Super-Resolution Pulse-Doppler Radar Sensing via One-Bit Sampling

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Motivation

Sampling and ***quantization*** is indispensable for signal acquisition and processing.

- *Classic*: Shannon-Nyquist sampling + high bit quantization (*e.g.*, ADC)
- *Modern*: Compressive sampling + high bit quantization (*e.g.*, AIC or Xampling)
- *Problem*: What is the effect of compressive sampling accompanied by low bit quantization?

Our Aim is

- ★ *Understand the effect of low bit quantization in compressive sampling;*
- ★ *Develop a super-resolution parameter estimation algorithm for radar sensing via low bit quantization.*

One-Bit Sampling Scheme

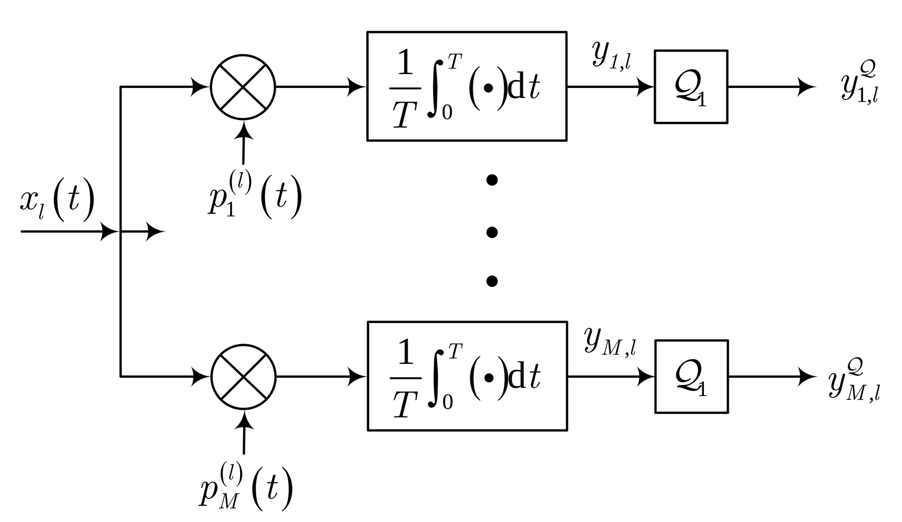

Figure 1: Multichannel one-bit sampling scheme

A multichannel one-bit sampling scheme is considered, where $p_m^{(l)}(t)$ is a weighted sum of complex sinusoids given by

$$p_m^{(l)}(t) = \sum_{n\in\mathcal{N}} p_{m,n}^{(l)} e^{-j2\pi nt/T}.$$

After the modulation and integration, the output of the m-th channel for the l-th pulse can be expressed as

$$\begin{aligned} y_{m,l} &= \frac{1}{T}\int_0^T p_m^{(l)}(t)x_l(t)dt \\ &= \sum_{n\in\mathcal{N}} p_{m,n}^{(l)} c_l[n], \end{aligned}$$

where $c_l[n]$ is the Fourier coefficient of $x_l(t)$:

$$\begin{aligned} c_l[n] &= \frac{1}{T}\int_0^T x_l(t)e^{-j2\pi nt/T}dt \\ &= \frac{1}{T}G(2\pi n/T)\sum_{k=1}^{K}\alpha_k e^{j2\pi\nu_k lT}e^{-j2\pi n\tau_k/T}. \end{aligned}$$

Therefore the one-bit quantization measurement is given as

$$y_{m,l}^{\mathcal{Q}} = \operatorname{sgn}(\Re\{y_{m,l}\}) + i\operatorname{sgn}(\Im\{y_{m,l}\}).$$

Signal Model

A typical pulse-Doppler radar transceiver transmits a pulse train

$$s(t) = \sum_{l=0}^{L-1} g(t-lT), 0 \le t \le LT,$$

where $g(t)$ is a known pulsed signal with bandwidth B. Then the received signal can be written as

$$x(t) = \sum_{l=0}^{L-1} x_l(t-lT),$$

where $x_l(t) = \sum_{k=1}^{K}\alpha_k g(t-\tau_k)e^{j2\pi\nu_k lT}$, α_k, τ_k and ν_k are the complex reflecting coefficient, time-delay and Doppler frequency of the k-th target, respectively.

Problem Formulation

Let $\mathbf{y}_l^{\mathcal{Q}} = [y_{1,l}^{\mathcal{Q}}, \cdots, y_{M,l}^{\mathcal{Q}}]^T$ and $\mathbf{y}_l = [y_{1,l}, \cdots, y_{M,l}]^T$, respectively. Then $\mathbf{y}_l$ can be represented as

$$\mathbf{y}_l = \mathbf{P}_l\mathbf{G}\mathbf{W}(\boldsymbol{\theta}_1)\mathbf{A}\mathbf{V}^H(\boldsymbol{\theta}_2)\mathbf{e}_l,$$

where $\mathbf{W}(\boldsymbol{\theta}_1) = [\mathbf{w}(\theta_{1,1}), \cdots, \mathbf{w}(\theta_{1,K})]$ and $\mathbf{V}(\boldsymbol{\theta}_2) = [\mathbf{v}(\theta_{2,1}), \cdots, \mathbf{v}(\theta_{2,K})]$ with with $\theta_{1,k} = -\tau_k/T$, $\theta_{2,k} = -\nu_k T$ and

$$\mathbf{w}(\theta) \triangleq [1, e^{j2\pi\theta}, \cdots, e^{j2\pi(N-1)\theta}]^T,$$

$$\mathbf{v}(\theta) \triangleq [1, e^{j2\theta}, \cdots, e^{j2\pi(L-1)\theta}]^T.$$

$\mathbf{P}_l$ and $\mathbf{G}$ are matrices determined by modulating signals $p_m^{(l)}(t)$ and transmitted signal $g(t)$, $\mathbf{A}$ are diagonal matrix determined by target reflecting coefficients.

Let $\mathbf{X} \triangleq \mathbf{W}(\boldsymbol{\theta}_1)\mathbf{A}\mathbf{V}^H(\boldsymbol{\theta}_2)$. The m-th element of $\mathbf{y}_l$ can be represented as

$$y_{m,l} = \langle \mathbf{X}, \mathbf{m}_{m,l}\mathbf{e}_l^H \rangle,$$

where $\mathbf{m}_{m,l}^H$ denotes the m-th row of the matrix $\mathbf{P}_l\mathbf{G}$. Therefore, we can define a linear operator $\mathcal{B}_l : \mathbb{C}^{N\times L} \mapsto \mathbb{C}^M$ which performs the linear mapping from the matrix $\mathbf{X}$ to the measurement vector $\mathbf{y}_l$, *i.e.*,

$$\mathbf{y}_l = \mathcal{B}_l(\mathbf{X}),$$

and the 1-bit measurements can be expressed as:

$$\mathbf{y}_l^{\mathcal{Q}} = \operatorname{sgn}(\Re(\mathcal{B}_l(\mathbf{X}))) + i\operatorname{sgn}(\Im(\mathcal{B}_l(\mathbf{X}))).$$

- ☐ $\mathbf{X}$ is low-rank if $K \ll \min\{L, N\}$;
- ☐ Delay-Doppler estimation ⇒ Structured low-rank matrix recovery from one-bit measurements.

Method

We define a set of atoms to enforce the structure constraint of $\mathbf{X}$:

$$\mathcal{A} \triangleq \{\mathbf{A}(\boldsymbol{\theta}, \phi) = e^{j\phi}\mathbf{w}(\theta_1)\mathbf{v}^H(\theta_2) : \theta_1, \theta_2 \in (0,1], \phi \in (0, 2\pi]\}.$$

Then our problem can be described as:

$$\min_{\mathbf{X}\in\mathbb{C}^{N\times L}} \|\mathbf{X}\|_{\mathcal{A}} + \lambda\sum_{l=0}^{L-1}\phi(\mathbf{y}_l^{\mathcal{Q}}, \operatorname{sgn}(\mathcal{B}_l(\mathbf{X}))),$$

where $\phi(\mathbf{y}_l^{\mathcal{Q}}, \operatorname{sgn}(\mathcal{B}_l(\mathbf{X})))$ is a penalty function evaluating the consistency between the one-bit measurements and the estimated value.
A surrogate matrix is constructed from the one-bit measurements as

$$\mathbf{S} = \sum_{l=0}^{L-1}\mathcal{B}_l^*(\mathbf{y}_l^{\mathcal{Q}}),$$

where $\mathcal{B}_l^* : \mathbb{C}^M \mapsto \mathbb{C}^{N\times L}$ denotes the adjoint operator of $\mathcal{B}_l$ and $\mathcal{B}_l^*(\mathbf{y}) = \sum_m y_m\mathbf{m}_{m,l}\mathbf{e}_l^H$. Then we define $\|\mathbf{S}-\mathbf{X}\|_F^2$ as the penalty function, and rewrite our problem as,

$$\min_{\mathbf{X}\in\mathbb{C}^{N\times L}} \|\mathbf{X}\|_{\mathcal{A}} + \lambda\|\mathbf{S}-\mathbf{X}\|_F^2.$$

We refer to this method as *One-Bit Atomic norm Soft-Thresholding* (OBAST) method.

Results

We compare our OBAST method with the one-bit sparse signal recovery (OBSSR) method based on discrete dictionary. A linear frequency modulated (LFM) signal with bandwidth 6.4MHz and pulsewidth $5\mu s$ is transmitted. Other parameters used are $T = 10\mu s$, $L = 30$ and $K = 4$.

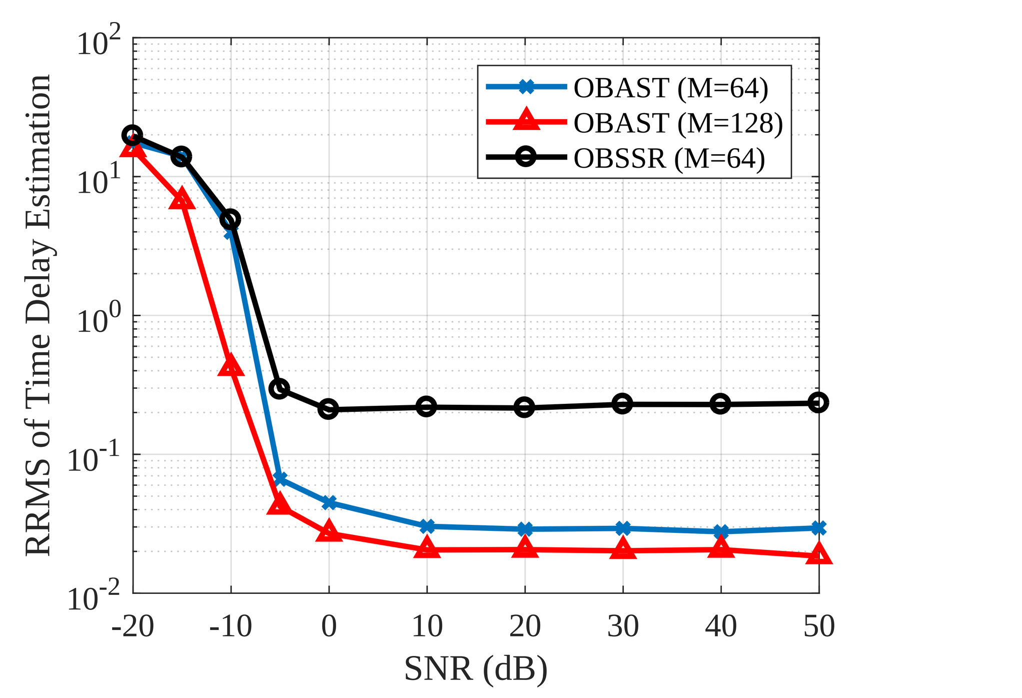

Figure 2: Time delay estimation performance with respect to SNR.

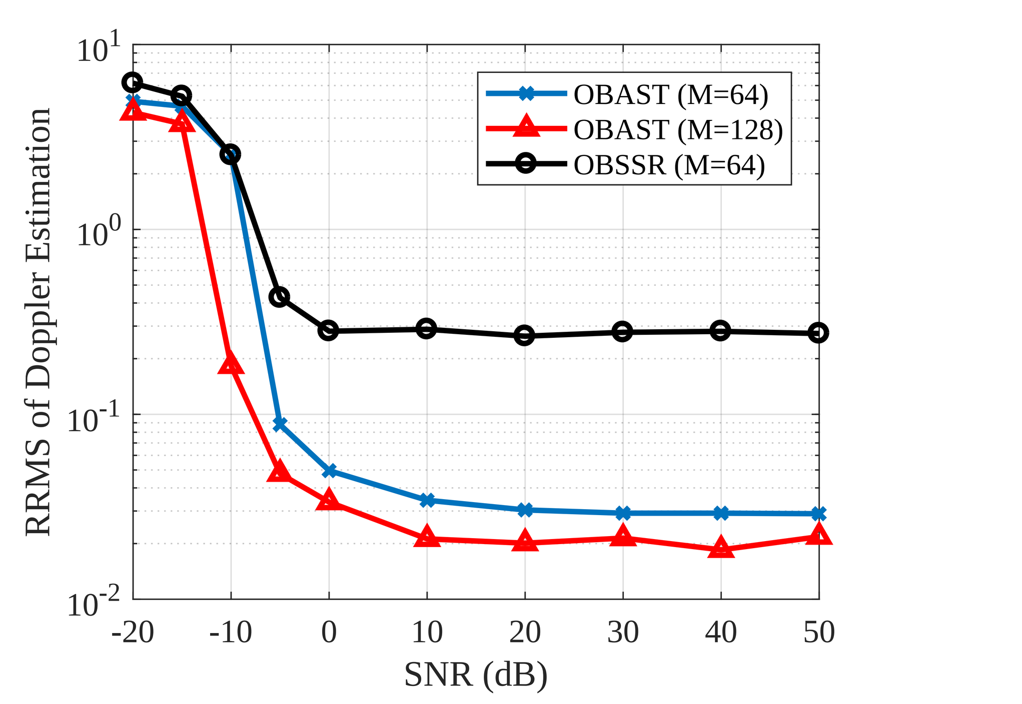

Figure 3: Doppler estimation performance with respect to SNR.

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