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Underwater Trajectory Estimation based on Grid Localization and Smoothing

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Introduction

- GPS fix: AUV obtain a GPS baseline at surface. It is often impractical.
- Acoustic beacon networks: triangulation. The cost of infrastructure is considered high.

We propose a trajectory estimation technique based on grid map localization and data smoothing approach. In the localization the receiver estimates the Channel State Information (CSI) and compares it with a CSI pre-computed on a grid of points covering the area of interest; the best match indicates the estimated location. For a dynamic receiver moving in the area of interest, with estimated locations of sample points along the receiver trajectory, smoothing approach based on P-splines is applied to recover the trajectory.

Grid Map Localization

To recover the receiver trajectory, the locations of the receiver at sample points are estimated via the grid map localization [1]. The algorithm compares the “signature” of the receiver location and that of the grid points to find the best match, which gives receiver location.

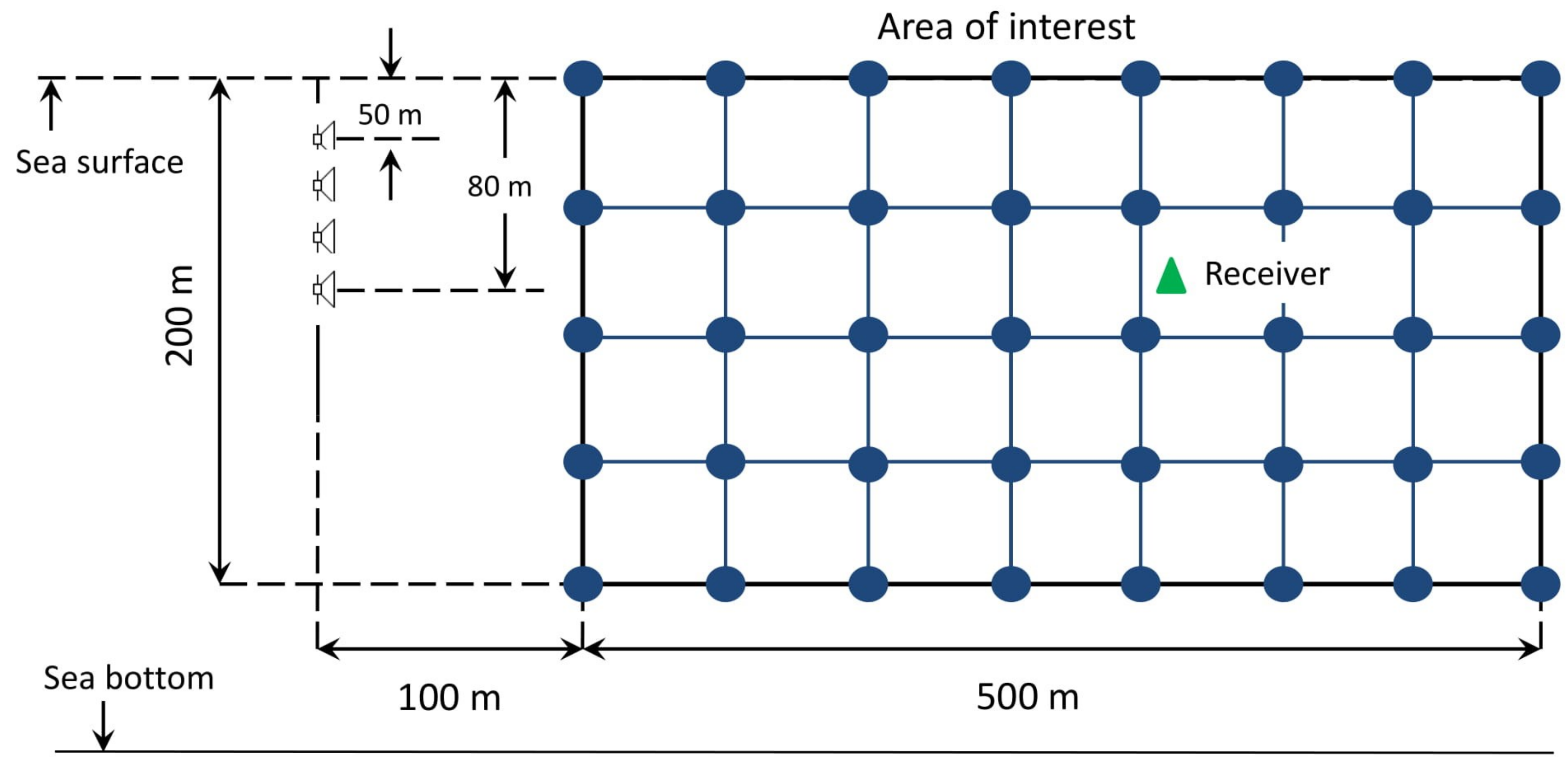

Fig. 1: Example scenario: the comparison of the receiver and the grid points in a grid map covering the area of interest.

- $\mathbf{g}_m$ is the CSI of the grid point, pre-computed by the ray-tracing program.
- △ $\hat{\mathbf{h}}$ is the CSI measured using a pilot signal transmitted from the array of transducers.

For the mth grid point of a map with M grid points, $\hat{\mathbf{h}}$ and $\mathbf{g}_m$ are compared to obtain a match level, and the estimate of the location is achieved by finding the maximum match level as

$$m_{\text{best}} = \arg \max_{m=1,\ldots M} c_m \;, \quad c_m = \frac{|\mathbf{g}^H \hat{\mathbf{h}}|}{\|g_m\|_2^2 \|\hat{h}\|_2^2} \;, \tag{1}$$

where c_m is also considered as a match level, of $\hat{\mathbf{h}}$ and $\mathbf{g}_m$.
To overcome the unknown delay between $\mathbf{g}_m$ and $\hat{\mathbf{h}}$, c_m is updated as

$$c_m = \sum_{t=1}^{N_T} \frac{\max_{\tau \in [\tau_{\min}, \tau_{\max}]} \left|\mathbf{g}_{t,m}^H \Lambda_\tau \hat{\mathbf{h}}_t\right|^2}{\|\mathbf{g}_{t,m}\|_2^2 \|\hat{\mathbf{h}}_t\|_2^2} \quad , \quad \Lambda_\tau = \operatorname{diag}\left[e^{-j2\pi f_0 \tau}, \ldots, e^{-j2\pi f_{K-1} \tau}\right], \tag{2}$$

Trajectory Estimation

After receiver localization, a set of location estimates and the corresponding match levels are obtained, which are used in the trajectory smoothing.
A curve $\hat{y}$ fitted to data (x, y) is given by $\hat{y}(x) = \sum_{j=1}^{n} a_j B_j(x)$. Let $B_j(x_i)$ be the value at the x_ith sample point of the jth B-spline. The objective function to minimize is given by:

$$S = \sum_{i=1}^{X} w_i \left[y_i - \sum_{j=1}^{n} a_j B_j(x_i)\right]^2 + \lambda \sum_{j=k+1}^{n} \left(\Delta^k a_j\right)^2, \tag{3}$$

$\Delta a_j = a_j - a_{j-1}$, $\Delta^2 a_j = \Delta(\Delta a_j) = a_j - 2a_{j-1} + a_{j-2}$ and in general $\Delta^k a_j = \Delta(\Delta^{k-1} a_j)$. λ is a regularization parameter that controls the smoothness of the fit, and w_i is the weight coefficient for (x_i, y_i), the estimated location at the ith sample point.

The system of equations with respect to $\mathbf{a}$ that follows from the minimization of S in (3) can be written in a matrix form as

$$\left(\mathbf{B}^T \mathbf{W} \mathbf{B} + \lambda \mathbf{D}_k^T \mathbf{D}_k\right) \mathbf{a} = \mathbf{B}^T \mathbf{W} \mathbf{y}, \tag{4}$$

where $\mathbf{D}_k$ is the matrix representation of the difference operation Δ^k, elements of $\mathbf{B}$ are $b_{ij} = B_j(x_i)$, and $\mathbf{W}$ is a diagonal matrix of weights w_i.

The match level c_m at the ith sample point is used as the weight coefficient w_i. λ can be set according to a max acceleration level.

Numerical Results

The area of interest is shown in Fig.1. The sound speed profile (SSP) and sea bottom parameters are shown in Fig. 2. The sea surface is assumed to be flat. The sea depth in the area of interest is 220 m. The transducers emit acoustic signals in the interval of vertical angles [−50◦, +50◦]. The receiver is equipped with a single receive antenna.
The pilot signal for channel estimation is transmitted at the carrier frequency 3072 Hz with a frequency bandwidth of 1024 Hz.

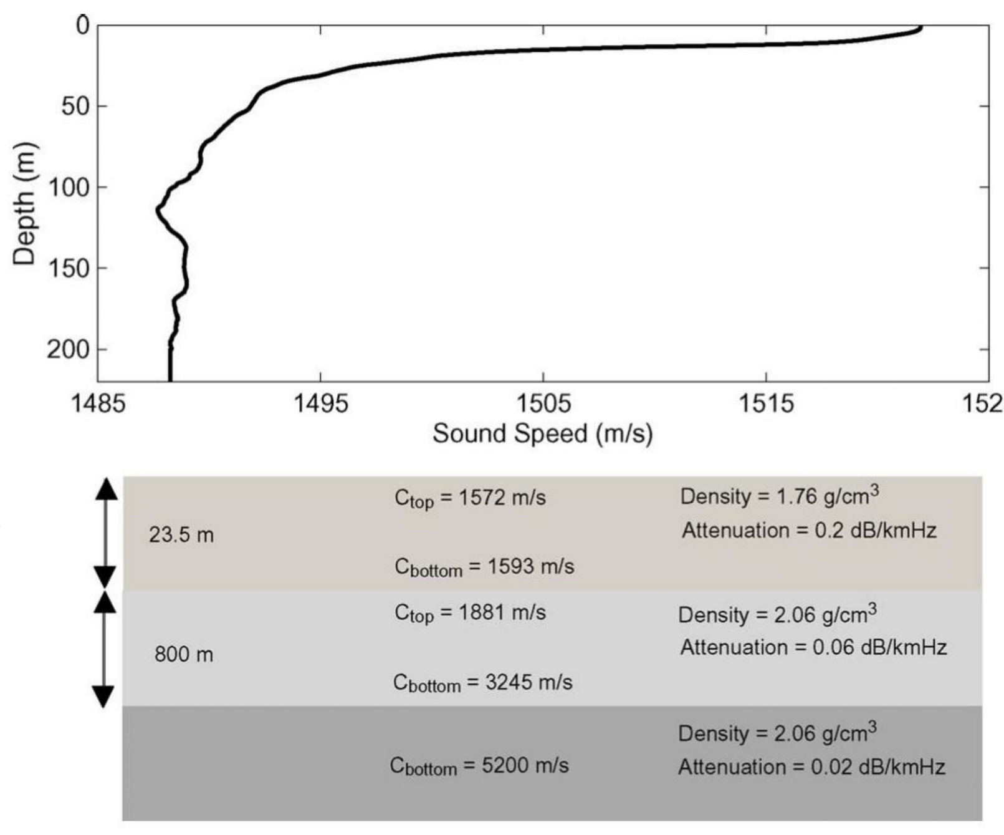

Fig. 2: The SSP and the layered sea bottom parameters.

The pilot transmission is performed using OFDM signals with 1024 subcarriers, an orthogonality interval of 1 s, and subcarrier spacing of 1 Hz.

Experiment 1:

- The receiver is moving at a constant speed of 0.5 m/s away from the transmitter.
- The true trajectory of the receiver is a sinusoid curve in space as in Fig. 3.
- A maximum acceleration is assumed to be 1 m/s^2.
- The depth difference between the true trajectory and its estimate is less than 2.5 m.

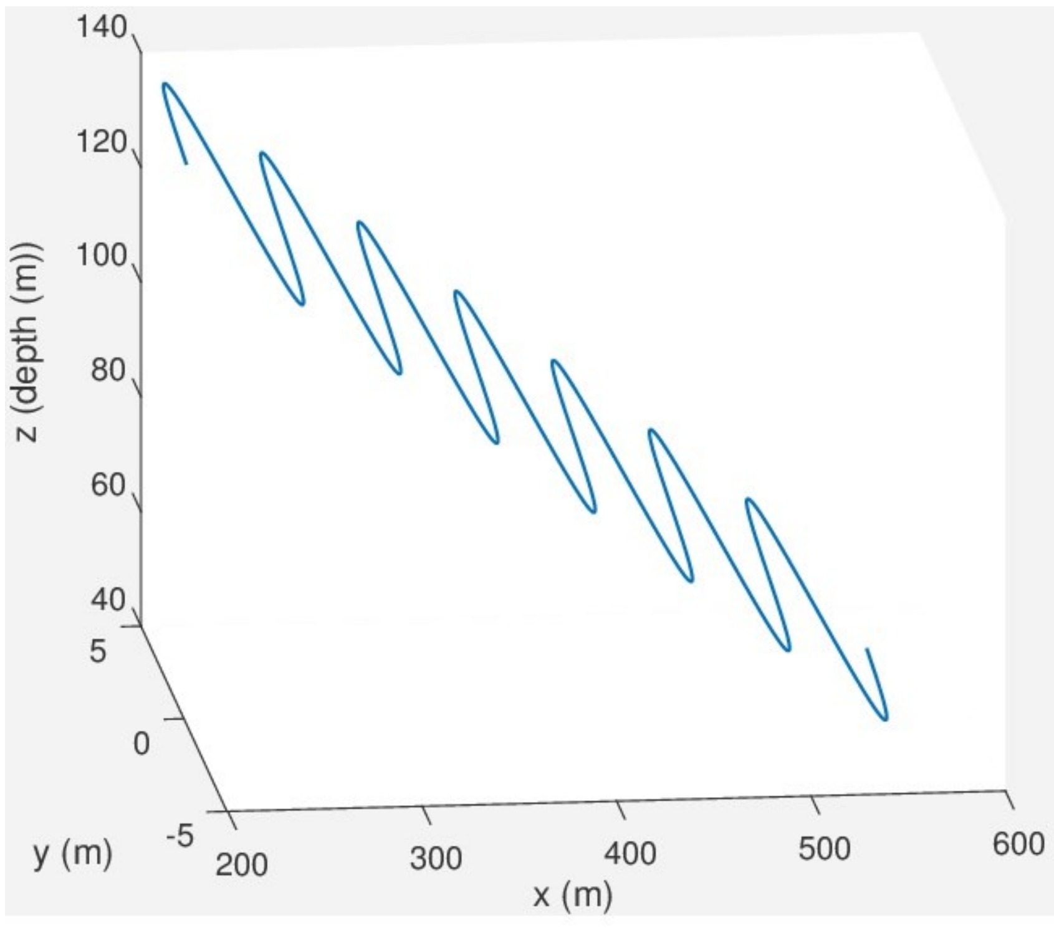

Fig. 3: The trajectory of the receiver, in 3D.

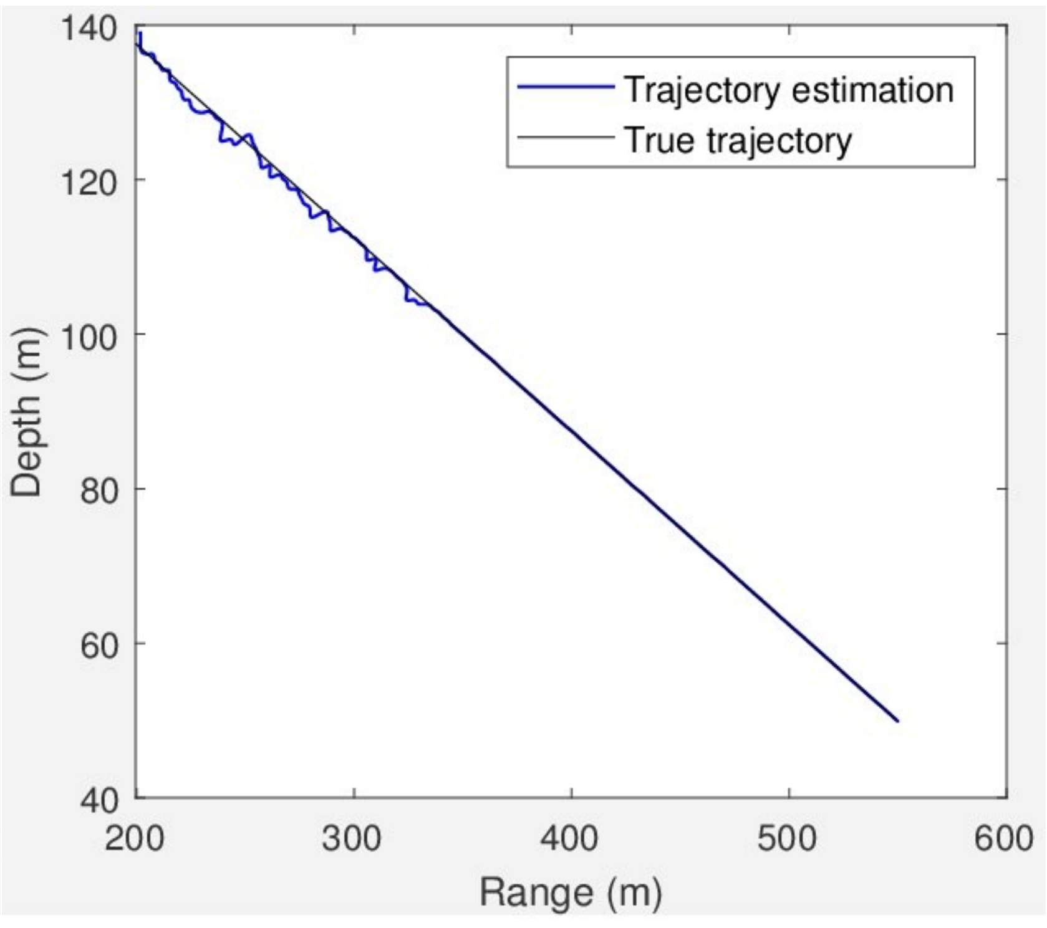

Fig. 4: The estimated trajectory and the true trajectory of the receiver, represented by the relative range to the transmitter and the depth of the receiver.

Experiment 2:

- The receiver is moving at a constant speed of 1 m/s in a sinusoid curve in the range-depth plane.
- The sampling rate is 1 Hz.
- The estimated and true trajectories are shown in Fig. 5.
- The maximum distance between the estimated trajectory and the true trajectory is less than 0.3 m.

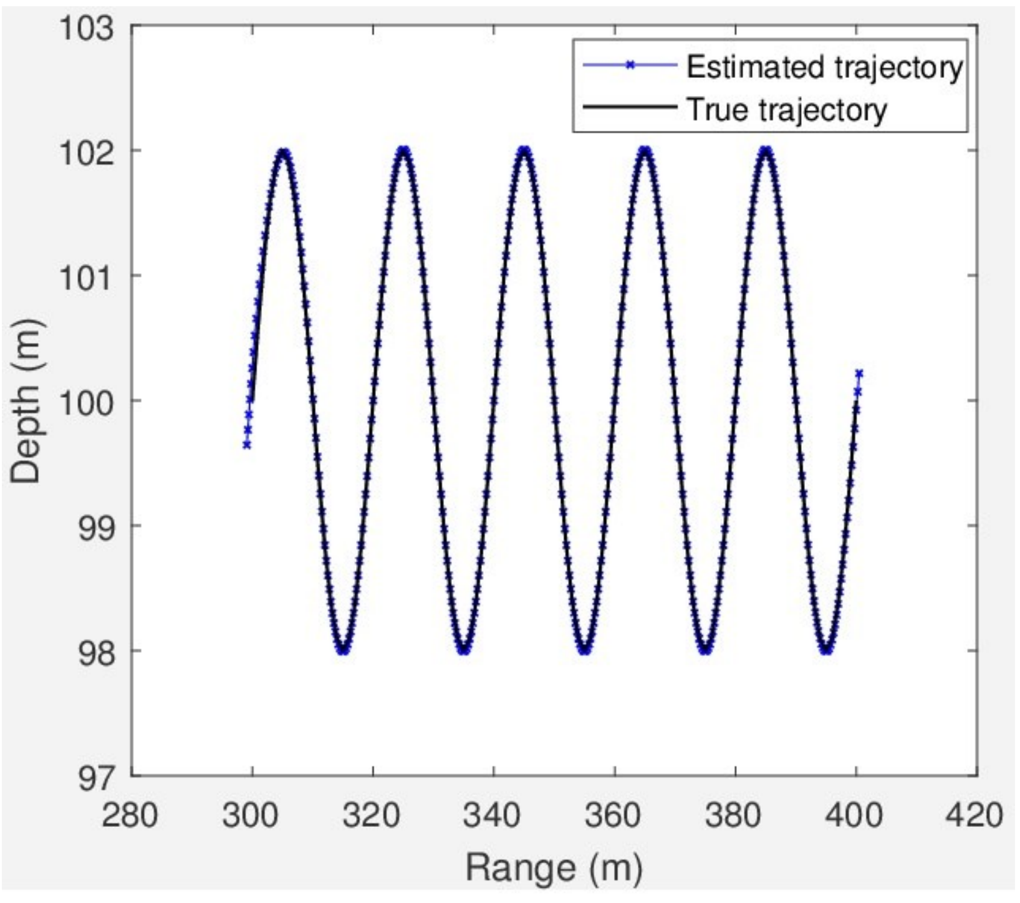

Fig. 5: The estimated trajectory and the true trajectory of the receiver moving in a sinusoid curve in the range-depth plane.

Reference

[1] L. Liao, Y. Zakharov, and P. D. Mitchell, “Underwater localization based on grid computation and its application to transmit beamforming in multiuser UWA communications,” IEEE Access, vol. 6, pp. 4297 – 4307, 2018.