Restoration of Multilayered Single-Photon 3D LiDAR Images

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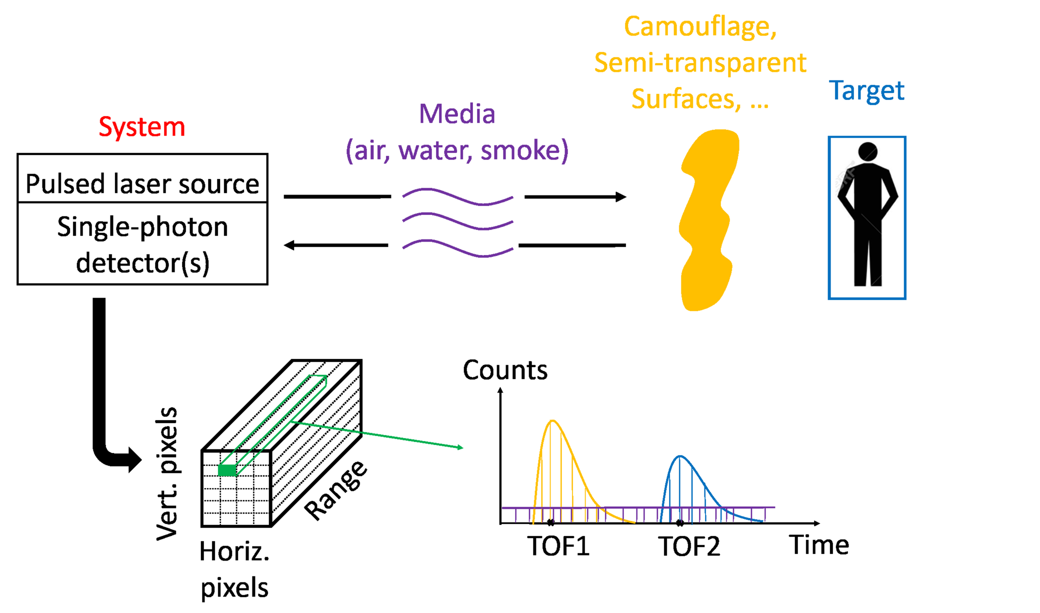
1. Introduction

Problem statement

- 3D imaging using a single-photon Lidar system
- Extreme conditions: reduced acquisition time, long-range, multilayered imaging, presence of obscurants ...
- Use of spatial correlations in the observed scene.



⇒ Objectives :
- Restore the target's returns under extreme conditions
- Propose a fast algorithm suitable for real life applications

2. Observation model

We observe N pixels $\boldsymbol{Y} = (\boldsymbol{y}_1, \cdots, \boldsymbol{y}_N)$, defined at each temporal gate k as follows:

$$y_{n,k} \sim \mathcal{P}\left(s_{n,k}\right)$$

- $\boldsymbol{y}_n$ and $\boldsymbol{s}_n$ are $K \times 1$ vectors representing the nth observed and noiseless pixels for K time bins
- $\mathcal{P}(.)$ denotes the Poisson distribution

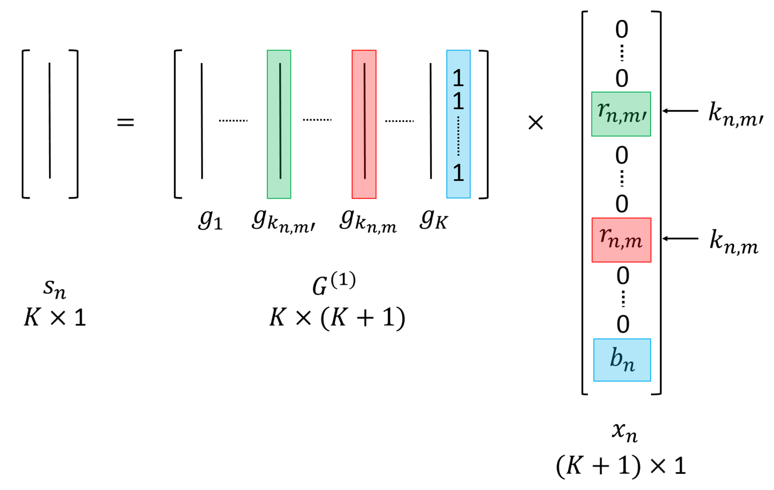
Parametric model

$$s_{n,k} = \sum_{m=1}^{M_n} \left[r_{n,m} g_0 \left(k - k_{n,m}\right)\right] + b_n \qquad (1)$$

- $r_{n,m}$, $k_{n,m}$ are the reflectivity and depth position of the mth object
- b_n is the background noise
- g_0 is the system impulse response
- M_n is the number of objects in the nth pixel

Equivalent formulation

$$\boldsymbol{s}_n = \boldsymbol{G}^{(1)}\, \boldsymbol{x}_n \left(\boldsymbol{r}_n, \boldsymbol{k}_n, b_n\right) \qquad (2)$$



- $g_{k_{n,m}}$ represents g_0 shifted by $k_{n,m}$
- $\boldsymbol{X}$ is a $(K+1) \times N$ matrix containing the parameters of interest

Prior Knowledge/Hypotheses on $\boldsymbol{X}$

- The elements of $\boldsymbol{X}$ are non-negative
- *Hyp. 1:* For local regions, a small number of depths are active with respect to the range window ($M_n << K, \forall n$)
- *Hyp. 2:* The observed objects present spatial correlations

3. Proposed Restoration Approach

Cost function

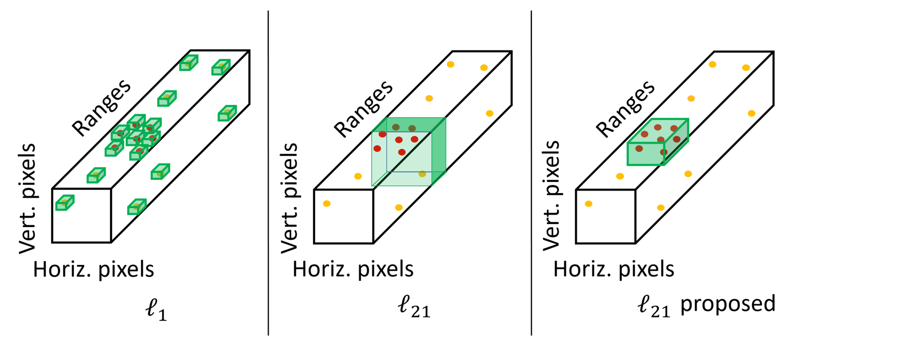
$$\underset{\boldsymbol{X}}{\operatorname{argmin}}\ \mathcal{L}(\boldsymbol{X}) + i_{\mathbb{R}_+}(\boldsymbol{X}) + \tau_1 \phi_{\boldsymbol{v}}(\boldsymbol{X}) + \tau_2 \psi_{\boldsymbol{w}}(\boldsymbol{X})$$

- $\tau_1 > 0, \tau_2 > 0$ are fixed regularization parameters
- $\boldsymbol{v} > 0, \boldsymbol{w} > 0$ are fixed weight vectors
- $i_{\mathbb{R}_+}(\boldsymbol{X})$ is the indicator function that imposes positivity on $\boldsymbol{X}$

Data statistics $\mathcal{L}(\boldsymbol{X})$ (negative log-likelihood)

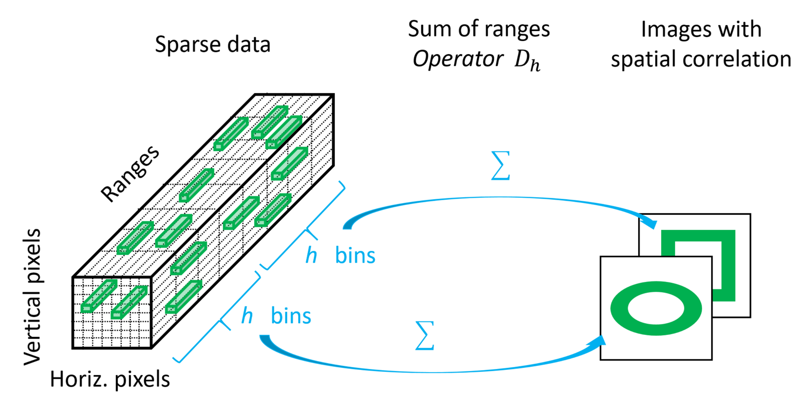
$$\mathcal{L}(\boldsymbol{X}) = \sum_{n=1}^{N} \sum_{k=1}^{K} \left\{ s_{n,k}(\boldsymbol{x}_n) - y_{n,k} \log\left[s_{n,k}(\boldsymbol{x}_n)\right] \right\}$$

Hyp. 1: Depth regularization $\phi_{\boldsymbol{v}}(\boldsymbol{X}) = ||\boldsymbol{K}_{\boldsymbol{v}} \boldsymbol{X}||_{2,1}$



(Left) model in [1], (middle) model in [2], (right) proposed model

Hyp. 2: Intensity regularization $\psi_{\boldsymbol{w}}(\boldsymbol{X}) = ||\boldsymbol{H}_{\boldsymbol{w}} \boldsymbol{D}_h \boldsymbol{X}||_1$



4. Estimation algorithm using ADMM

Our Problem

$$\underset{\boldsymbol{X}}{\operatorname{argmin}}\ \mathcal{H}\left(\boldsymbol{G}^{(1)}\boldsymbol{X}\right) + i_{\mathbb{R}_+}(\boldsymbol{X}) + \tau_1 \phi_{\boldsymbol{v}}(\boldsymbol{X}) + \tau_2 \psi_{\boldsymbol{w}}(\boldsymbol{X})$$

with $\mathcal{L}(\boldsymbol{X}) = \mathcal{H}\left(\boldsymbol{G}^{(1)}\boldsymbol{X}\right)$

Equivalent formulation

$$\underset{\boldsymbol{X},\boldsymbol{V}}{\operatorname{argmin}}\ \mathcal{H}(\boldsymbol{V}_1) + i_{\mathbb{R}_+}(\boldsymbol{V}_2) + \tau_1 ||\boldsymbol{V}_3||_{2,1} + \tau_2 ||\boldsymbol{V}_5||_1$$

subject to $\boldsymbol{AX} + \boldsymbol{BV} = 0$ and

$$\boldsymbol{A} = \begin{bmatrix} \boldsymbol{G}^{(1)} \\ \mathbb{I}_{K+1} \\ \boldsymbol{K}_v \\ \boldsymbol{D}_h \\ 0 \end{bmatrix}, \quad \boldsymbol{B} = \begin{bmatrix} -\mathbb{I} & 0 & 0 & 0 & 0 \\ 0 & -\mathbb{I} & 0 & 0 & 0 \\ 0 & 0 & -\mathbb{I} & 0 & 0 \\ 0 & 0 & 0 & -\mathbb{I} & 0 \\ 0 & 0 & 0 & \boldsymbol{H}_w & -\mathbb{I} \end{bmatrix}.$$

Alternating direction method of multipliers algorithm

Initialize $\boldsymbol{V}_j^{(0)}, \boldsymbol{F}^{(0)}, \forall j, \mu$. Set $i \leftarrow 0$, conv← 0
while conv= 0 **do**
  Linear system of equations
  Update $\boldsymbol{X}^{(i+1)}$ by solving a linear system of equations
  Moreau proximity operators
  Update $\boldsymbol{V}_j^{(i+1)}, \forall j \in \{1, \cdots, 5\}$ by evaluating their analytical proximal operators
  Update Lagrange multipliers
  $\boldsymbol{F}^{(i+1)} \leftarrow \boldsymbol{F}^{(i)} - \boldsymbol{AX}^{(i+1)} - \boldsymbol{BV}^{(i+1)}$
  conv← 1, if the stopping criterion is satisfied.
  $i = i + 1$
end while

where $\boldsymbol{F}$ denotes the Lagrange multipliers

5. Results on synthetic data

Synthetic bowling scene (see Fig. 1-left)

- 123×139 pixels, and 300 time bins
- Interval of averaged signal Photon-Per-Pixel (PPP): [0.2, 5] ppp
- Interval of Signal to Background (SBR) level: [0.05, 1.25]
- Comparison with:
  - Class.: classical cross-correlation algorithm
  - BFC: Class. algorithm applied to background-free data
  - NR3D: Proposed Nonlocal Restoration of 3D images

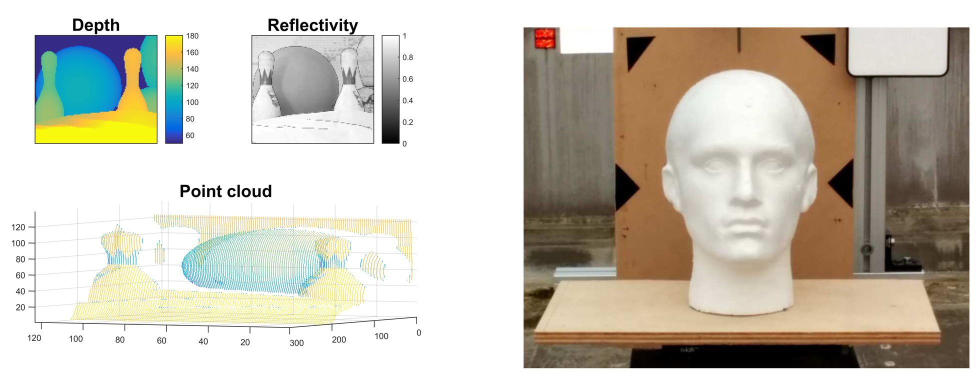


Fig. 1: (Left) Synthetic bowling scene. (Right) Picture of the real target (acquired with 142×142 pixels).

SRE (in dB) results with respect to SBR and PPP

Signal PPP		5	2	0.8	0.4	0.2
SBR		1.25	0.5	0.2	0.1	0.05
Depth	BFC	17.9	8.7	3.6	1.9	1.0
	Class.	11.5	5.3	2.9	2.2	1.9
	NR3D	**19.8**	**14.0**	**11.0**	**7.5**	**5.0**
Reflect.	BFC	7.3	3.5	−0.4	−3.5	−6.9
	Class.	6.6	1.4	−6.5	−13.1	−19.6
	NR3D	**13.3**	**13.0**	**8.4**	**9.8**	**3.2**

6. Results on real data

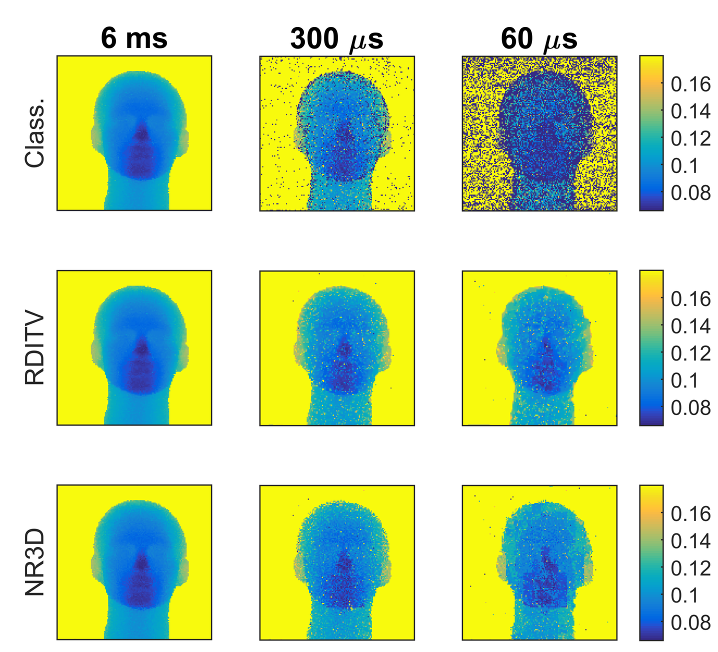


Fig. 2: Depth maps for reduced acquisition times-per-pixel (comparison with RDITV [3])

7. Future work

- Generalize to high-dimensional data (multi-frames, multi-wavelengths)
- Set the weights $\boldsymbol{v}, \boldsymbol{w}$ using other acquisition modalities to perform multi-modal data fusion

References

[1] D. Shin, F. Xu, F. N. C. Wong, J. H. Shapiro, and V. K. Goyal, "Computational multi-depth single-photon imaging," *Opt. Express*, vol. 24, no. 3, pp. 1873–1888, Feb 2016.

[2] A. Halimi, R. Tobin, A. McCarthy, S. McLaughlin, and G. S. Buller, "Restoration of multilayered single-photon 3D lidar images," in *Proc. EUSIPCO*, 2017, pp. 708–712.

[3] A. Halimi, Y. Altmann, A. McCarthy, X. Ren, R. Tobin, G. S. Buller, and S. McLaughlin, "Restoration of intensity and depth images constructed using sparse single-photon data," in *Proc. EUSIPCO*, 2016, pp. 86–90.