# Two-Way Full-Duplex MIMO with Hybrid TX-RX MSE Minimization and Interference Cancellation



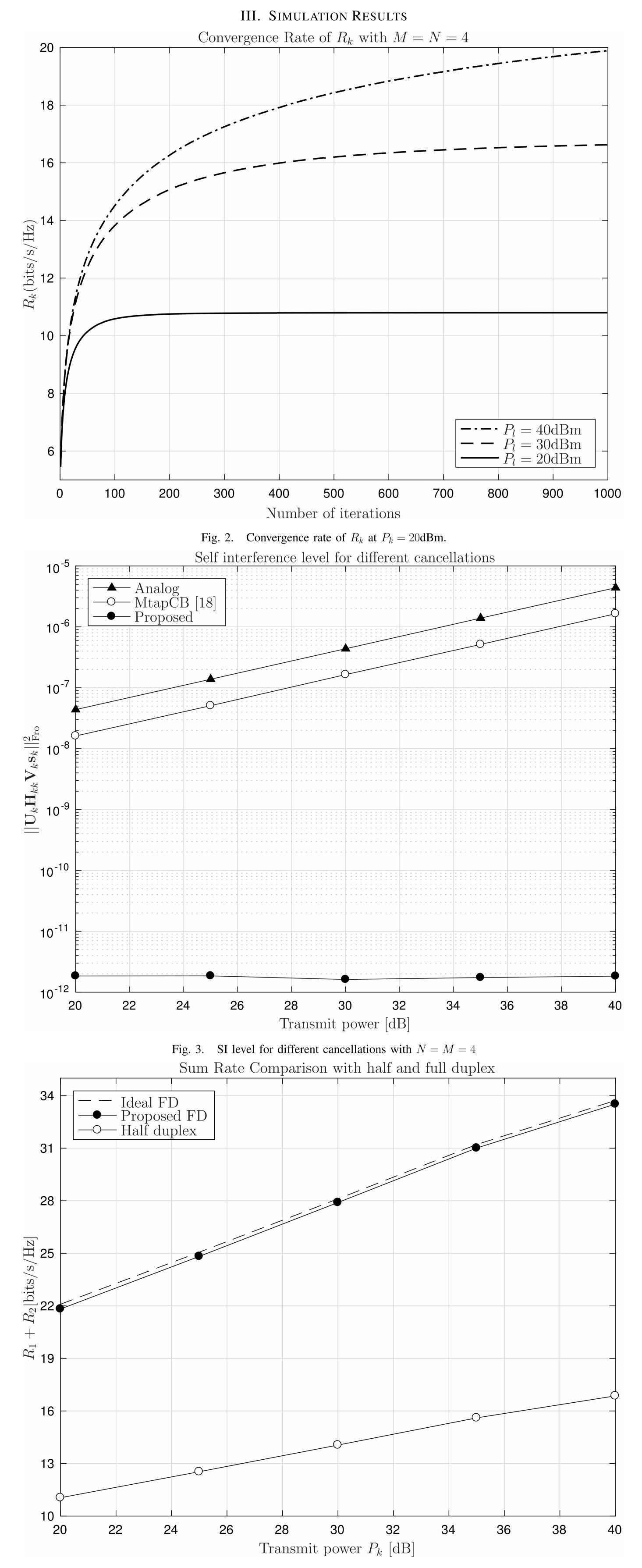
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*Abstract* - We consider a two-way full-duplex (FD) multiple-input multiple-output (MIMO) communication system in which devices are equipped both with multi-tap analog interference cancellers and TX-RX beamforming capabilities, and propose a joint analog and digital algorithm to simultaneously maximize the rate and minimize the self-interference (SI) in such a system. Simulation results demonstrate that the proposed scheme is capable of suppressing residual SI down to background noise levels typical of wireless systems, significantly outperforming similar methods previously proposed.

## I. SYSTEM MODEL

Consider the model illustrated in Figure 1, of a FD MIMO communication system in which two nodes with M transmit and N receive antennas, respectively, transmit and receive simultaneously in the same frequency



such that the signals of interest at both nodes are jammed by the SI signal from their own transmit signals.

#### Signal Model with TX-RX Beamforming

The signal arriving at the k-th node,  $k \in \{1,2\}$ , after multi-tap analog SI cancellation and the corresponding signal vector estimate  $\tilde{s}_k \in \mathbb{C}^{d \times 1}$  are respectively given by  $y_k = H_{\ell k} V_{\ell} s_{\ell} + (H_{kk} - C_k) V_k s_k + n_k(1)$  $= \underbrace{H_{\ell k} V_{\ell} s_{\ell}}_{\text{Intended}} + \underbrace{\tilde{H}_{kk} V_k s_k}_{\text{SI}} + \underbrace{n_k}_{\text{Noise}}, (2)$  $= U_k H_{\ell k} V_{\ell} s_{\ell} + \underbrace{U_k \tilde{H}_{kk} V_k s_k + U_k n_k}_{\text{SI}}, (2)$ where  $v_k$  is the interference-plus-noise component at the k-th node,  $C_k \in \mathbb{C}^{N \times M}$ denotes the analog cancellation matrix [1], [2] and  $\tilde{H}_{kk} = H_{kk} - C_k.$ 

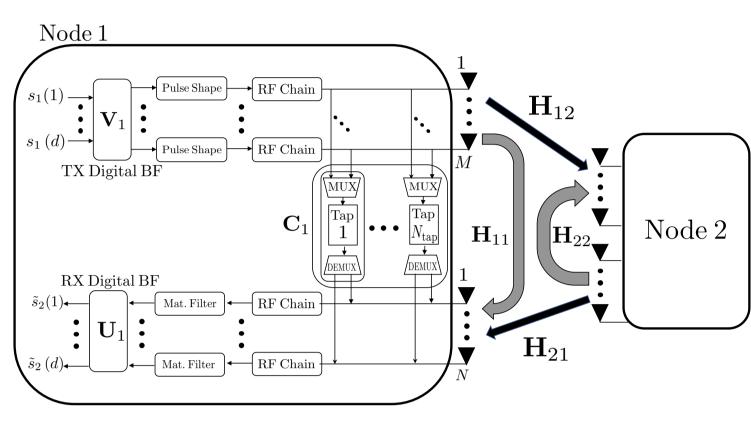


Fig. 1. System model of two-way full duplex MIMO with limited multi-tap analog cancellation.

From equations (2), it follows that the achievable rates in bits/sec/Hz at the k-th node are given as follows.

$$\boldsymbol{R}_{k} = \log \det \left( \boldsymbol{I}_{d} + \boldsymbol{U}_{k} \boldsymbol{H}_{\ell k} \boldsymbol{V}_{\ell} \boldsymbol{V}_{\ell}^{\mathrm{H}} \boldsymbol{H}_{\ell k}^{\mathrm{H}} \boldsymbol{U}_{k}^{\mathrm{H}} \boldsymbol{\Sigma}_{k}^{-1} \right),$$
(3)

where  $\Sigma_k$  is the covariance matrix of  $v_k$  given by

$$\boldsymbol{\Sigma}_{k} = \boldsymbol{U}_{k} \tilde{\boldsymbol{H}}_{kk} \boldsymbol{V}_{k} \boldsymbol{V}_{k}^{\mathrm{H}} \tilde{\boldsymbol{H}}_{kk}^{\mathrm{H}} \boldsymbol{U}_{k}^{\mathrm{H}} + \sigma^{2} \boldsymbol{U}_{k} \boldsymbol{U}_{k}^{\mathrm{H}}.$$
(4)

### **Decomposition of** $V_k$

Because of the fact that the capability of

analog SI cancellation is limited by the number

of taps in  $C_k$ , it was proposed in [1] to

split the transmit beamforming matrix into

LMMSE RX Beamforming Matrix  $U_k$ 

The linear MMSE RX beamformer is adopted in this article based on the discussion in [3] about the optimality of LMMSE in interference MIMO channels, which is given by

 $V_k = F_k G_k, \forall k \in \{1,2\}$ , such that the  $F_k$ component of  $V_k$  can be focused also on SI suppression, leaving the component  $G_k \in \mathbb{C}^{\alpha \times d}$ to be designed with aim at maximizing the downlink rate.

# $\boldsymbol{U}_{k} = \boldsymbol{G}_{\ell}^{\mathrm{H}} \boldsymbol{F}_{\ell}^{\mathrm{H}} \boldsymbol{H}_{\ell k}^{\mathrm{H}} \left( \hat{\boldsymbol{\Sigma}}_{k} + \boldsymbol{H}_{\ell k} \boldsymbol{F}_{\ell} \boldsymbol{G}_{\ell} \boldsymbol{G}_{\ell}^{\mathrm{H}} \boldsymbol{F}_{\ell}^{\mathrm{H}} \boldsymbol{H}_{\ell k}^{\mathrm{H}} \right),$ (5) with $\hat{\boldsymbol{\Sigma}}_{k} = \tilde{\boldsymbol{H}}_{k k} \boldsymbol{F}_{k} \boldsymbol{G}_{k} \boldsymbol{G}_{k}^{\mathrm{H}} \boldsymbol{F}_{k}^{\mathrm{H}} \tilde{\boldsymbol{H}}_{k k}^{\mathrm{H}} + \sigma^{2} \boldsymbol{I},$ (6)

(8)

## II. PROPOSED MMSE TX-RX BEAMFORMING DESIGN

For the two-way FD MIMO system with multi-tap analog canceller described above, consider the following minimum mean square error (MMSE) minimization problem subject to transmit power constraint,

$$\min_{\alpha_k, \mathbf{G}_k} \sum_{k=1}^{2} \operatorname{Tr} \{ \boldsymbol{\epsilon}_k \}$$
s.t.  $\operatorname{Tr} \{ \boldsymbol{G}_k^{\mathrm{H}} \boldsymbol{G}_k \} = P_k.$ 
(7a)
(7b)

where  $\epsilon_k$  is MSE vector of the intended signals given by

$$\boldsymbol{\epsilon}_{k} = \mathbb{E}\left[\left(\boldsymbol{s}_{\ell} - \alpha_{k}^{-1} \tilde{\boldsymbol{s}}_{\ell}\right) \left(\boldsymbol{s}_{\ell} - \alpha_{k}^{-1} \tilde{\boldsymbol{s}}_{\ell}\right)^{\mathrm{H}}\right].$$

### Lagrange Multipliers and the Karush-Kuhn-Tucker conditions

From equation (7a), the Lagrangean of the problem is

$$\mathcal{L}(\{\boldsymbol{G}_{k}, \alpha_{k}, \beta_{k}\}) = \underbrace{\sum_{k=1}^{2} \operatorname{Tr}\{\boldsymbol{\epsilon}_{k}\} - \beta_{k}}_{\text{Objective}} \underbrace{\left(\operatorname{Tr}\{\boldsymbol{G}_{k}^{\mathrm{H}}\boldsymbol{G}_{k}\} - P_{k}\right)}_{\text{Constraint}}, \tag{9}$$

where  $\beta_k$  is the Lagrange multiplier and its KKT conditions are given by [4]

$$\frac{\partial \mathcal{L}\left(\{\boldsymbol{G}_{k}, \alpha_{k}, \beta_{k}\}\right)}{\partial \alpha_{k}} = 0, \ \operatorname{Tr}\left\{\boldsymbol{G}_{k}^{\mathrm{H}} \frac{\partial \mathcal{L}\left(\{\boldsymbol{G}_{k}, \alpha_{k}, \beta_{k}\}\right)}{\partial \boldsymbol{G}_{k}^{*}}\right\} = 0, \ \frac{\partial \mathcal{L}\left(\{\boldsymbol{G}_{k}, \alpha_{k}, \beta_{k}\}\right)}{\partial \boldsymbol{G}_{k}^{*}} = 0$$

Consequently, the optimized scaling parameters  $\alpha_k, \beta_k$  and RX beamformer  $G_k$  are respectively obtained by

Optimized scaling parameters 
$$\alpha_k, \beta_k$$
 and RX beamformer  $G_k$   

$$\alpha_k = 2 \frac{\text{Tr} \left\{ U_k H_{\ell k} F_\ell G_\ell G_\ell^H F_\ell^H H_{\ell k}^H U_k^H + U_k \tilde{H}_{k k} F_k G_k G_k^H F_k^H \tilde{H}_{k k}^H U_k^H + \sigma^2 U_k U_k^H \right\}}{\text{Tr} \left\{ G_\ell^H F_\ell^H H_{\ell k}^H U_k^H + U_k H_{\ell k} F_\ell G_\ell \right\}}, \quad (10)$$

$$\beta_k = \frac{\text{Tr} \left\{ \alpha_k^{-2} U_k \tilde{H}_{k k} F_k G_k G_k^H F_k^H \tilde{H}_{k k}^H U_k^H + \alpha_\ell^{-2} U_\ell H_{k \ell} F_k G_k G_k^H F_k^H H_{k \ell}^H U_\ell^H - \alpha_\ell^{-1} G_k^H F_k^H H_{k \ell}^H U_\ell^H \right\}}{P_k}, \quad (11)$$

$$G_k = \left( \frac{F_k^H \tilde{H}_{k k}^H U_k^H U_k \tilde{H}_{k k} F_k}{\alpha_k^2} + \frac{F_k^H H_{k \ell}^H U_\ell^H U_\ell H_{\ell \ell} H_k \ell F_k}{\alpha_\ell^2} - \beta_k I \right)^{-1} \cdot \frac{F_k^H H_{k \ell}^H U_\ell^H}{\alpha_\ell}. \quad (12)$$

Algorithm 1 Joint design of TX-RX beamforming.

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Input:
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•  $P_k, \boldsymbol{H}_{kk}, \boldsymbol{H}_{\ell k}, \forall k$ 

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• C_k, \forall k given by [1]
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#### **Output:**

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• The optimized beamformers F_k, G_k, U_k, \forall k
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#### Steps:

1. Set  $\mathbf{F}_k$ ,  $\forall k$  to be the right singular vectors of  $\tilde{\mathbf{H}}_{kk}$  corresponding to the minimum  $\frac{N_{\text{tap}}}{N}$  singular values. 2. Make an arbitrary initial TX-precoding matrices  $\mathbf{G}_k$ ,  $\forall k$  and do the following iterations until convergence **repeat** 

## - Compute $U_k$ , $\forall k$ by (5) for a given $G_k$ , $F_k$ , $\forall k$

- Find an optimal  $G_k$ ,  $\forall k$  for fixed  $U_k$ ,  $F_k$ ,  $\forall k$  by equation (12)

until convergence

Fig. 4. Sum rate comparison for different transmit power with M = N = 4 for a fixed  $P_{\ell} = 20$  dBm

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