

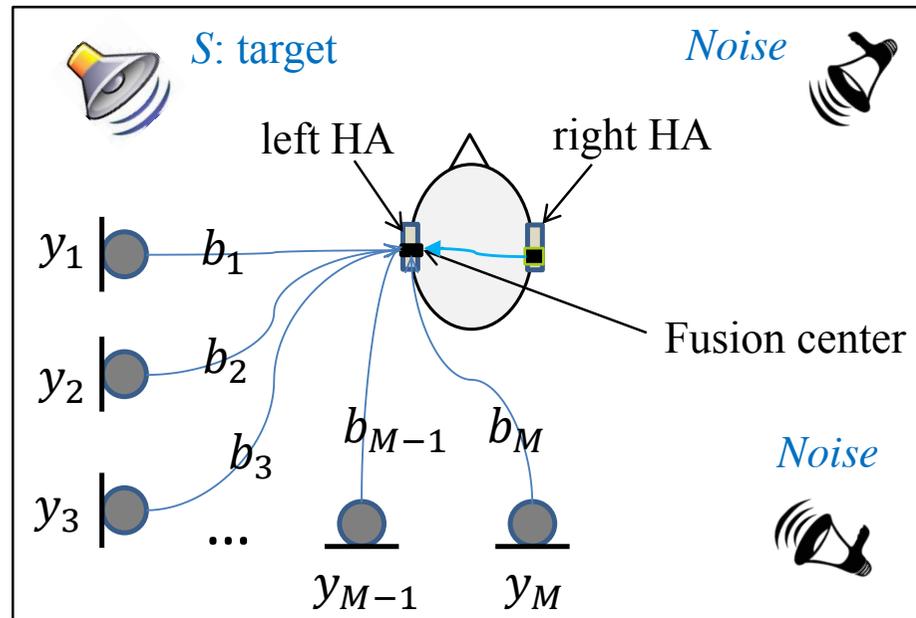
# Rate-Distributed Binaural LCMV Beamforming for Assistive Hearing in Wireless Acoustic Sensor Networks

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# Introduction



General binaural HAs configuration: HAs are a part of a bigger wireless acoustic sensor network.

Problem statement:

minimise total transmission costs  
 subject to noise reduction performance  
 spatial cue preservation

# Motivation

- Energy efficiency is essential in the design of algorithms in WASNs since usually each sensor has a limited energy budget (e.g., battery).
- In general, there are two ways to reduce energy costs:
  - [Microphone subset selection](#)
  - [Bit-rate allocation](#)
- Rate allocation is more general than sensor selection, i.e., sensor selection ([hard/binary decision](#)) can be seen as a special case of rate allocation ([soft/multiple decision](#)).

# Fundamentals

- Signal model in STFT domain:

$$\hat{\mathbf{y}} = \mathbf{y} + \mathbf{q} \in \mathbb{C}^M,$$

where

$$\begin{aligned}\mathbf{y} &= \mathbf{x} + \mathbf{z} + \mathbf{v} \\ &= \sum_{i=1}^{\mathcal{I}} \mathbf{a}_i s_i + \sum_{j=1}^{\mathcal{J}} \mathbf{h}_j u_j + \mathbf{v} \\ &= \mathbf{A}\mathbf{s} + \mathbf{H}\mathbf{u} + \mathbf{v},\end{aligned}$$

where  $\mathbf{a}_i = [a_{i1}, a_{i2}, \dots, a_{iM}]^T$ ,  $\mathbf{h}_j = [h_{j1}, h_{j2}, \dots, h_{jM}]^T$ ,  $\mathbf{A} = [\mathbf{a}_1, \dots, \mathbf{a}_{\mathcal{I}}]$ ,  $\mathbf{s} = [s_1, \dots, s_{\mathcal{I}}]^T$ ,  $\mathbf{H} = [\mathbf{h}_1, \dots, \mathbf{h}_{\mathcal{J}}] \in \mathbb{C}^{M \times \mathcal{J}}$ ,  $\mathbf{u} = [u_1, \dots, u_{\mathcal{J}}]^T$ .

# Fundamentals

- Second-order statistics:

$$\mathbf{R}_{yy} = \mathbb{E}\{\mathbf{y}\mathbf{y}^H\} = \mathbf{R}_{xx} + \underbrace{\mathbf{R}_{zz} + \mathbf{R}_{vv}}_{\mathbf{R}_{nn}} \in \mathbb{C}^{M \times M},$$

where  $\mathbf{R}_{xx} = \sum_{i=1}^{\mathcal{I}} \mathbb{E}\{\mathbf{x}_i \mathbf{x}_i^H\}$  and  $\mathbf{R}_{zz} = \sum_{j=1}^{\mathcal{J}} \mathbb{E}\{\mathbf{z}_j \mathbf{z}_j^H\}$ .

- **Assumption:** the target sources, interfering sources and quantisation noise are mutually uncorrelated, such that

$$\mathbf{R}_{n+q} = \mathbf{R}_{nn} + \mathbf{R}_{qq}.$$

- Uniform quantisation:

$$\mathbf{R}_{qq} = \frac{1}{12} \text{diag} \left( \left[ \frac{\mathcal{A}_1^2}{4^{b_1}}, \frac{\mathcal{A}_2^2}{4^{b_2}}, \dots, \frac{\mathcal{A}_M^2}{4^{b_M}} \right] \right),$$

where  $\mathcal{A}_k = \max\{|y_k|\}$  and  $b_k, \forall k$  denotes the rate for the  $k$ th sensor node.

# Fundamentals

- Transmission energy model:

- SNR over communication channels:  $\text{SNR}_k = d_k^{-2} E_k / V_k$ .

- Channel capacity:

$$b_k = \frac{1}{2} \log_2 (1 + \text{SNR}_k).$$

- Transmission energy:

$$E_k = d_k^2 V_k (4^{b_k} - 1),$$

which holds under two conditions: 1) for spectrum-limited applications; 2) quantize at the channel capacity (i.e., the upper bound).

# Binaural LCMV beamforming

- **Binaural cue preservation:**  $\mathbf{w}_{\text{BLCMV}} = [\mathbf{w}_L^T \ \mathbf{w}_R^T]^T$

- Preserving target sources:

$$\left. \begin{array}{l} \mathbf{w}_L^H \mathbf{a}_i = a_{iL} \\ \mathbf{w}_R^H \mathbf{a}_i = a_{iR} \end{array} \right\} \implies \text{ITF}_{\mathbf{x}_i}^{\text{in}} = \text{ITF}_{\mathbf{x}_i}^{\text{out}} \implies \mathbf{\Lambda}_1^H \mathbf{w} = \mathbf{f}_1$$

- Preserving interfering sources:

$$\text{ITF}_{\mathbf{n}_j}^{\text{in}} = \text{ITF}_{\mathbf{n}_j}^{\text{out}} \implies \frac{h_{jL}}{h_{jR}} = \frac{\mathbf{w}_L^H \mathbf{h}_j}{\mathbf{w}_R^H \mathbf{h}_j}, \forall j,$$

$$\implies \mathbf{w}_L^H \mathbf{h}_j h_{jR} - \mathbf{w}_R^H \mathbf{h}_j h_{jL} = 0 \implies \mathbf{\Lambda}_2^H \mathbf{w} = \mathbf{f}_2$$

- Binaural cues:  $\text{ILD} = |\text{ITF}|^2$ ,  $\text{IPD} = \angle \text{ITF}$ .

# Binaural LCMV beamforming

- **Binaural LCMV (BLCMV) beamforming** for joint noise reduction and spatial cue preservation can be formulated as

$$\hat{\mathbf{w}}_{\text{BLCMV}} = \arg \min_{\mathbf{w}} \mathbf{w}^H \tilde{\mathbf{R}}_{\mathbf{n}+\mathbf{q}} \mathbf{w}, \quad \text{s.t.} \quad \mathbf{\Lambda}^H \mathbf{w} = \tilde{\mathbf{f}},$$

where

$$\tilde{\mathbf{R}}_{\mathbf{n}+\mathbf{q}} = \begin{bmatrix} \mathbf{R}_{\mathbf{n}+\mathbf{q}} & \mathbf{0} \\ \mathbf{0} & \mathbf{R}_{\mathbf{n}+\mathbf{q}} \end{bmatrix} \in \mathbb{C}^{2M \times 2M},$$

$$\mathbf{\Lambda} = \left[ \mathbf{\Lambda}_1 \mid \mathbf{\Lambda}_2 \right] \in \mathbb{C}^{2M \times (2\mathcal{I} + \mathcal{J})},$$

$$\tilde{\mathbf{f}} = \left[ \mathbf{f}_1^H \mid \mathbf{f}_2^H \right]^T \in \mathbb{R}^{2\mathcal{I} + \mathcal{J}}.$$

- **BLCMV beamformer:**  $\hat{\mathbf{w}}_{\text{BLCMV}} = \tilde{\mathbf{R}}_{\mathbf{n}+\mathbf{q}}^{-1} \mathbf{\Lambda} (\mathbf{\Lambda}^H \tilde{\mathbf{R}}_{\mathbf{n}+\mathbf{q}}^{-1} \mathbf{\Lambda})^{-1} \tilde{\mathbf{f}}.$

# Problem formulation

- **General problem formulation:**

$$\begin{aligned} \min_{\mathbf{w}, \mathbf{b}} \quad & \sum_{k=1}^M d_k^2 V_k (4^{b_k} - 1) \\ \text{s.t.} \quad & \mathbf{w}^H \mathbf{R}_{\mathbf{n}+\mathbf{q}} \mathbf{w} \leq \frac{\beta}{\alpha} \\ & \mathbf{\Lambda}^H \mathbf{w} = \mathbf{f}, \\ & b_k \in \mathbb{Z}_+, \quad b_k \leq b_0, \forall k, \end{aligned} \tag{P1}$$

where

- $\beta$ : minimum output noise power;
- $\alpha \in (0, 1]$ : performance controller;
- $b_0$ : maximum rate, e.g., 16 bits per sample.

# Rate-distributed BLCMV beamforming

- Substitution of the BLCMV beamformer, we can obtain

$$\begin{aligned}
 \min_{\mathbf{b}} \quad & \sum_{k=1}^M d_k^2 V_k(4^{b_k} - 1) \\
 \text{s.t.} \quad & \tilde{\mathbf{f}}^H (\mathbf{\Lambda}^H \tilde{\mathbf{R}}_{\mathbf{n}+\mathbf{q}}^{-1} \mathbf{\Lambda})^{-1} \tilde{\mathbf{f}} \leq \frac{\beta}{\alpha} \\
 & b_k \in \mathbb{Z}_+, \quad b_k \leq b_0, \forall k.
 \end{aligned} \tag{P2}$$

- Convex optimisation:

$$\mathbf{\Lambda}^H \tilde{\mathbf{R}}_{\mathbf{n}+\mathbf{q}}^{-1} \mathbf{\Lambda} = \mathbf{Z}, \tag{1}$$

$$\tilde{\mathbf{f}}^H \mathbf{Z}^{-1} \tilde{\mathbf{f}} \leq \frac{\beta}{\alpha}, \tag{2}$$

where  $\mathbf{Z} \in \mathbb{S}_+^{2\mathcal{I}+\mathcal{J}}$  is Hermitian.

# Rate-distributed BLCMV beamforming

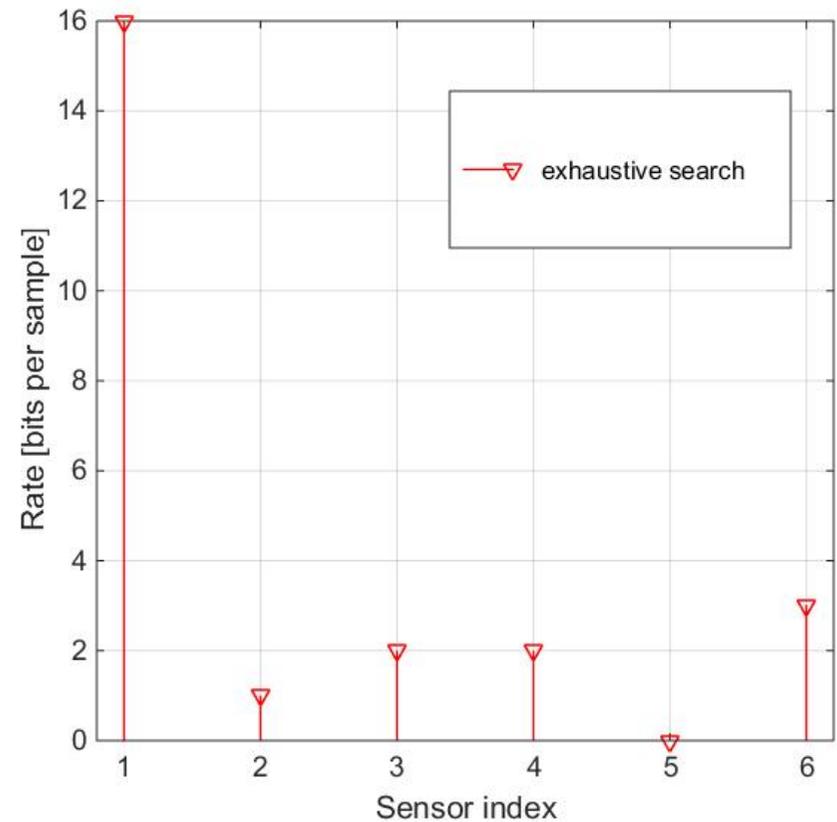
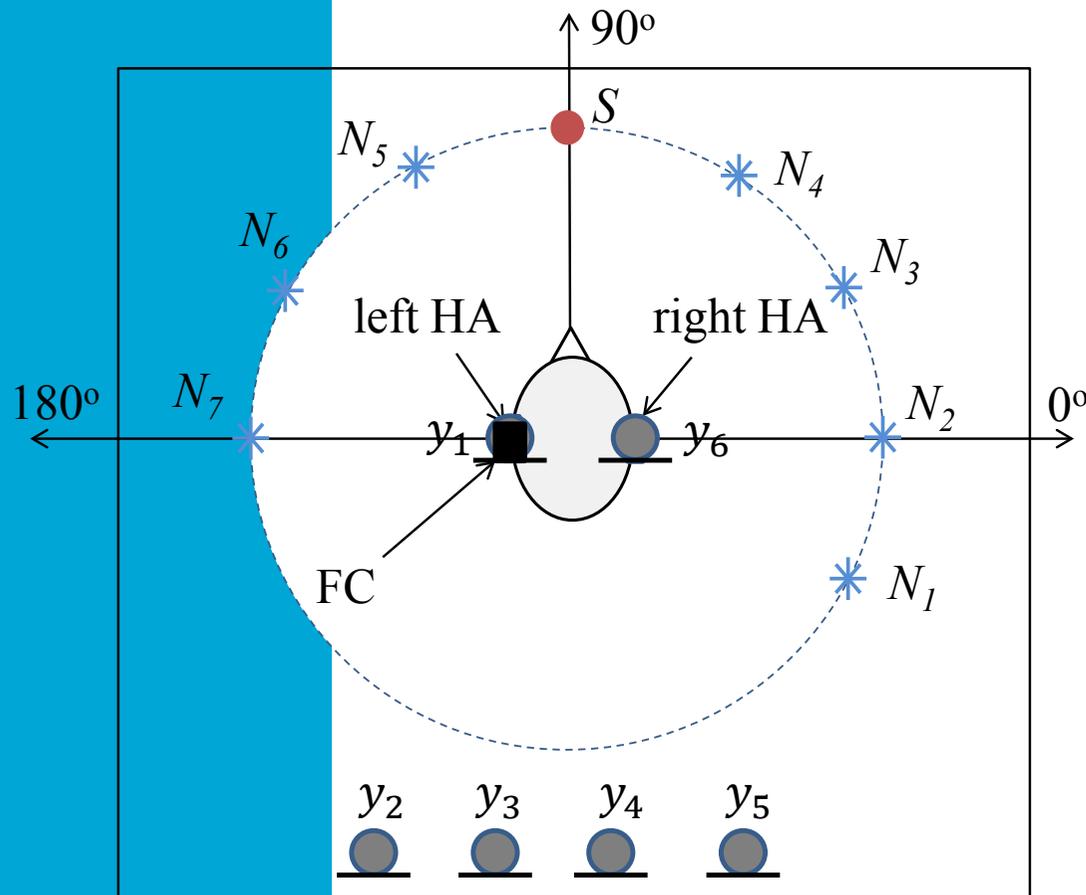
- Define a constant vector  $\mathbf{e} = \left[ \frac{12}{\mathcal{A}_1^2}, \dots, \frac{12}{\mathcal{A}_M^2} \right]$  and introduce a variable change  $t_k = 4^{b_k} \in \mathbb{Z}_+, \forall k$ , such that  $\mathbf{R}_{\mathbf{q}\mathbf{q}}^{-1} = \text{diag}(\mathbf{e} \odot \mathbf{t})$ ,

$$\begin{aligned} \min_{\mathbf{t}, \mathbf{Z}} \quad & \sum_{k=1}^M d_k^2 V_k(t_k - 1) \\ \text{s.t.} \quad & \begin{bmatrix} \mathbf{Z} & \mathbf{f} \\ \mathbf{f}^H & \frac{\beta}{\alpha} \end{bmatrix} \succeq \mathbf{O}_{2I+J+1} \\ & \begin{bmatrix} \tilde{\mathbf{R}}_{\mathbf{nn}}^{-1} + \tilde{\mathbf{R}}_{\mathbf{qq}}^{-1} & \tilde{\mathbf{R}}_{\mathbf{nn}}^{-1} \mathbf{\Lambda} \\ \mathbf{\Lambda}^H \tilde{\mathbf{R}}_{\mathbf{nn}}^{-1} & \mathbf{\Lambda}^H \tilde{\mathbf{R}}_{\mathbf{nn}}^{-1} \mathbf{\Lambda} - \mathbf{Z} \end{bmatrix} \succeq \mathbf{O}_{2M+2I+J} \\ & 1 \leq t_k \leq 4^{b_0}, \forall k. \end{aligned}$$

- The integer rates can be resolved by  $b_k = \log_4 t_k, \forall k$  and randomised rounding technique.

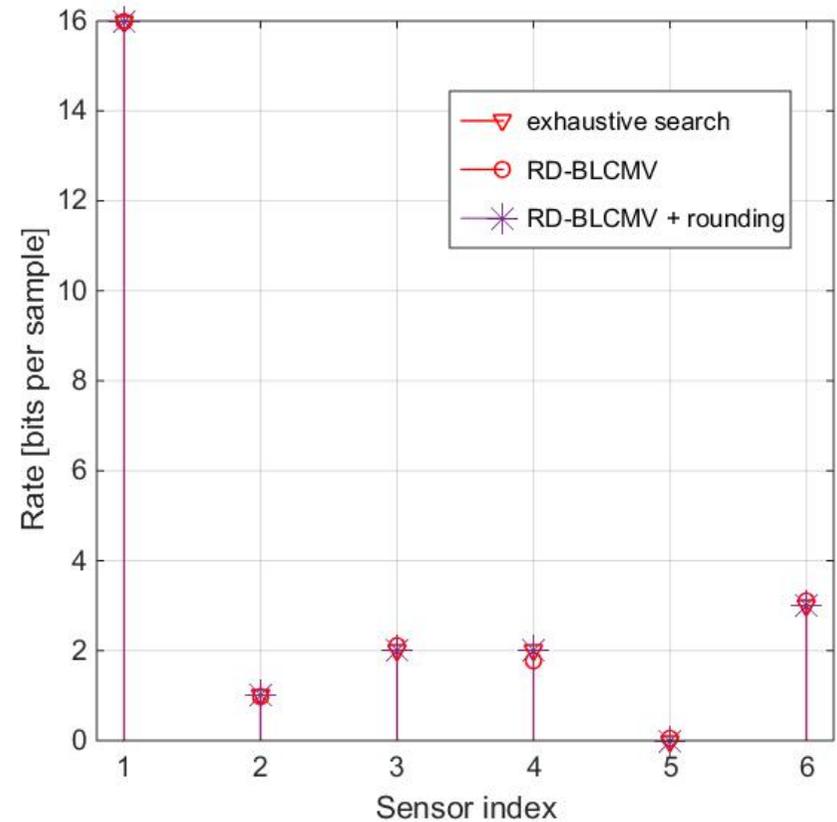
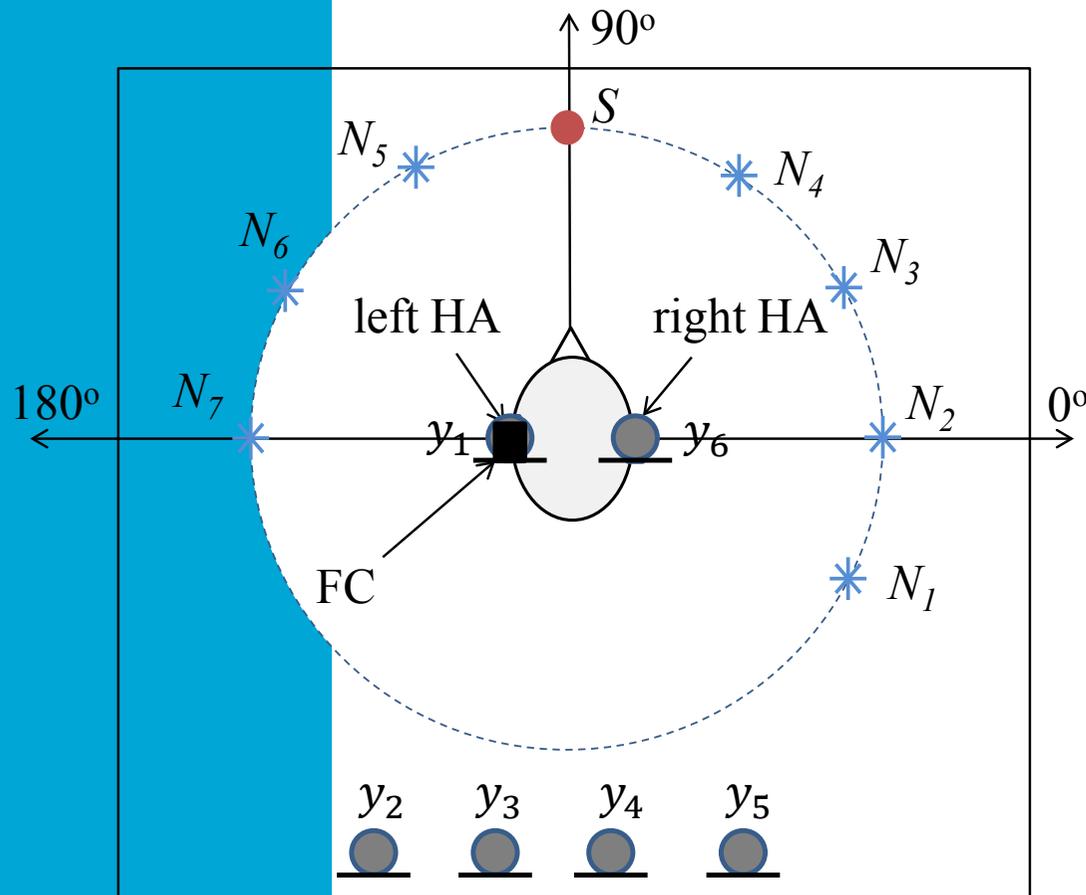
# Simulation results

- Setting: 6 mics in  $(4 \times 3)\text{m}$  2D room,  $f_s=16\text{kHz}$ ,  $T_{60}=200\text{ms}$ ,  $\alpha = 0.8$



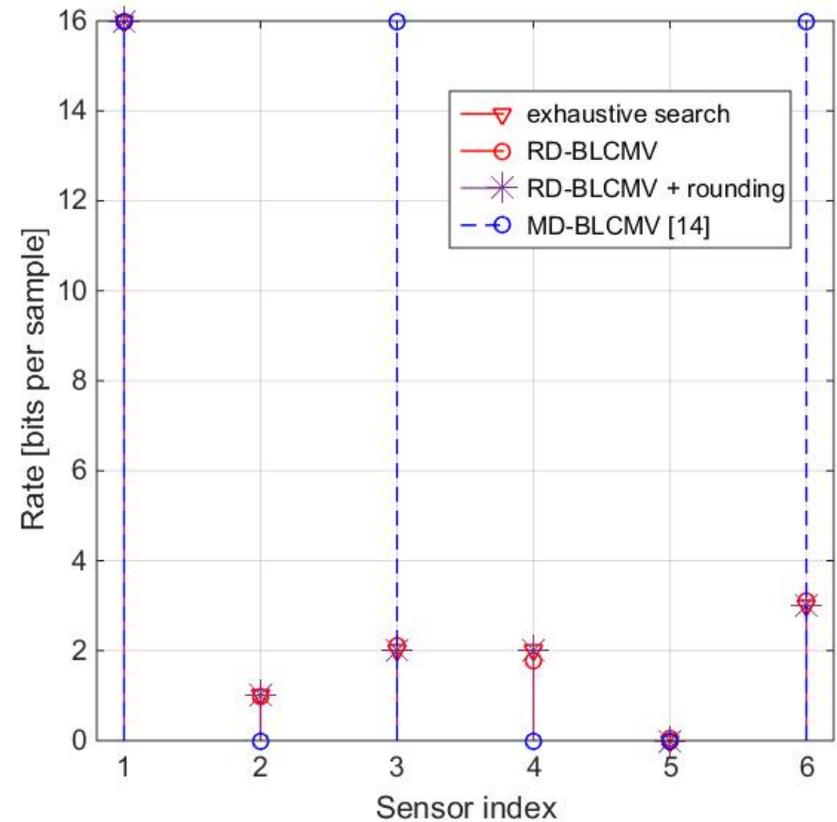
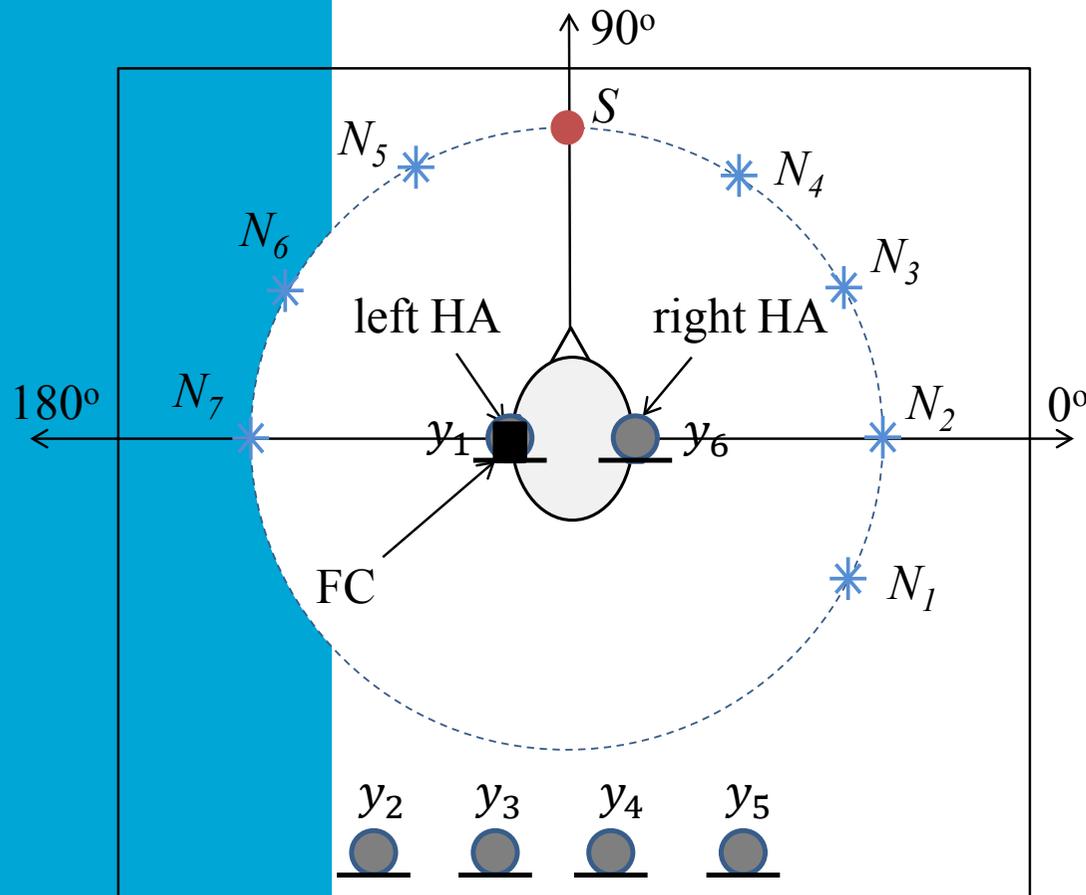
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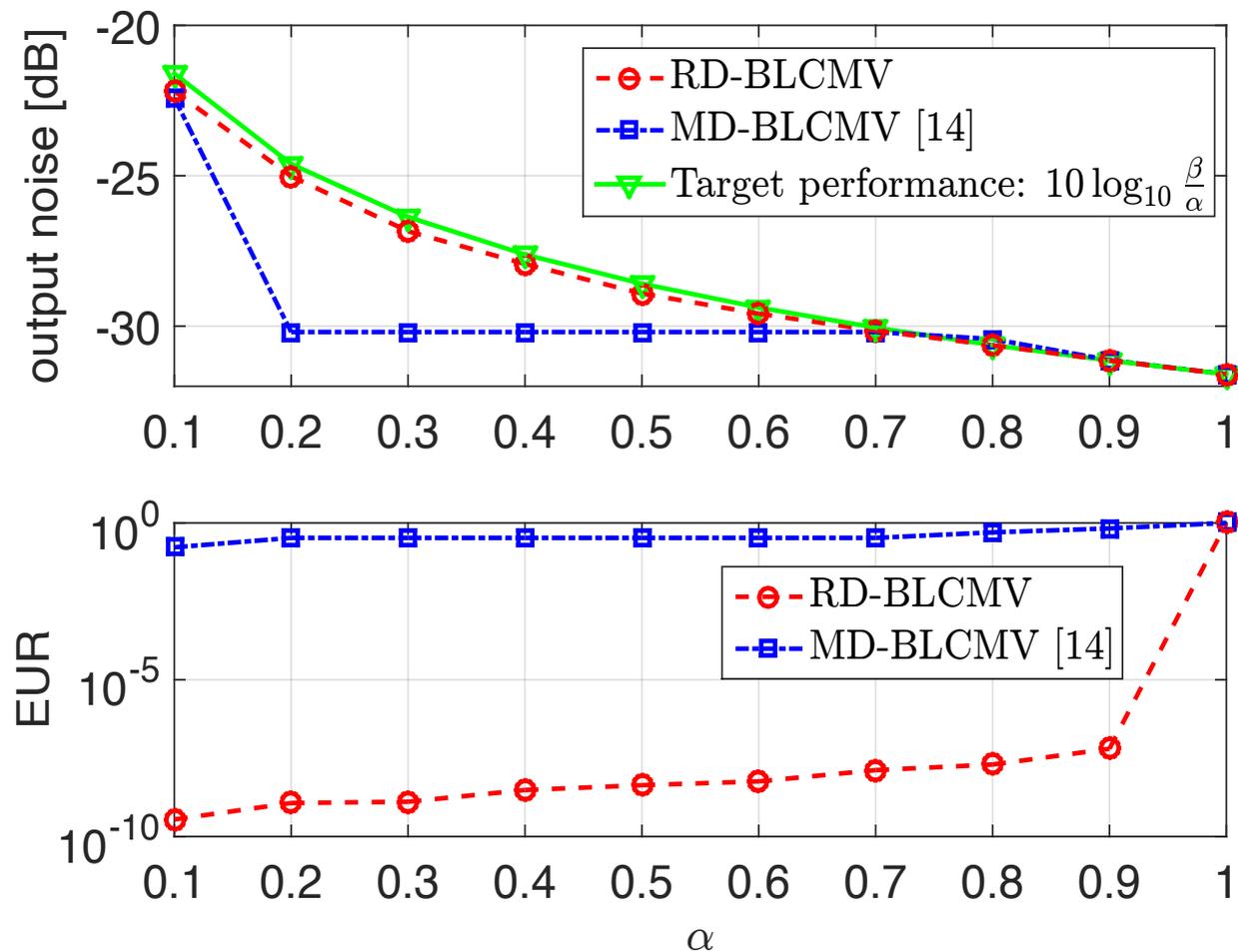
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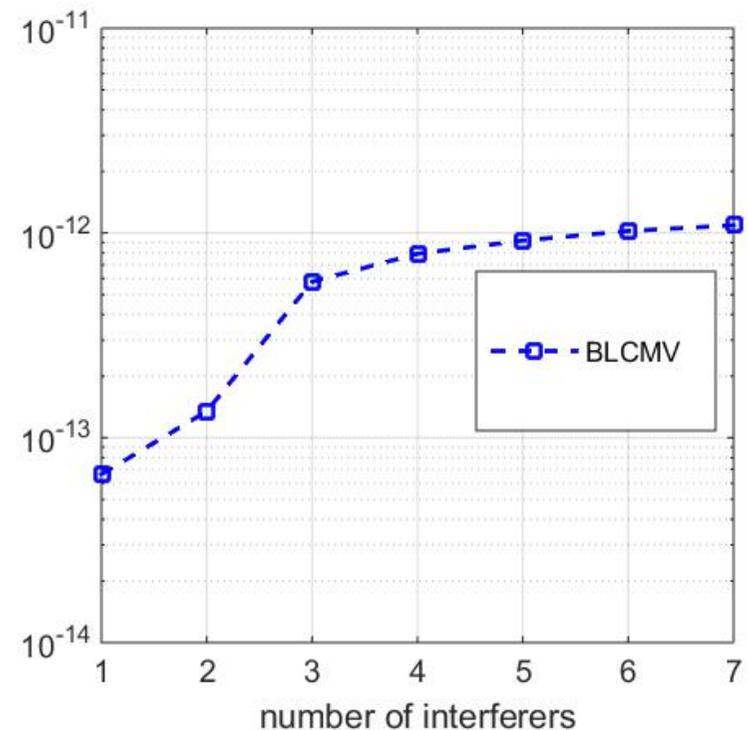
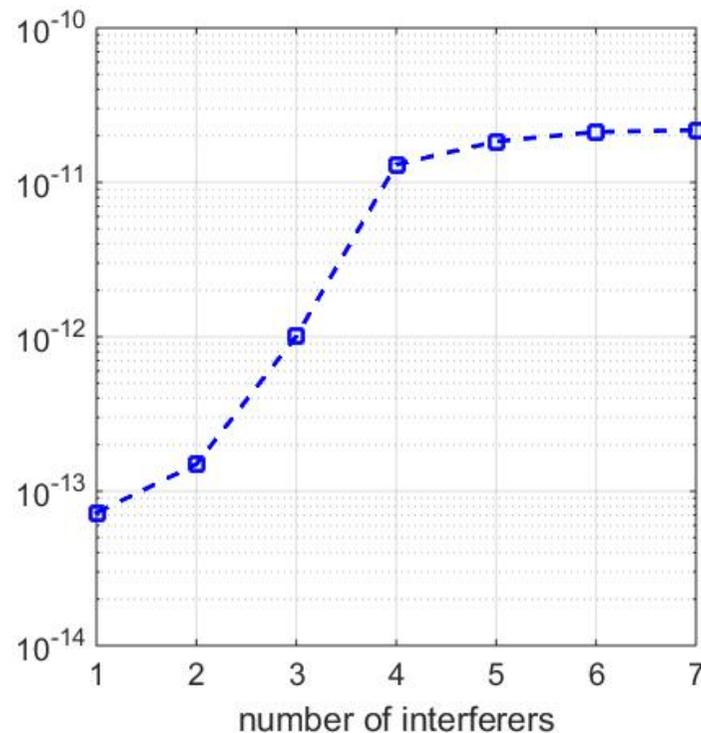
# Simulation results

- Noise reduction performance and energy usage ratio (EUR):



# Simulation results

- Performance of spatial cue preservation:

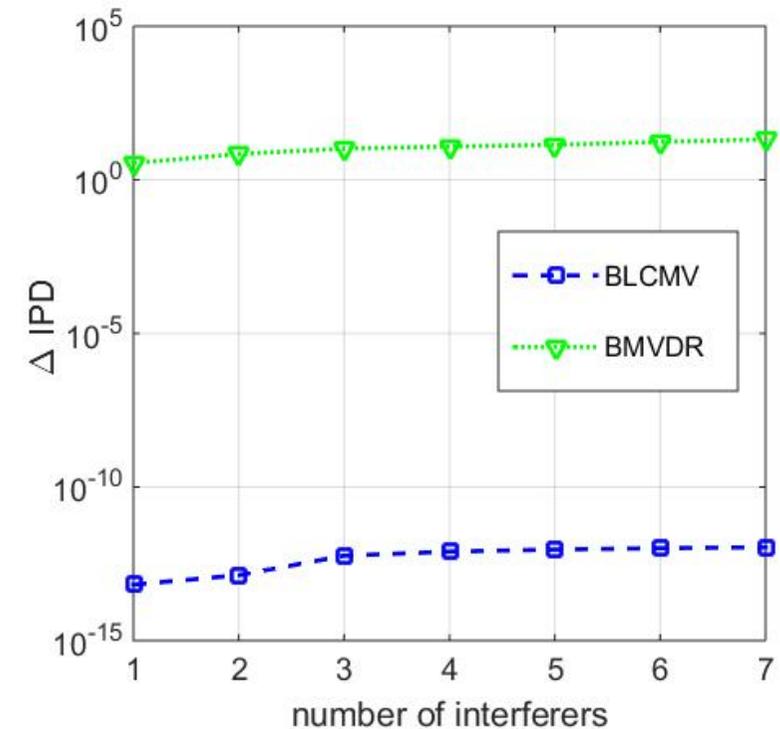
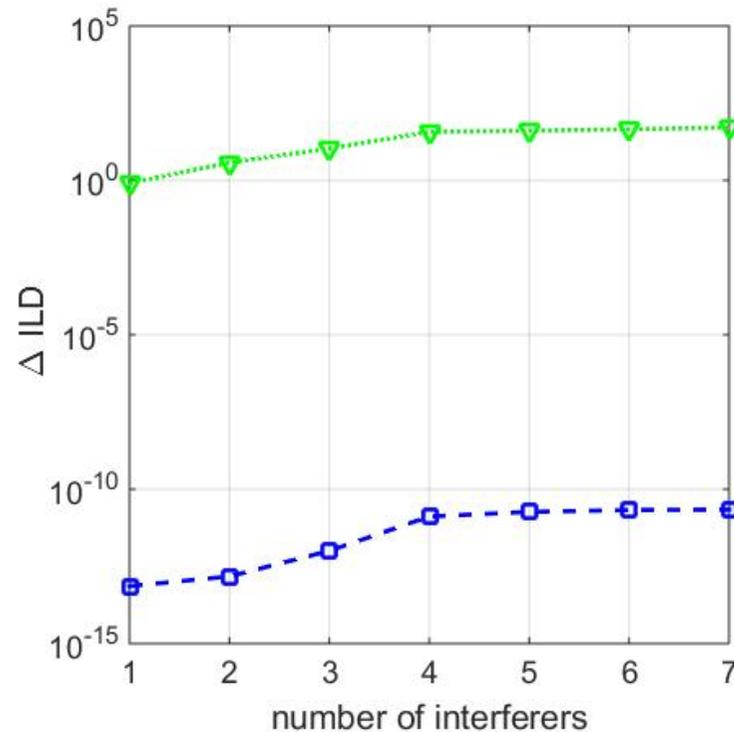


$$\Delta\text{ILD} = \sum_{j=1}^J \sum_{\omega} \left( \text{ILD}_j(\omega) - \tilde{\text{ILD}}_j(\omega) \right)^2;$$

$$\Delta\text{IPD} = \sum_{j=1}^J \sum_{\omega} \left( \text{IPD}_j(\omega) - \tilde{\text{IPD}}_j(\omega) \right)^2$$

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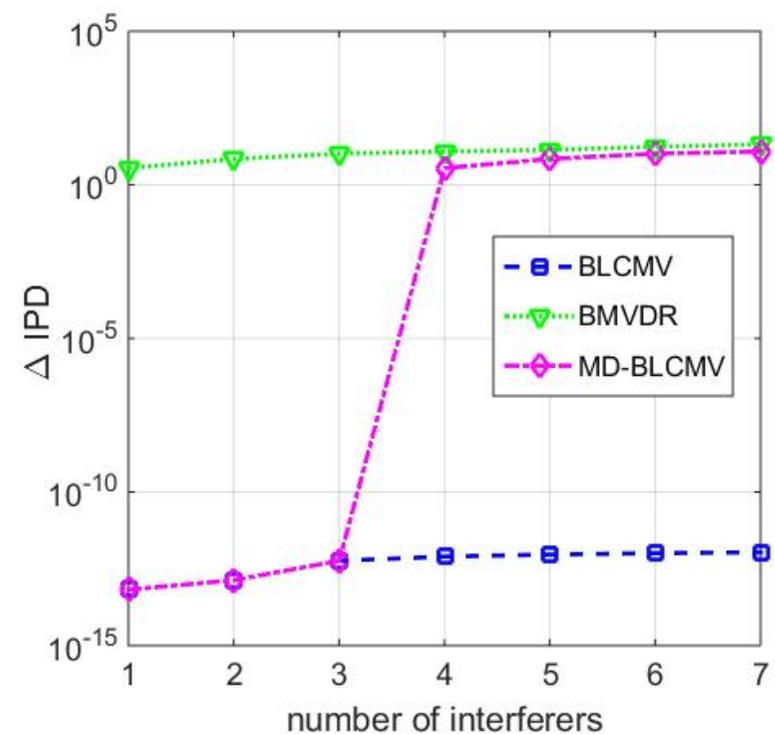
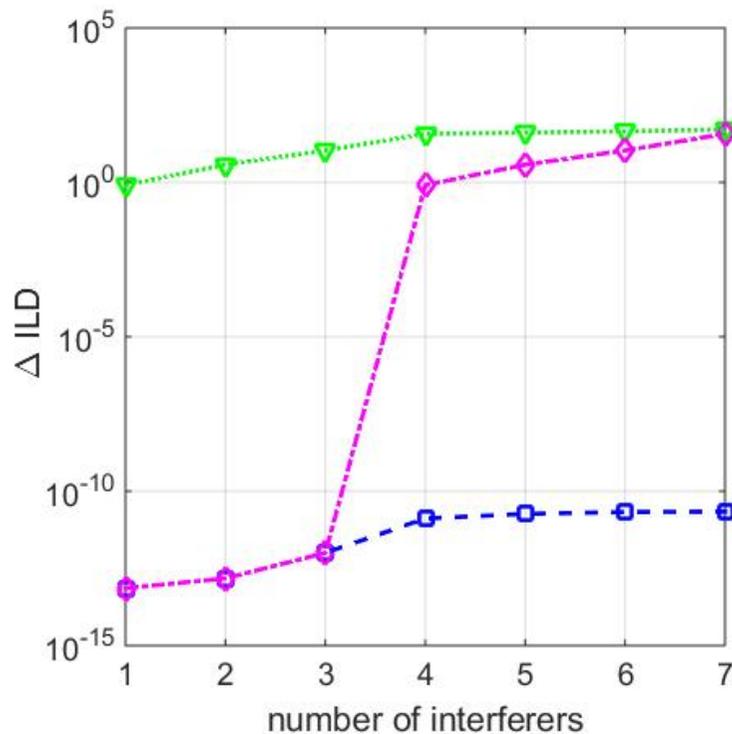


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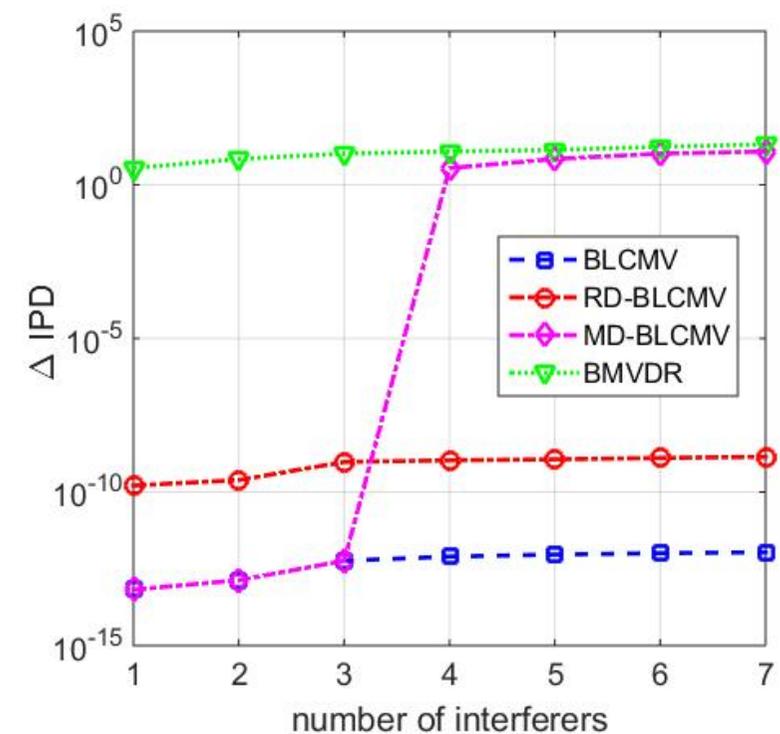
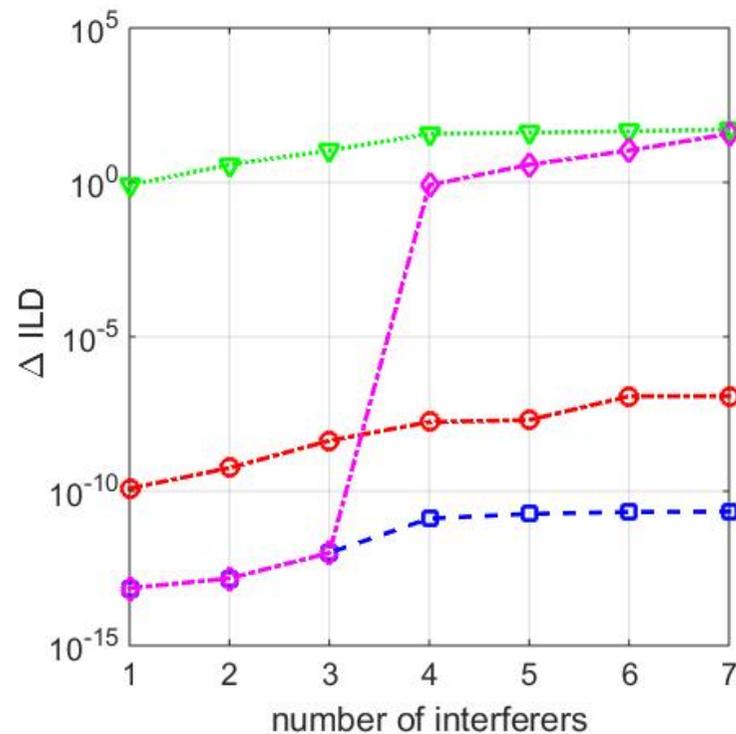


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# Conclusion

- We studied rate-distributed BLCMV beamforming for wireless binaural hearing aids. The proposed method was formulated by minimizing the energy usage and constraining the noise reduction performance.
- Under the utilization of a BLCMV beamformer, the problem was solved by semi-definite programming with the capability of joint noise reduction and binaural cue preservation.
- The proposed method can achieve better energy efficiency by distributing bit rates, and preserve more interferers' spatial cues by activating more sensors as compared to sensor selection approaches.

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**Thank you!**

# Appendix

- Matrix inversion lemma:

$$\tilde{\mathbf{R}}_{\mathbf{n}+\mathbf{q}}^{-1} = (\tilde{\mathbf{R}}_{\mathbf{nn}} + \tilde{\mathbf{R}}_{\mathbf{qq}})^{-1} = \tilde{\mathbf{R}}_{\mathbf{nn}}^{-1} - \tilde{\mathbf{R}}_{\mathbf{nn}}^{-1}(\tilde{\mathbf{R}}_{\mathbf{nn}}^{-1} + \tilde{\mathbf{R}}_{\mathbf{qq}}^{-1})^{-1}\tilde{\mathbf{R}}_{\mathbf{nn}}^{-1},$$

where

$$\tilde{\mathbf{R}}_{\mathbf{nn}} = \begin{bmatrix} \mathbf{R}_{\mathbf{nn}} & \mathbf{0} \\ \mathbf{0} & \mathbf{R}_{\mathbf{nn}} \end{bmatrix}, \quad \tilde{\mathbf{R}}_{\mathbf{qq}} = \begin{bmatrix} \mathbf{R}_{\mathbf{qq}} & \mathbf{0} \\ \mathbf{0} & \mathbf{R}_{\mathbf{qq}} \end{bmatrix}.$$

- Convex relaxation:

$$\Lambda^H \mathbf{R}_{\mathbf{n}+\mathbf{q}}^{-1} \Lambda \succeq \mathbf{Z}$$

$$\implies \Lambda^H \tilde{\mathbf{R}}_{\mathbf{nn}}^{-1} \Lambda - \mathbf{Z} \succeq \Lambda^H \tilde{\mathbf{R}}_{\mathbf{nn}}^{-1} (\tilde{\mathbf{R}}_{\mathbf{nn}}^{-1} + \tilde{\mathbf{R}}_{\mathbf{qq}}^{-1})^{-1} \tilde{\mathbf{R}}_{\mathbf{nn}}^{-1} \Lambda$$

$$\implies \begin{bmatrix} \tilde{\mathbf{R}}_{\mathbf{nn}}^{-1} + \tilde{\mathbf{R}}_{\mathbf{qq}}^{-1} & \tilde{\mathbf{R}}_{\mathbf{nn}}^{-1} \Lambda \\ \Lambda^H \tilde{\mathbf{R}}_{\mathbf{nn}}^{-1} & \Lambda^H \tilde{\mathbf{R}}_{\mathbf{nn}}^{-1} \Lambda - \mathbf{Z} \end{bmatrix} \succeq \mathbf{O}_{2M+2L+J}.$$