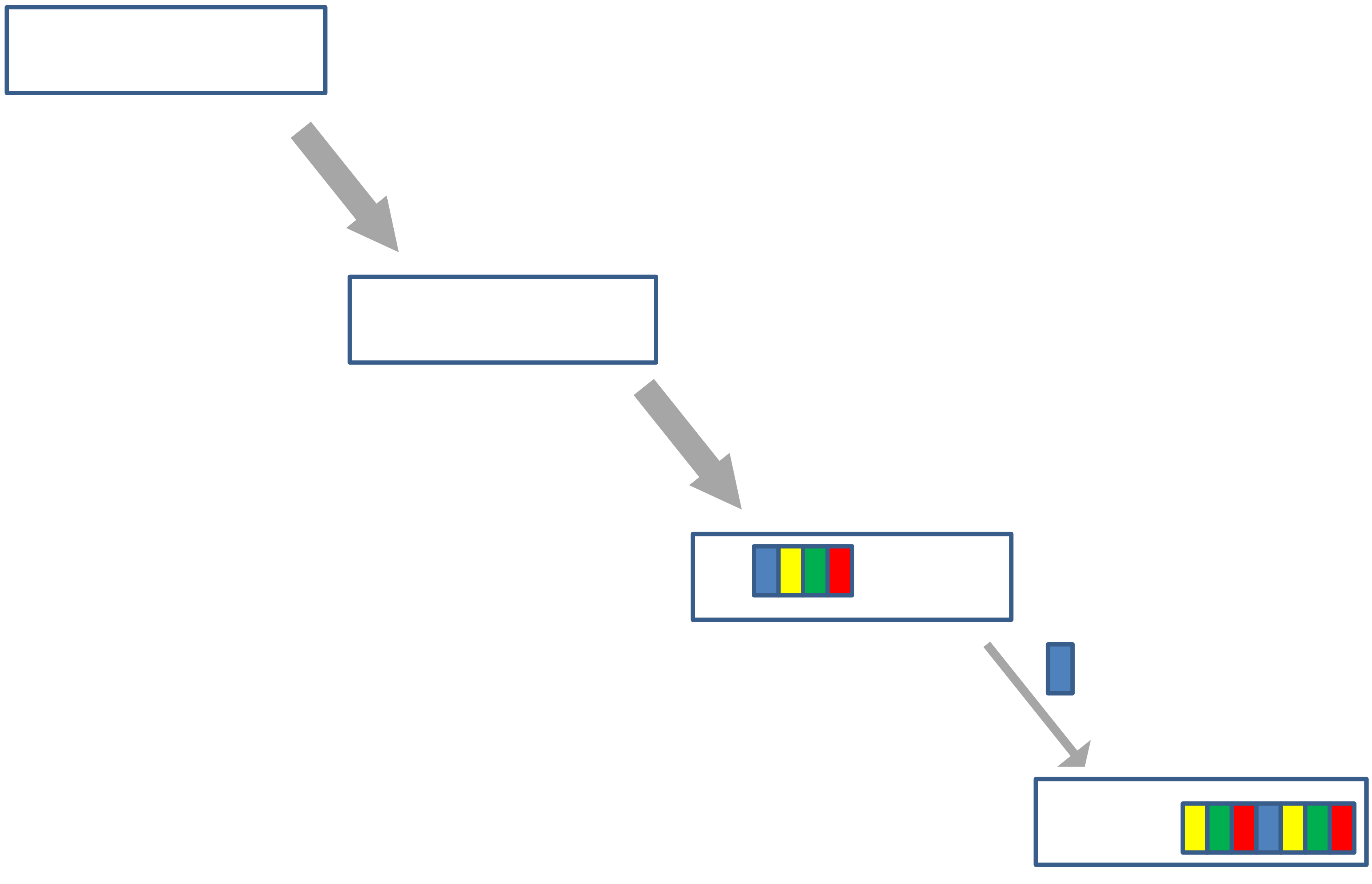


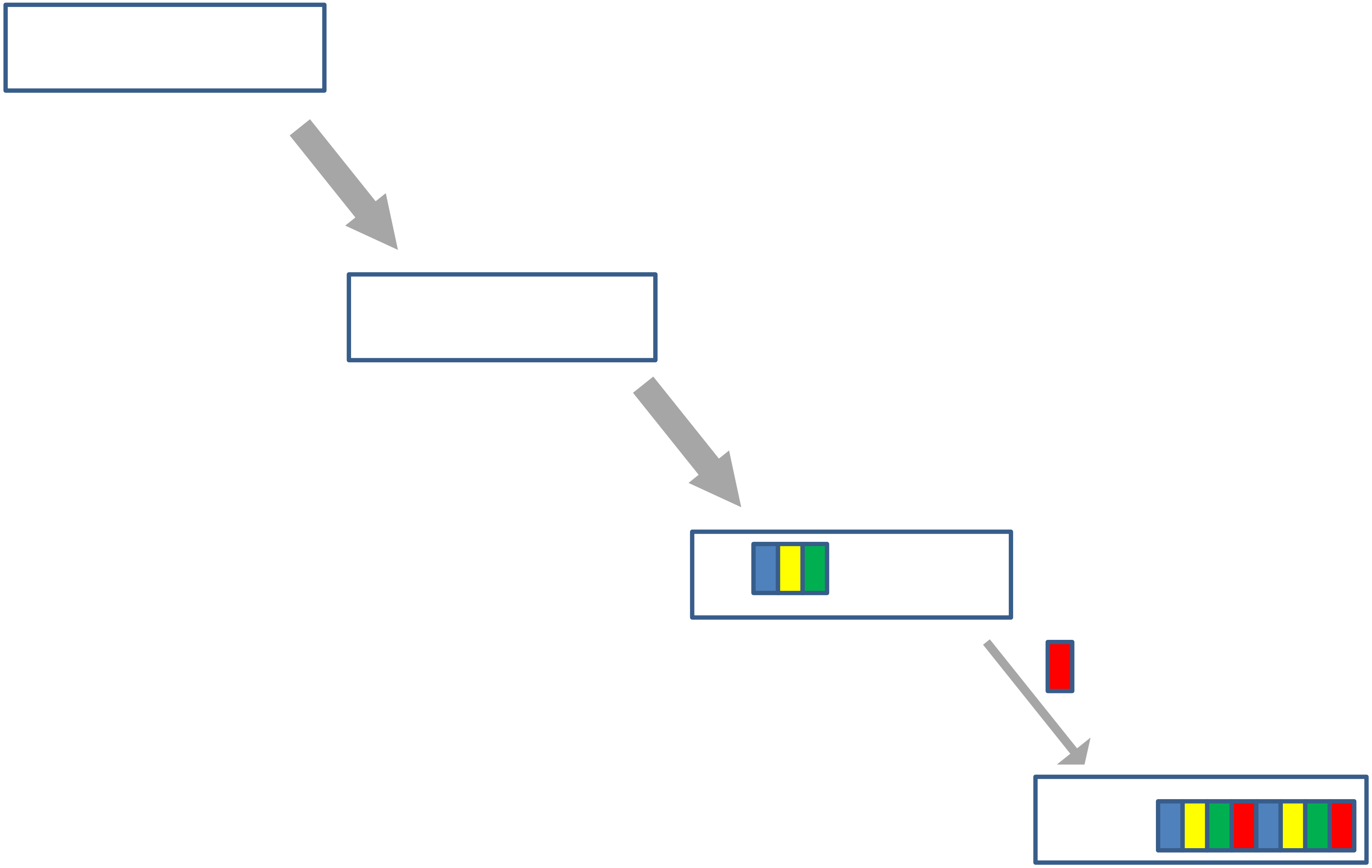


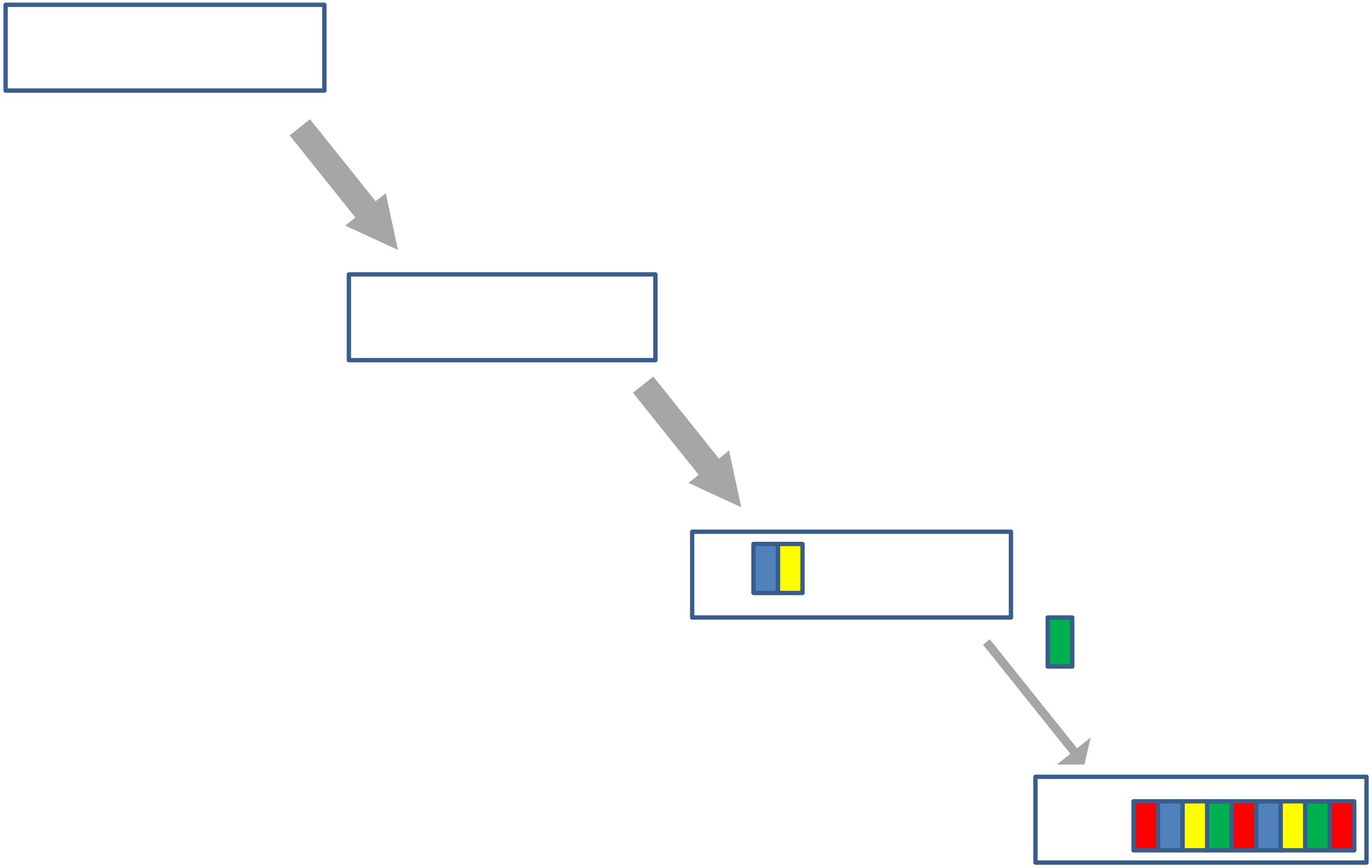


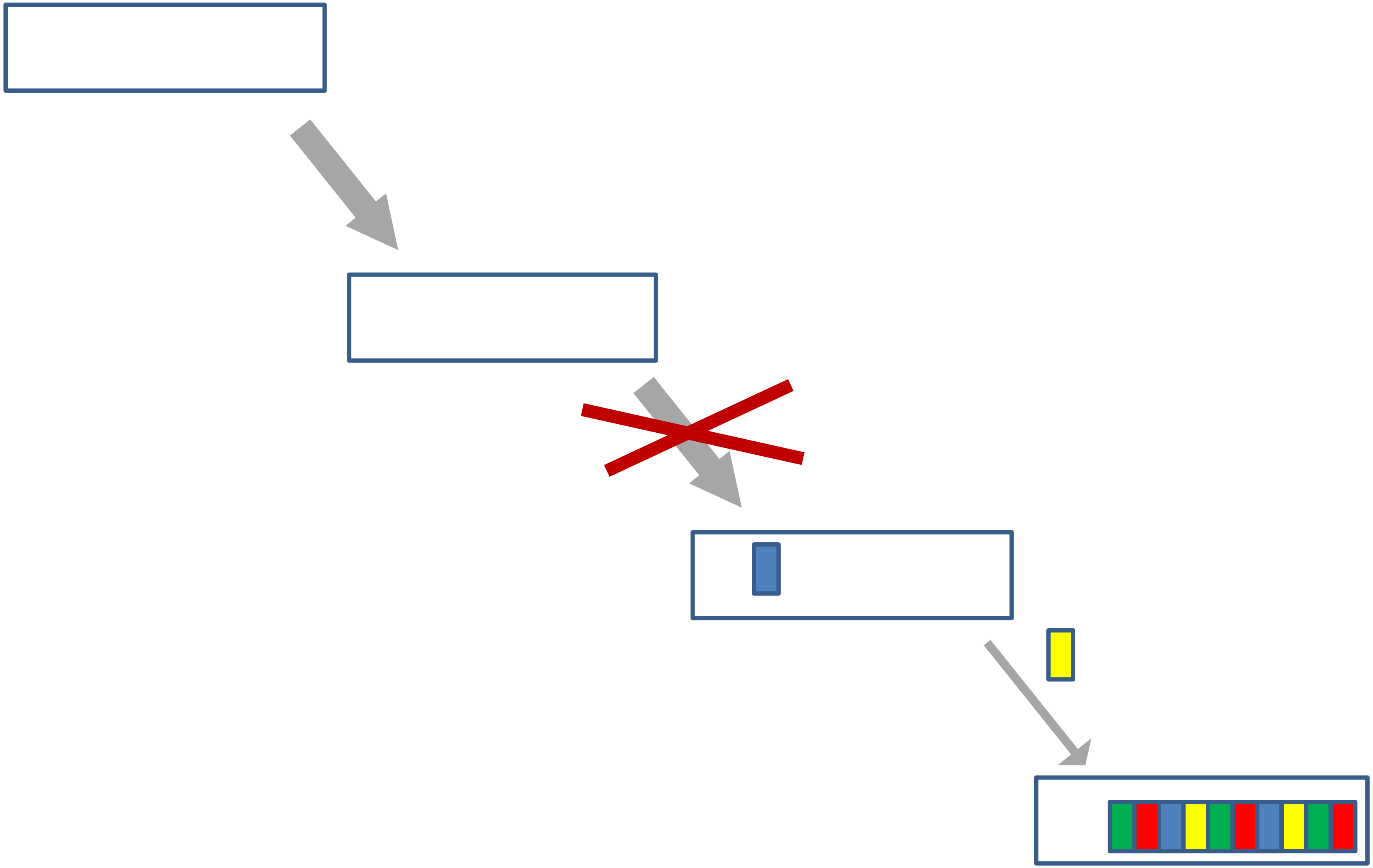


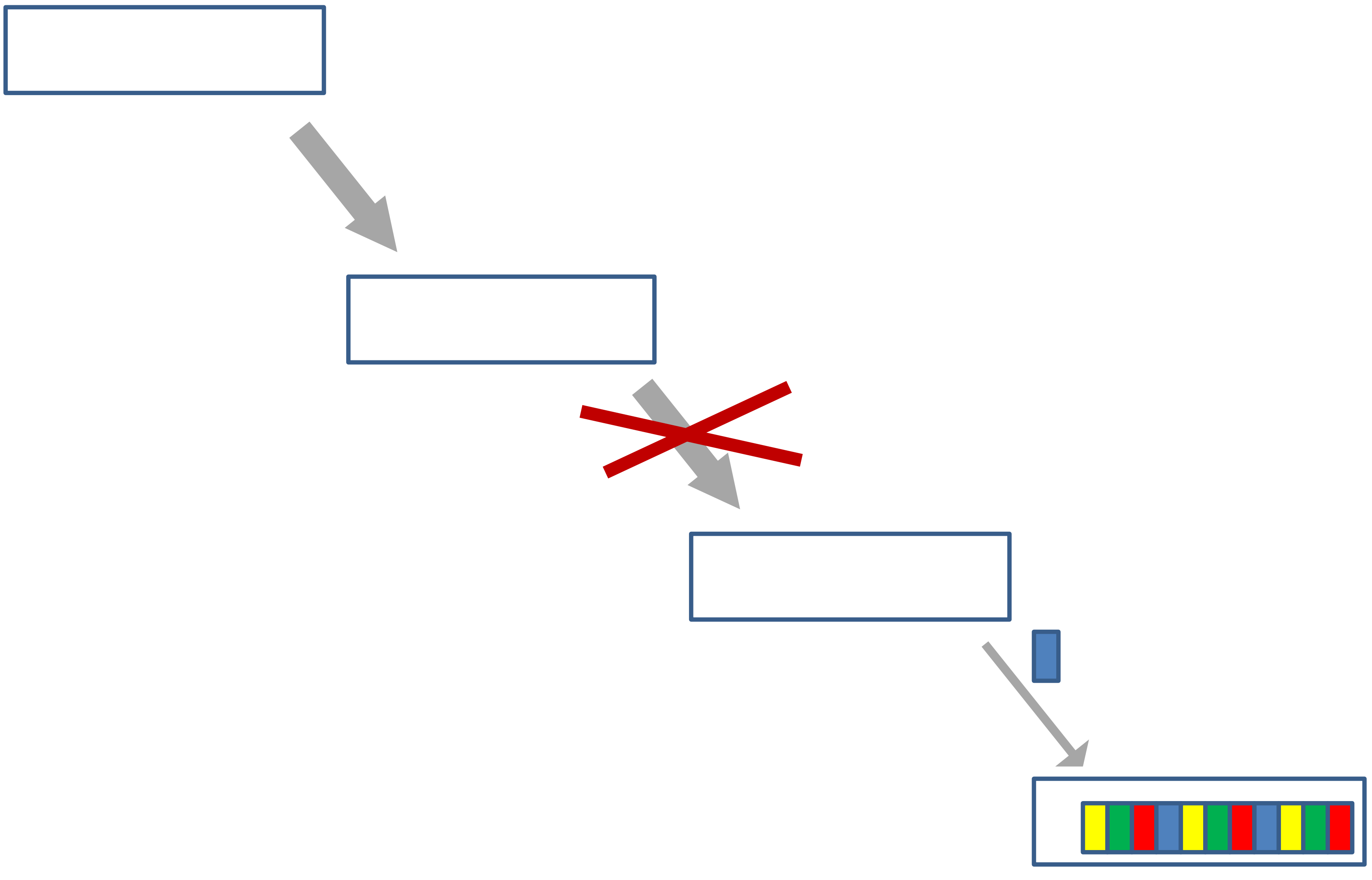


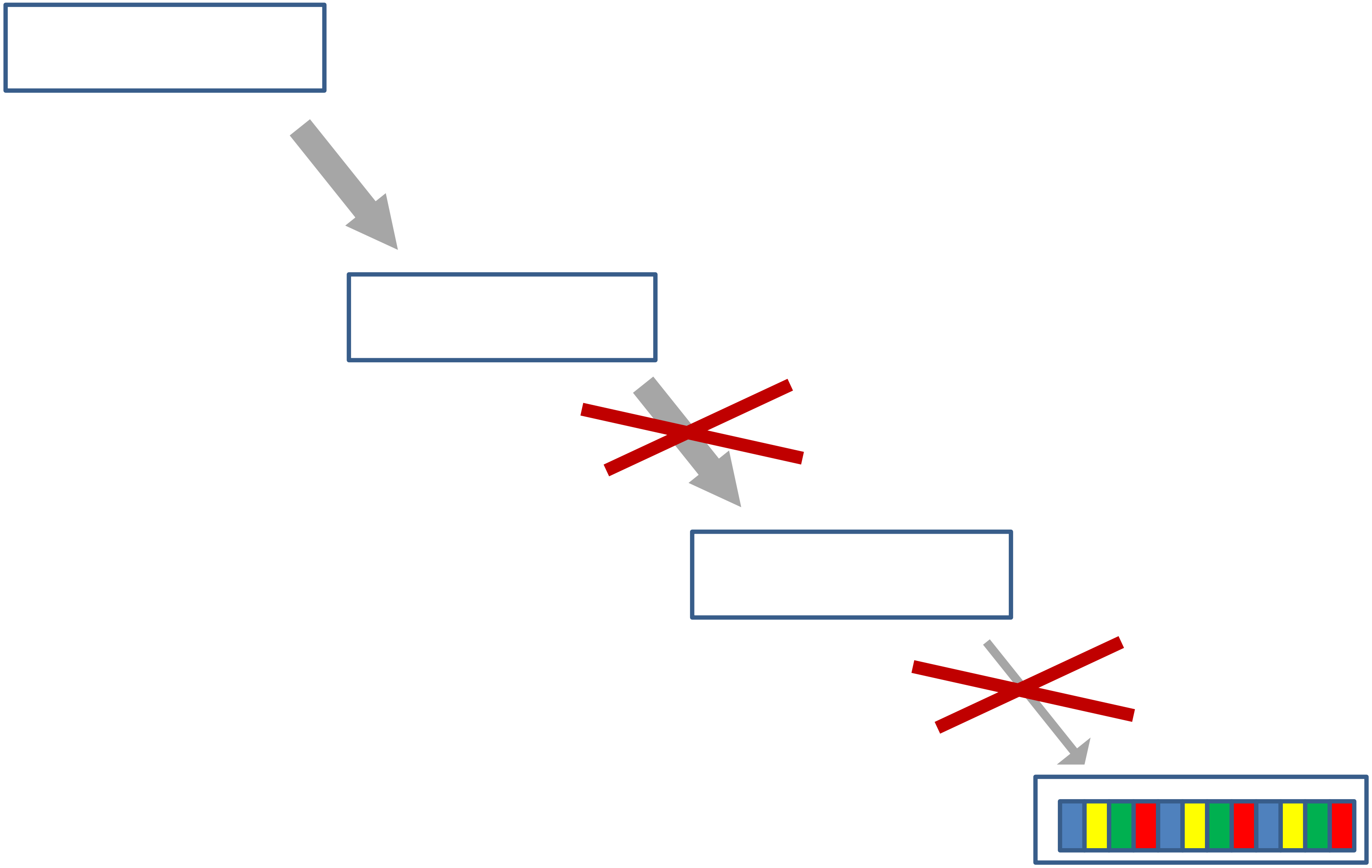


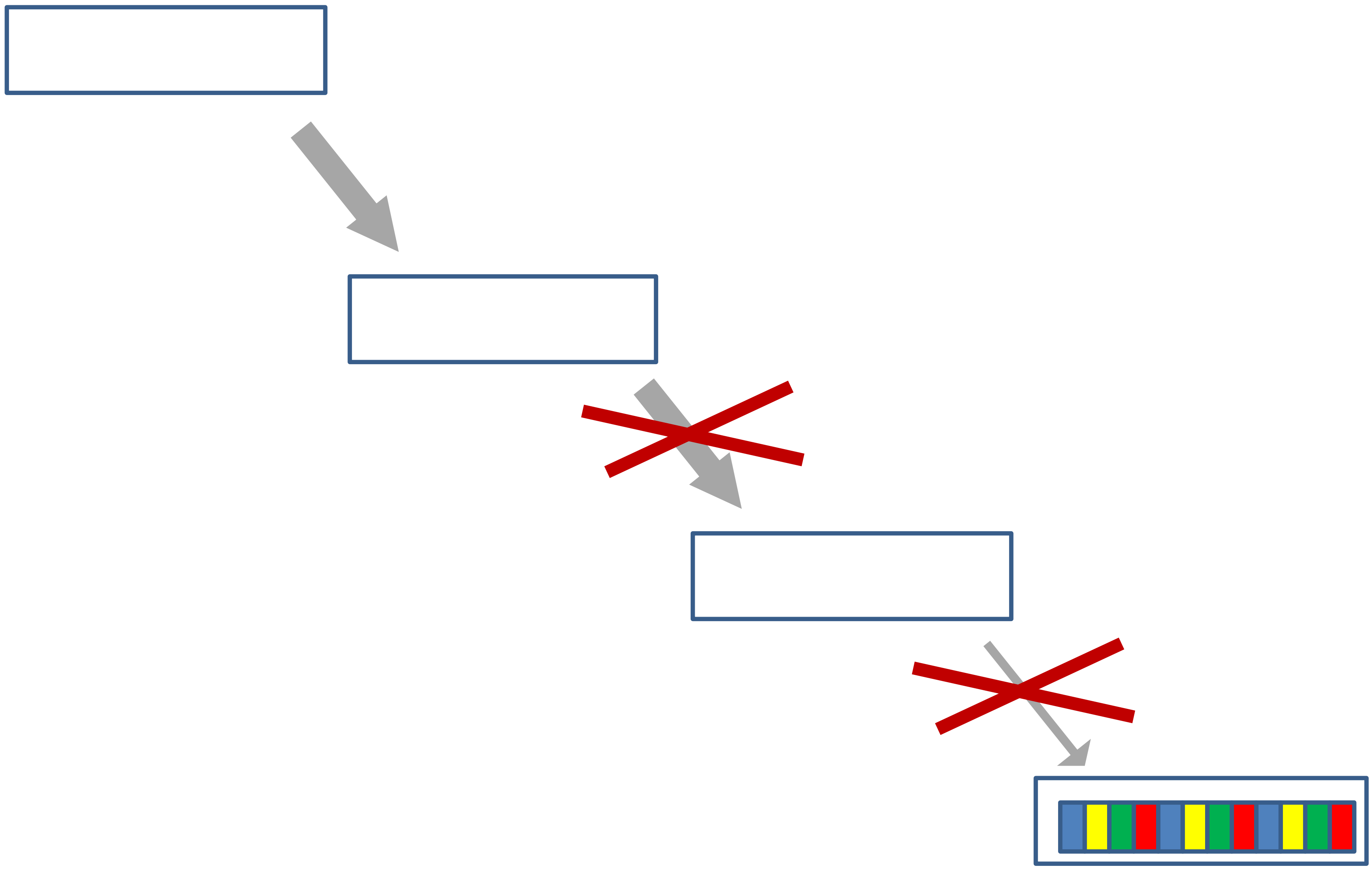












Observations

- Transmission time over a **single link** is quite well understood [Jelenkovic-Tan ’08, Nair et al. ’10 & ’16, Ginzboorg et al. ’11, …]
- We **cannot** just **sum up** the transmission times over individual links…why?
- There tends to be a **queue** ahead of the bottleneck…why?

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  - Add estimate for the whole message over the single bottleneck link

Logic (cont'd)

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- Now there is certainly a queue all the time ahead of the bottleneck…why?
- But how does the rest of the chain look like when the **last** block has finally passed the bottleneck?…..probably looks fairly empty…
- Let us assume that, indeed, the last block does **not** have to **wait** in any queue…
- We get the same estimate…why?

Logic (cont'd)

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- Then we can combine both tricks:
  - Count time for the **first** block to reach the bottleneck
  - Add time for the **whole** message to pass the bottleneck
  - Add time for the **last** block to pass the rest of the chain (assuming it is empty)

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  - Count time for the **first** block to reach the bottleneck
  - Add time for the **whole** message to pass the bottleneck
  - Add time for the **last** block to pass the rest of the chain (assuming it is empty)
  - **Same result again!**

Approximation

- T = (expected) transmission time of whole message (of k blocks) over whole chain
- $T_o^{(1)}$ = (expected) transmission time of one block over an ordinary link
- $T_b^{(k)}$ = (expected) transmission time of k blocks over a bottleneck link

$$\mathbf{T = (n-1)\, T_o^{(1)} + T_b^{(k)}}$$

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If the ordinary links are different then the first term to be replaced by a sum

Formulas

- We have derived closed formulas for this approximation (see paper)
- Special attention devoted for
  - Uniform distribution
  - Exponential distribution

Variance

- We have also an approximation for variance of transmission time
- Logic:
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Variance

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  - Let bottleneck be the first link
  - If passing bottleneck takes long then the last block goes fast over the rest of the chain
  - If passing bottleneck goes faster then the progress of last block is slowed down by preceding blocks

Variance

- Conclusion: It makes sense to use the bottleneck variance as an approximation for the transmission time over the whole chain

$$\mathbf{V = V_b}$$

Simulations

- We have tested our approximations by simulations
  - Uniform distribution
  - Exponential distribution
- Different number of links
- Different block lengths
- Severity of bottleneck varies also

Results 1/2

		exponentially distributed disruptions							
nr. of links n:		1		3		5		8	
$E(Y)$/(s)	f/(s)	ϵ_1	ϵ_2	ϵ_1	ϵ_2	ϵ_1	ϵ_2	ϵ_1	ϵ_2
1	0.01	-2.9	-2.2	-20.1	-13.1	-38.0	-19.1	-45.2	-24.9
	0.1	-2.8	-1.9	-28.2	-14.7	-37.3	-20.3	-43.9	-25.9
	1	-1.7	1.5	-25.8	-29.4	-31.4	-35.8	-33.9	-42.1
2	0.01	-4.9	-2.6	-8.9	5.7	-25.5	9.3	-32.4	11.3
	0.1	-4.9	-2.2	-18.0	5.0	-25.1	8.5	-31.6	10.7
	1	-3.1	-0.6	-16.4	-7.5	-21.6	-7.5	-25.1	-10.0
4	0.01	-7.2	-2.8	-5.4	1.9	-17.0	4.8	-21.4	8.7
	0.1	-6.8	-2.4	-12.7	1.8	-16.5	4.9	-20.7	8.9
	1	-4.4	-1.0	-10.1	-2.5	-13.2	0.1	-16.0	3.0
8	0.01	-8.9	-3.0	-4.6	-0.7	-13.3	0.0	-19.5	1.8
	0.1	-8.5	-2.7	-11.0	-1.1	-12.8	0.2	-14.9	2.0
	1	-5.3	-0.9	-7.6	-2.2	-9.0	-0.8	-10.2	1.0
16	0.01	-9.7	-2.8	-4.3	-1.9	-11.9	-1.8	-12.9	-1.1
	0.1	-9.4	-2.5	-10.7	-2.0	-11.5	-1.5	-12.4	-0.8
	1	-6.0	-1.2	-6.9	-1.9	-7.4	-1.3	-7.9	-0.6

Results 2/2

		uniformly distributed disruptions							
nr. of links n:		1		3		5		8	
$E(Y)$/(s)	f/(s)	ϵ_1	ϵ_2	ϵ_1	ϵ_2	ϵ_1	ϵ_2	ϵ_1	ϵ_2
1	0.01	-1.0	-2.7	-28.4	-4.7	-28.5	-8.3	-35.5	-12.9
	0.1	-0.8	-2.4	-19.9	-7.4	-27.9	-11.2	-34.5	-16.0
	1	2.7	0.2	-16.5	-30.4	-22.4	-36.3	-26.0	-42.2
2	0.01	-1.8	-2.9	-18.0	5.1	-14.2	10.8	-19.9	16.1
	0.1	-1.5	-2.6	-8.7	4.1	-13.7	9.3	-19.2	14.0
	1	2.6	0.1	-4.3	-11.4	-8.8	-11.2	-12.9	-12.8
4	0.01	-2.5	-2.8	-12.7	-0.7	-7.8	1.3	-10.5	3.8
	0.1	-2.1	-2.7	-5.1	-0.9	-7.2	1.0	-9.8	3.3
	1	2.5	0.1	0.8	-7.2	-0.7	-6.4	-2.6	-5.4
8	0.01	-3.3	-2.9	-11.2	-2.0	-5.7	-1.4	-6.9	-0.6
	0.1	-2.8	-2.6	-4.2	-2.1	-5.1	-1.4	-6.3	-0.7
	1	2.3	0.3	2.0	-4.0	1.6	-3.8	1.0	-3.8
16	0.01	-3.6	-3.1	-10.9	-2.5	-4.9	-2.2	-5.6	-2.0
	0.1	-3.1	-2.5	-3.9	-2.6	-4.3	-2.2	-4.9	-2.0
	1	2.2	0.3	2.1	-2.0	2.1	-1.9	1.9	-2.1

Thanks!