

# Blind Source Separation under the Langmuir model for chemical sensors

IEEE SAM

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July 10, 2018



## Electronic what ?

An electronic nose = bio-inspired instrument which is able to identify and recognise Volatile Organic Compounds (VOCs) [1]



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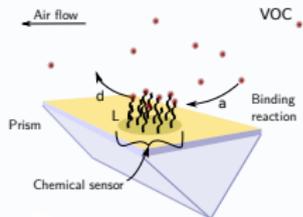
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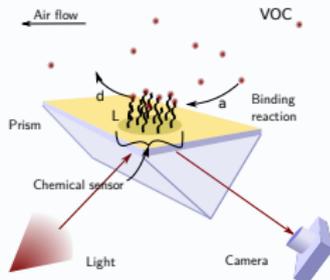




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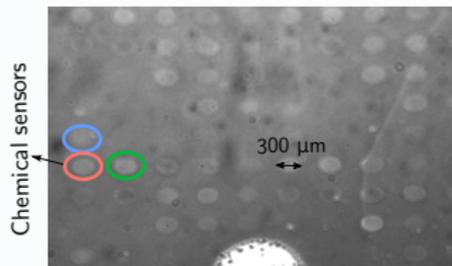
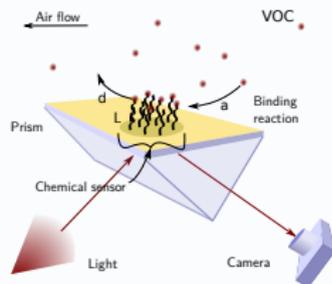
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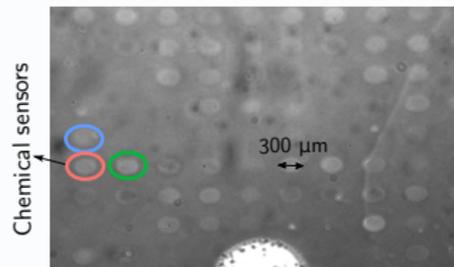
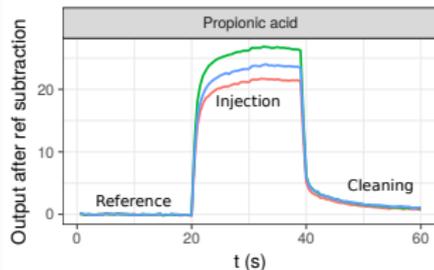




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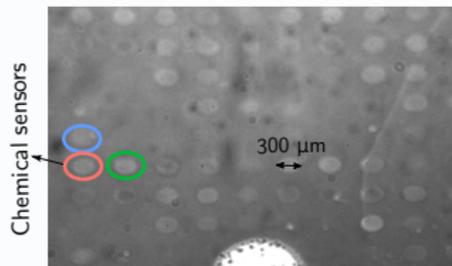
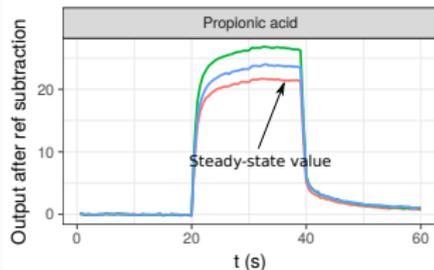




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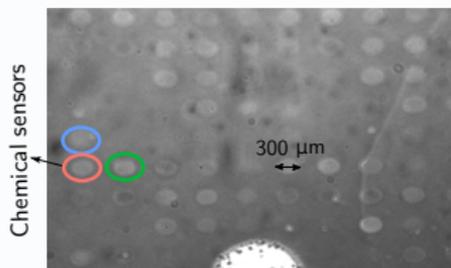
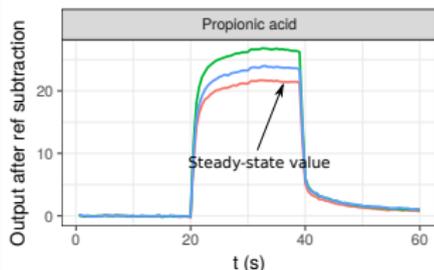




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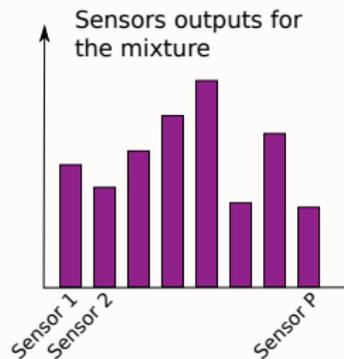
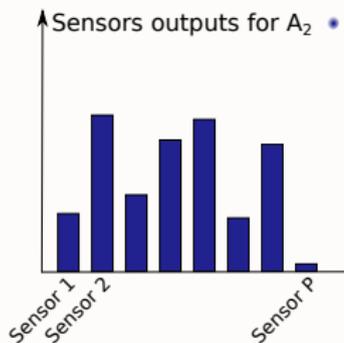
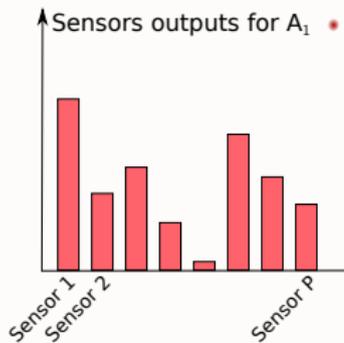
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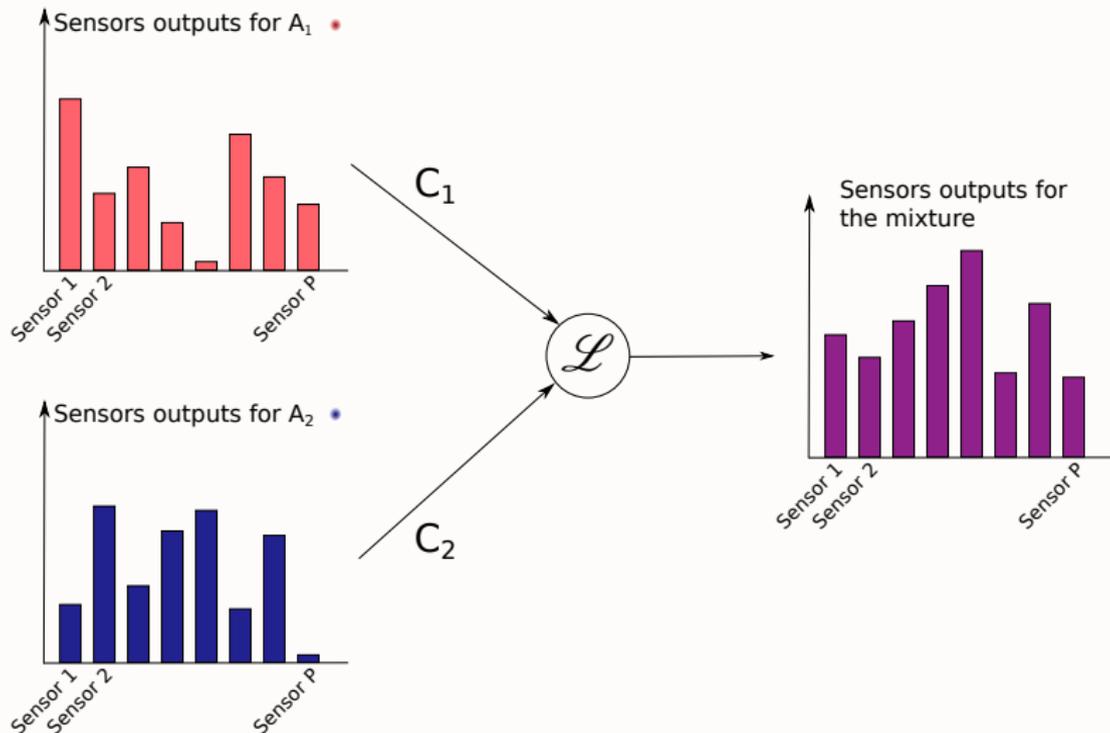
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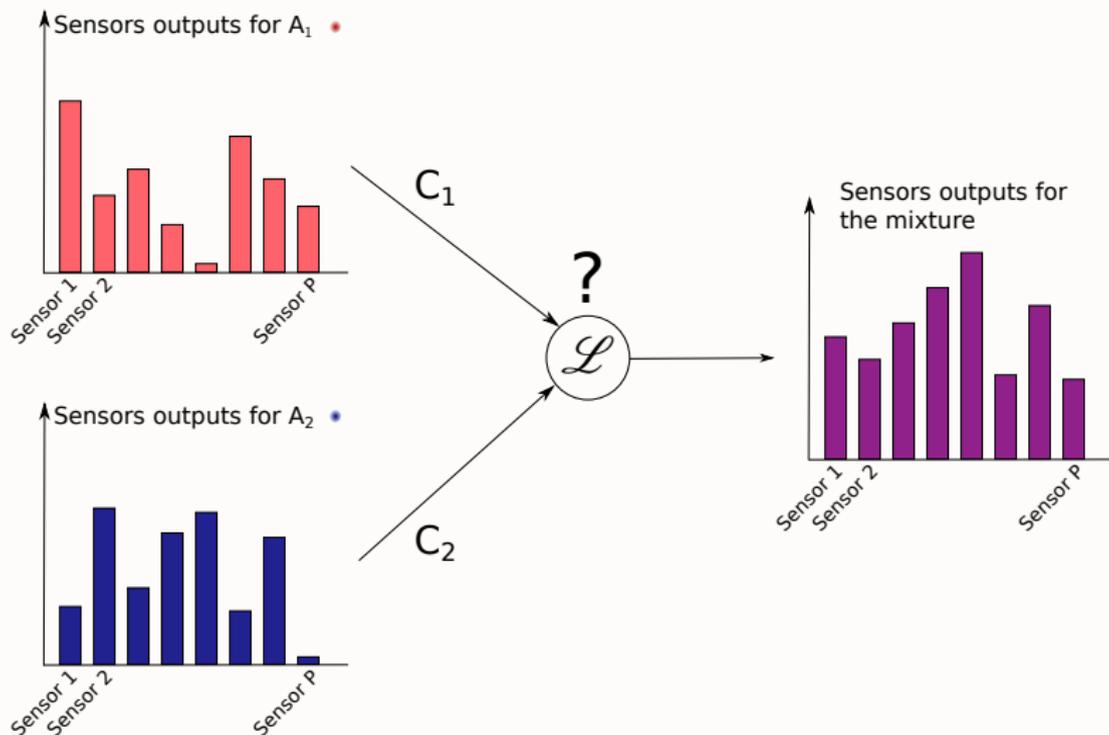


## For what purpose ?

Biomedical, in/outdoor air monitoring, security, navigation, ...





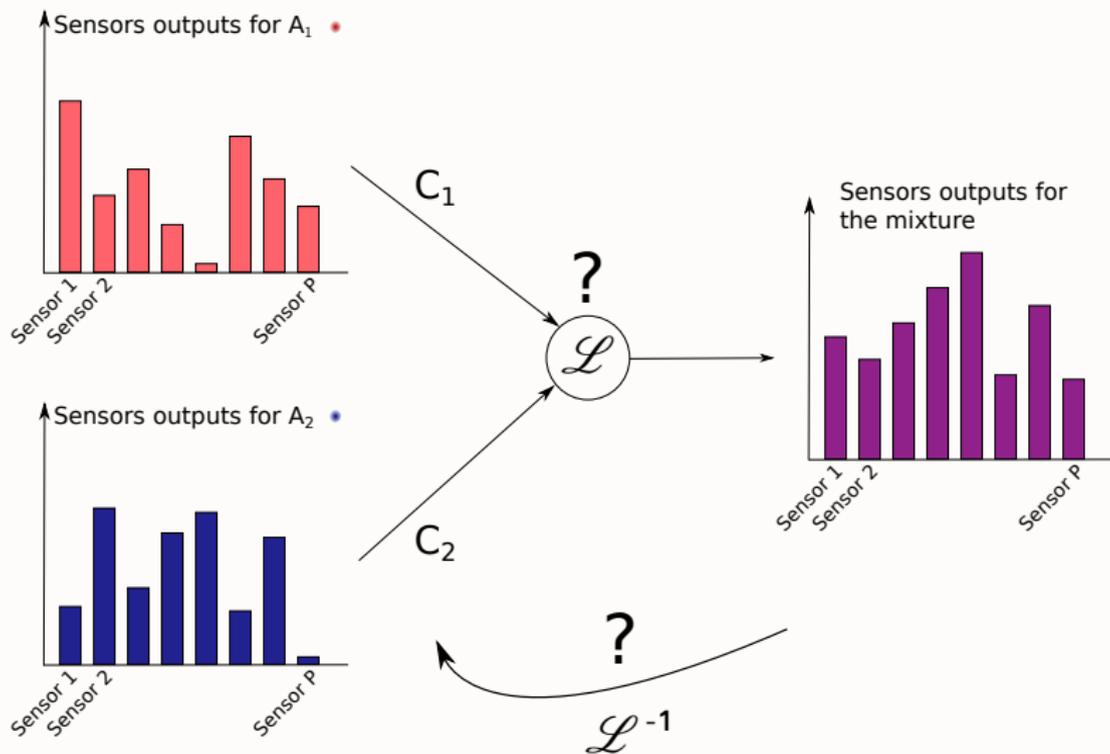


# Electronic nose

Source Separation issue



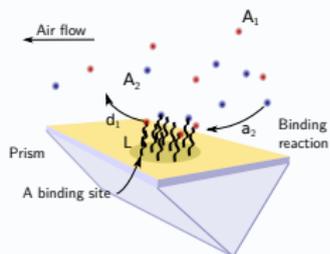
1



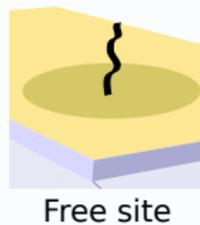
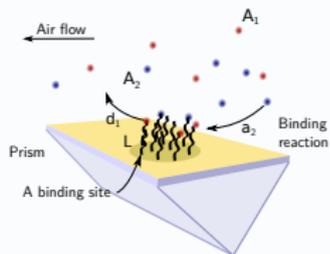


## Langmuir isotherm

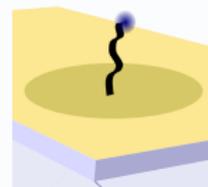
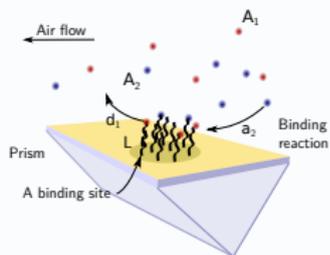
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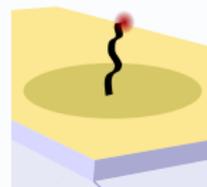
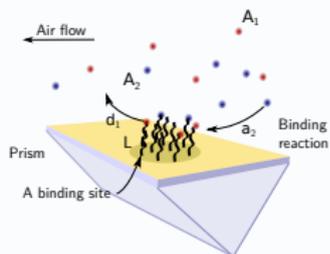


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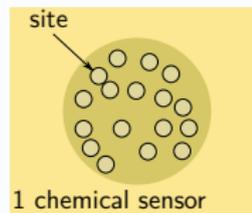
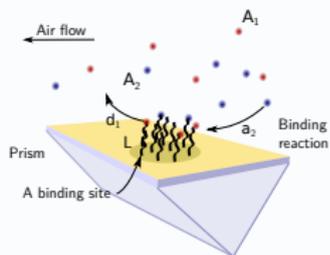
Occupied site

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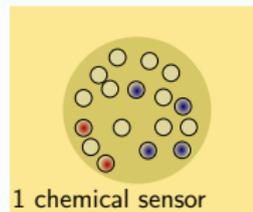
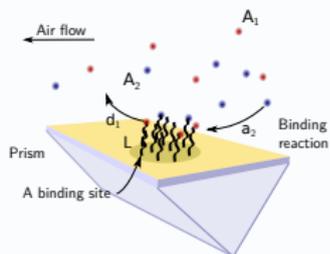


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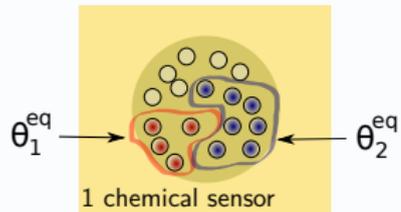
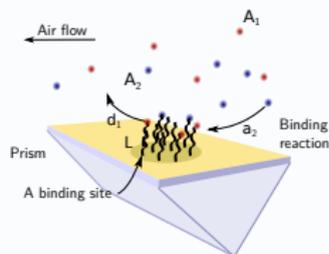
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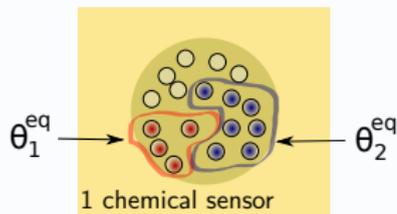
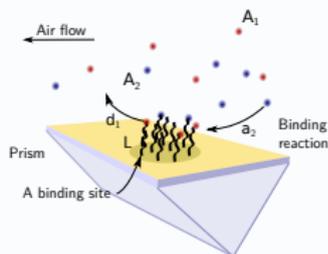


## Langmuir isotherm



$\theta^{eq}$  : fraction of occupied sites

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$\theta^{eq}$  : fraction of occupied sites

Langmuir isotherm [2] for a multi-component gas, noting :

$\left\{ \begin{array}{l} r \text{ the VOC} \\ p \text{ the chemical sensor} \\ n \text{ the mixture} \end{array} \right.$

$$\theta_{rpn}^{eq} = \frac{k_{rp}c_{rn}}{1 + \sum_{r=1}^R k_{rp}c_{rn}}$$

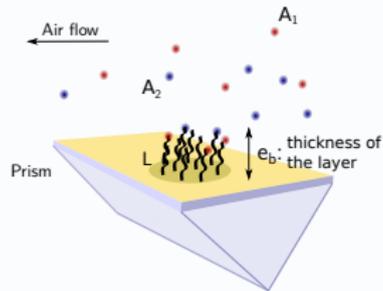
with  $k_{rp}$  the **affinity** and  $c_{rn}$  the **concentration**



## Surface Plasmon Resonance model

## Additivity assumption

## Surface Plasmon Resonance model

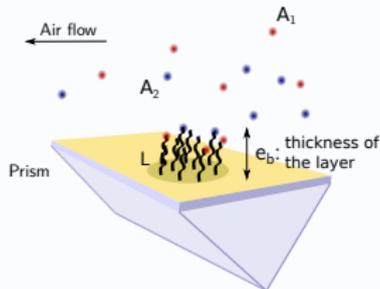


Assuming that  $e_b$  is in the nanometer range, the measure  $y_{rpn}$  is **proportional** to the fraction of occupied sites  $\theta_{rpn}^{eq}$  (with  $m_r$  the mass of  $\mathcal{A}_r$ ) [3, 4]:

$$y_{rpn} = \gamma m_r \theta_{rpn}^{eq}$$

## Additivity assumption

## Surface Plasmon Resonance model



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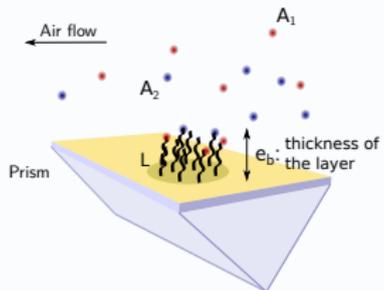
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## Additivity assumption

The measure of the mixture is assumed to be the **sum** of the individual contributions:

$$y_{pn} = \sum_{r=1}^R y_{rpn} = \gamma \frac{\sum_{r=1}^R m_r k_{rp} c_{rn}}{1 + \sum_{r=1}^R k_{rp} c_{rn}}$$

## Surface Plasmon Resonance model



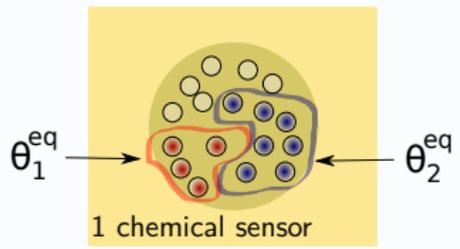
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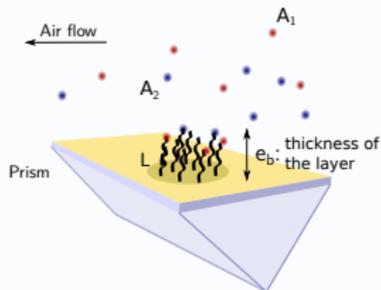
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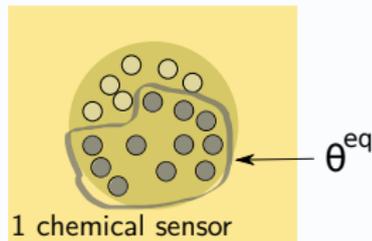
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## Problem dimensions



## Problem dimensions

R VOCs



$A_1$



$A_2$

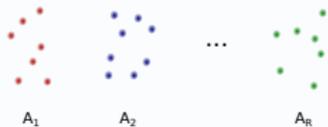
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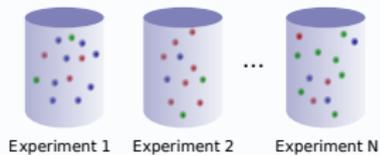
$A_R$

## Problem dimensions

R VOCs



N experiments



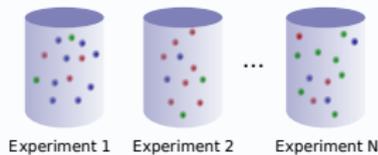


### Problem dimensions

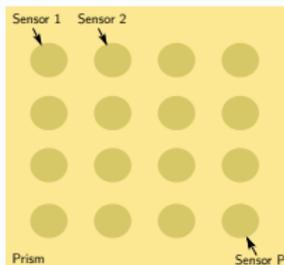
R VOCs



N experiments



P sensors



chemical sensor, mixture VOC

$$y_{pn} = \gamma \frac{\sum_{r=1}^R m_r k_{rp} c_{rn}}{1 + \sum_{r=1}^R k_{rp} c_{rn}}$$

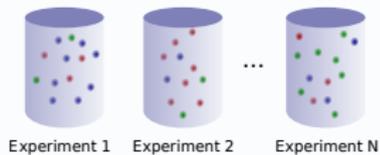
VOC, chemical sensor   VOC, mixture

## Problem dimensions

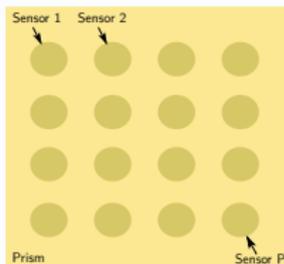
R VOCs



N experiments



P sensors



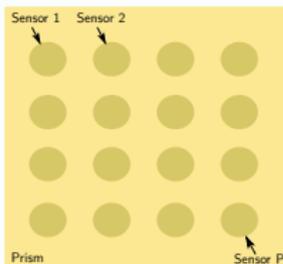
## Matrix formulation

## Problem dimensions

R VOCs



P sensors



## Matrix formulation

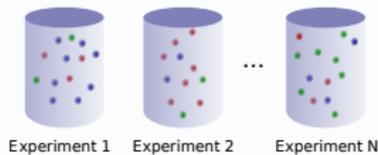
$$\left[ \begin{array}{c} \mathbf{K} \\ \text{Signal.} \end{array} \right] \left. \vphantom{\begin{array}{c} \mathbf{K} \\ \text{Signal.} \end{array}} \right\} \begin{array}{l} \text{P sensors} \\ \text{R VOCs} \end{array}$$

## Problem dimensions

R VOCs



N experiments



## Matrix formulation

$$\underbrace{\begin{bmatrix} \mathbf{K} \\ \text{Signal.} \end{bmatrix}}_{R \text{ VOCs}} \left. \vphantom{\begin{bmatrix} \mathbf{K} \\ \text{Signal.} \end{bmatrix}} \right\} P \text{ sensors} \quad \underbrace{\begin{bmatrix} \mathbf{C} \\ \text{Conc.} \end{bmatrix}}_{R \text{ VOCs}} \left. \vphantom{\begin{bmatrix} \mathbf{C} \\ \text{Conc.} \end{bmatrix}} \right\} N \text{ exp.}$$

## Problem dimensions

R VOCs



$A_1$



$A_2$

...



$A_R$

## Matrix formulation

$$\left[ \begin{array}{c} \mathbf{K} \\ \text{Signat.} \end{array} \right] \left. \vphantom{\begin{array}{c} \mathbf{K} \\ \text{Signat.} \end{array}} \right\} \begin{array}{l} \text{P sensors} \\ R \text{ VOCs} \end{array} \quad \left[ \begin{array}{c} \mathbf{C} \\ \text{Conc.} \end{array} \right] \left. \vphantom{\begin{array}{c} \mathbf{C} \\ \text{Conc.} \end{array}} \right\} \begin{array}{l} N \text{ exp.} \\ R \text{ VOCs} \end{array}$$

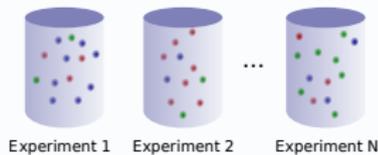
$$\left[ \begin{array}{c} \mathbf{M} \\ \text{Masses} \end{array} \right] \left. \vphantom{\begin{array}{c} \mathbf{M} \\ \text{Masses} \end{array}} \right\} \begin{array}{l} R \text{ VOCs} \\ R \text{ VOCs} \end{array}$$



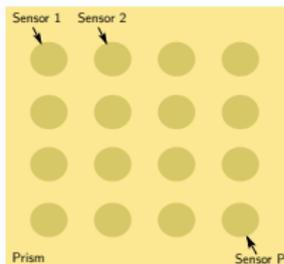
Diagonal

## Problem dimensions

N experiments



P sensors



## Matrix formulation

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 \left[ \begin{array}{c} \mathbf{K} \\ \text{Signal.} \end{array} \right] \left. \vphantom{\begin{array}{c} \mathbf{K} \\ \text{Signal.} \end{array}} \right\} \begin{array}{l} \text{P sensors} \\ R \text{ VOCs} \end{array}
 \end{array}
 \begin{array}{c}
 \left[ \begin{array}{c} \mathbf{C} \\ \text{Conc.} \end{array} \right] \left. \vphantom{\begin{array}{c} \mathbf{C} \\ \text{Conc.} \end{array}} \right\} \begin{array}{l} N \text{ exp.} \\ R \text{ VOCs} \end{array}$$
  

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 \left[ \begin{array}{c} \mathbf{M} \\ \text{Masses} \end{array} \right] \left. \vphantom{\begin{array}{c} \mathbf{M} \\ \text{Masses} \end{array}} \right\} \begin{array}{l} R \text{ VOCs} \\ R \text{ VOCs} \end{array}
 \end{array}
 \begin{array}{c}
 \left[ \begin{array}{c} \mathbf{Y} \\ \text{Measures} \end{array} \right] \left. \vphantom{\begin{array}{c} \mathbf{Y} \\ \text{Measures} \end{array}} \right\} \begin{array}{l} P \text{ sensors} \\ N \text{ exp.} \end{array}$$
  

$$\Downarrow$$

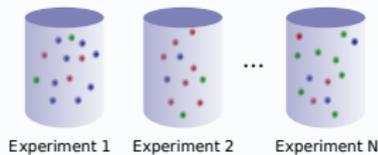
Diagonal

## Problem dimensions

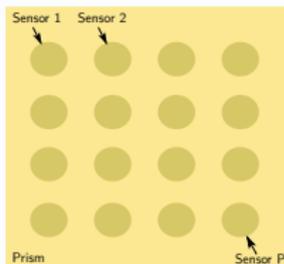
R VOCs



N experiments



P sensors



## Matrix formulation

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$$\underbrace{\begin{bmatrix} \mathbf{M} \\ \text{Masses} \end{bmatrix}}_{R \text{ VOCs}} \left. \vphantom{\begin{bmatrix} \mathbf{M} \\ \text{Masses} \end{bmatrix}} \right\} R \text{ VOCs} \quad \underbrace{\begin{bmatrix} \mathbf{Y} \\ \text{Measures} \end{bmatrix}}_{N \text{ exp.}} \left. \vphantom{\begin{bmatrix} \mathbf{Y} \\ \text{Measures} \end{bmatrix}} \right\} P \text{ sensors}$$

⇓  
Diagonal

Mixture model

$$\mathbf{Y} = \mathbf{KMC}^t \square (\mathbf{1}_P \mathbf{1}_N^t + \mathbf{KC}^t)$$

# Blind Source Separation

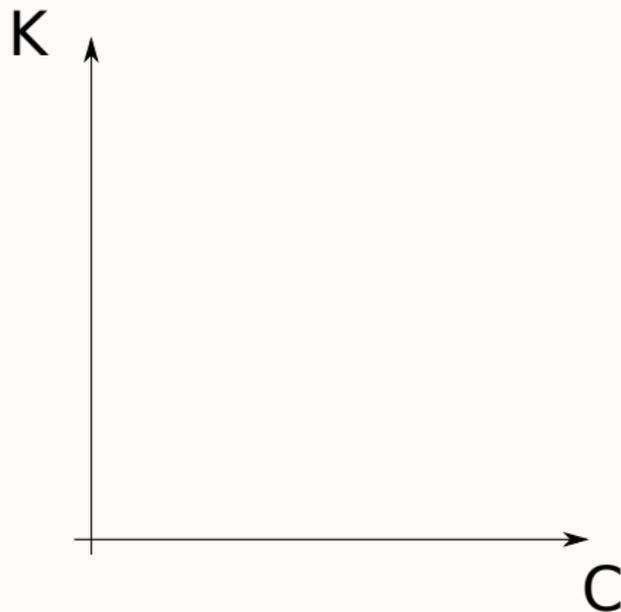
Theoretical results



Mixture model

$$\mathcal{L}(\mathbf{K}, \mathbf{C}) = \mathbf{Y} = \mathbf{K}\mathbf{M}\mathbf{C}^t \square (\mathbf{1}_P\mathbf{1}_N^t + \mathbf{K}\mathbf{C}^t)$$

Particular cases





Mixture model

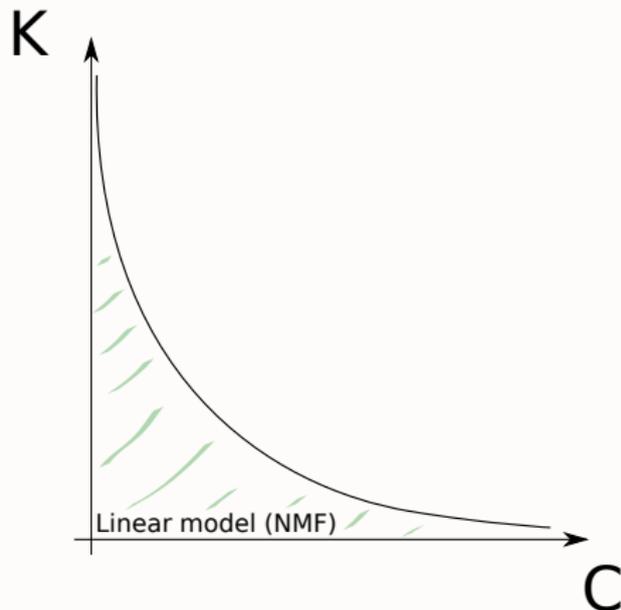
$$\mathcal{L}(K, C) = Y = KMC^t \square (1_P 1_N^t + KC^t)$$

### Particular cases

- ① Low concentration: assuming  $KC^t \ll 1$ , then

$$Y \approx KMC^t$$

→ NMF [5]





Mixture model

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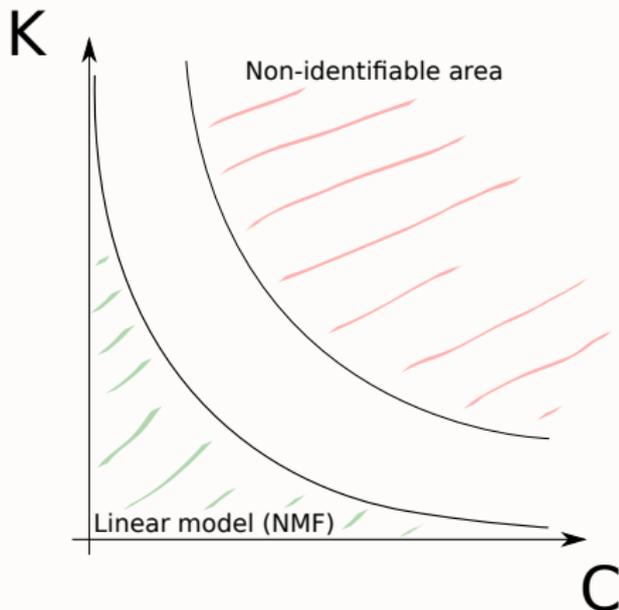
→ NMF [5]

- ② Saturation: assuming high concentrations and/or high affinities, then  $KC^t \gg 1$

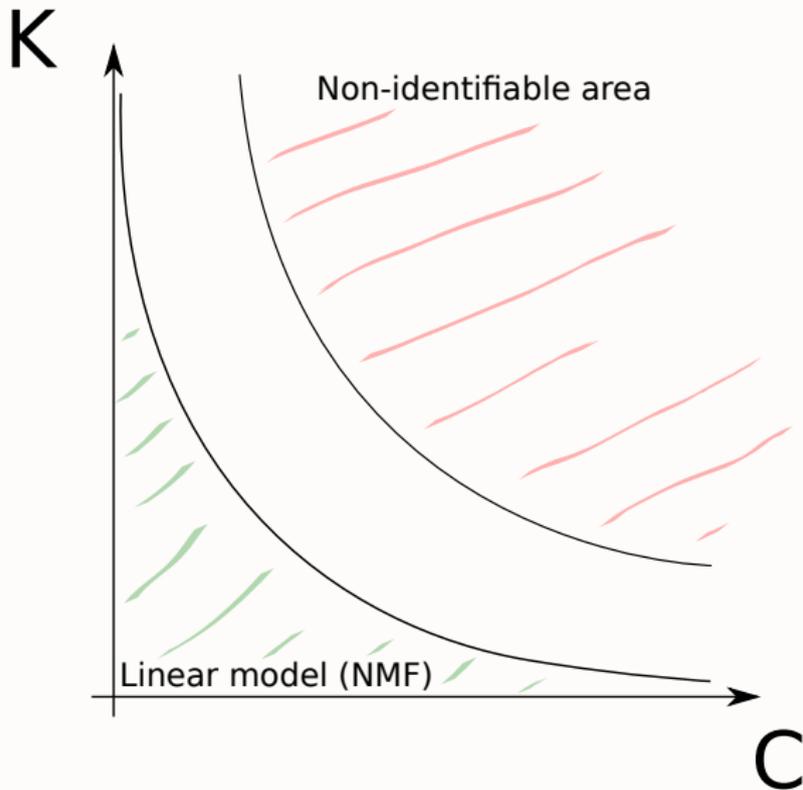
$$Y \approx KMC^t \square KC^t$$

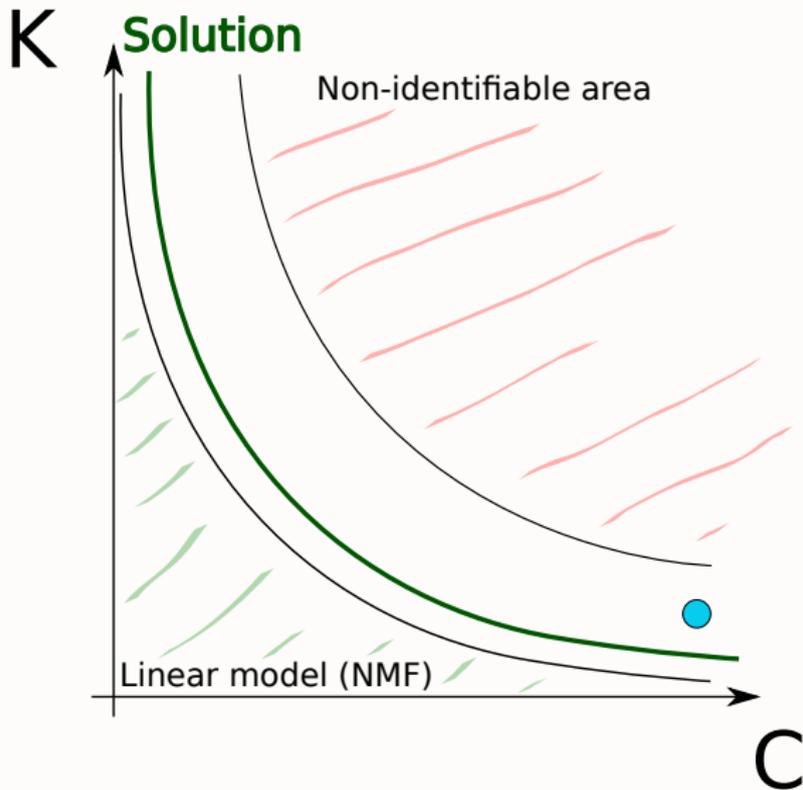
→ highly non-identifiable:

$$Y \approx D_1 KMC^t D_2 \square D_1 KC^t D_2$$



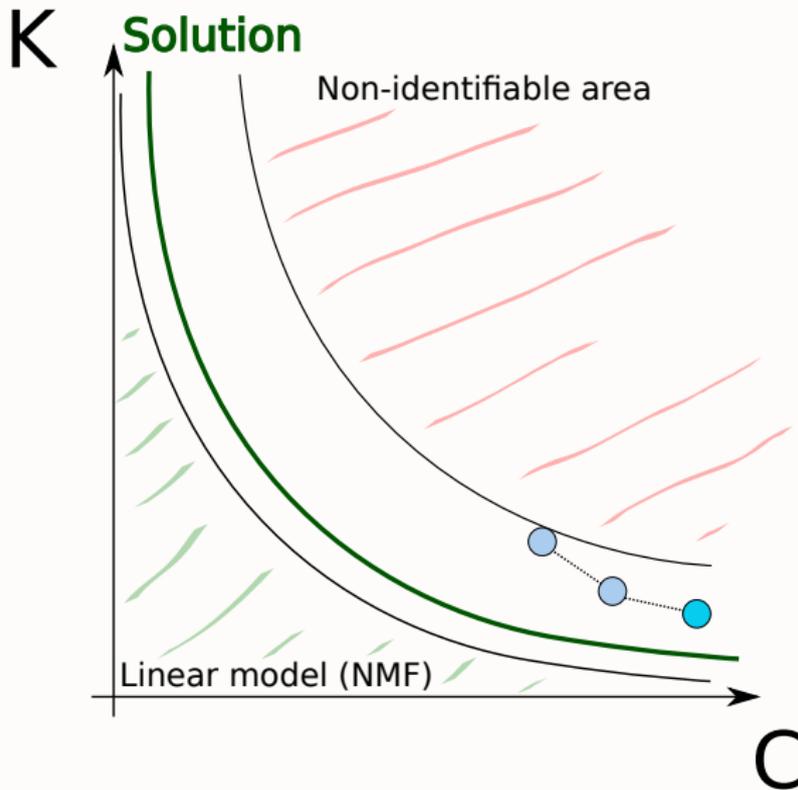
# Blind Source Separation Algorithm





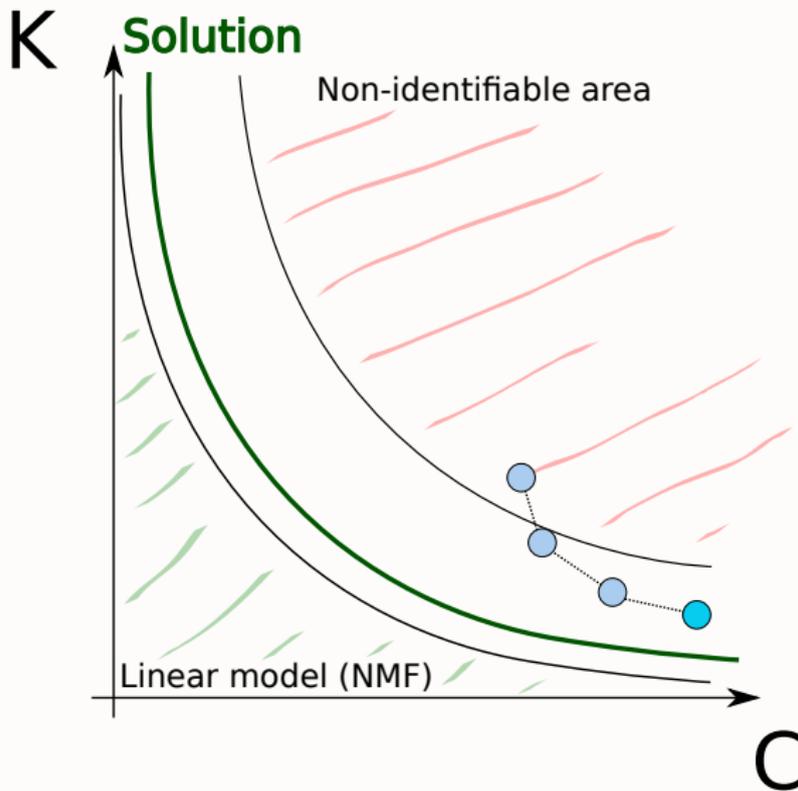
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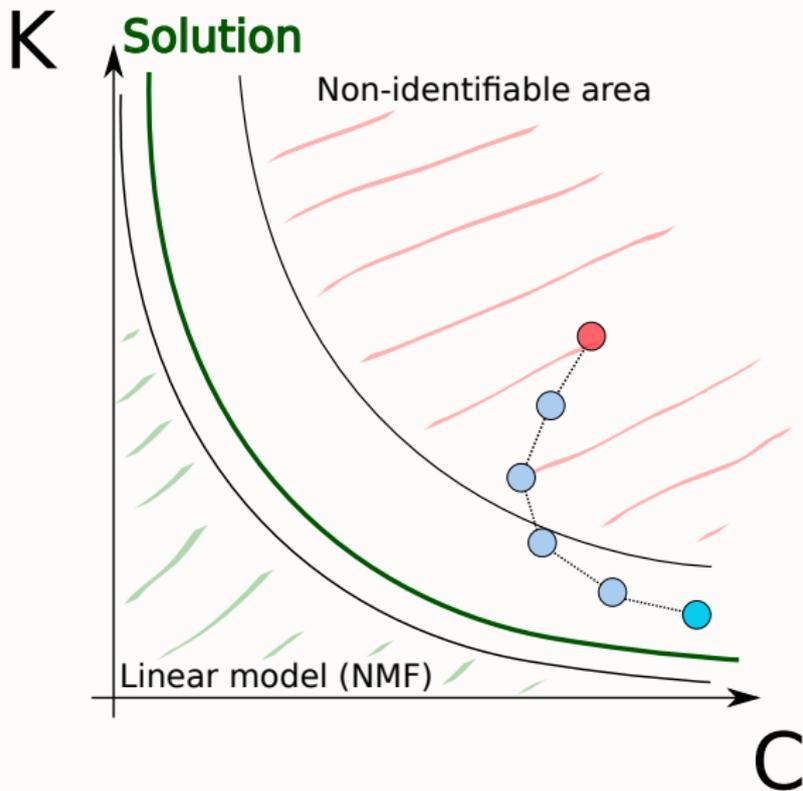
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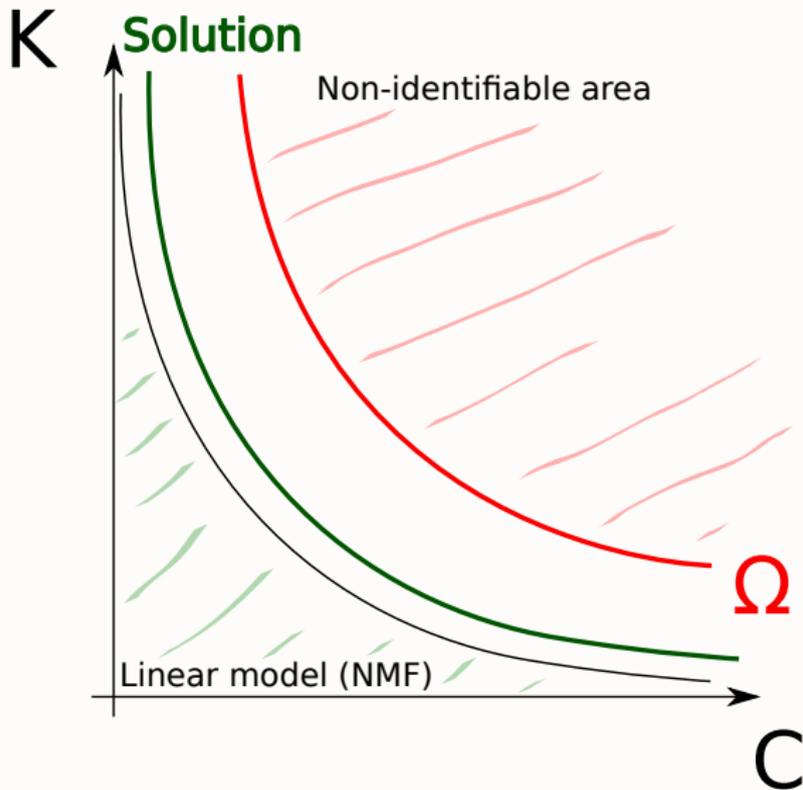
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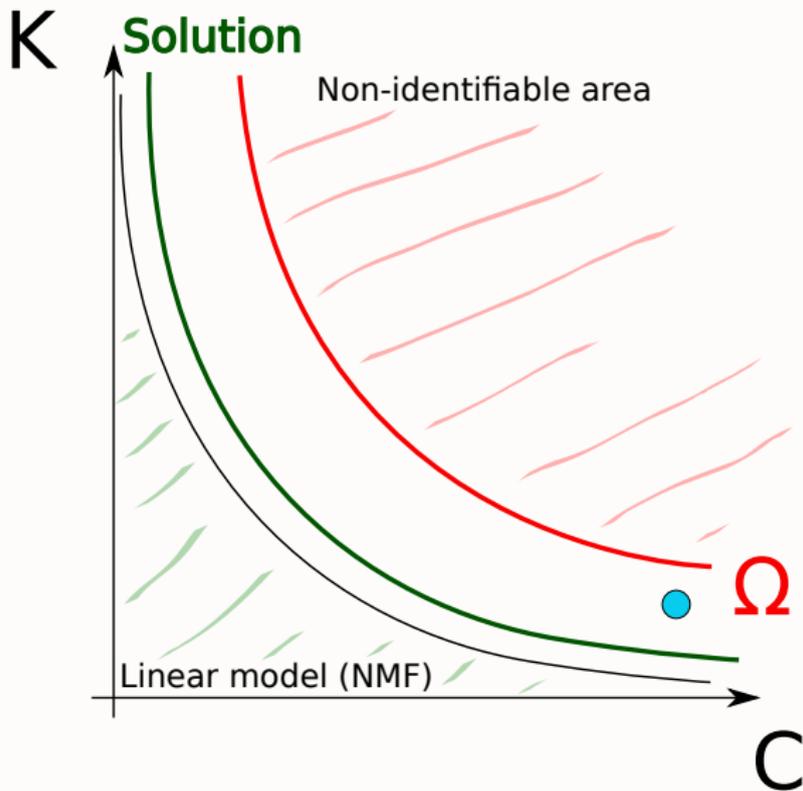
# Blind Source Separation

## Algorithm



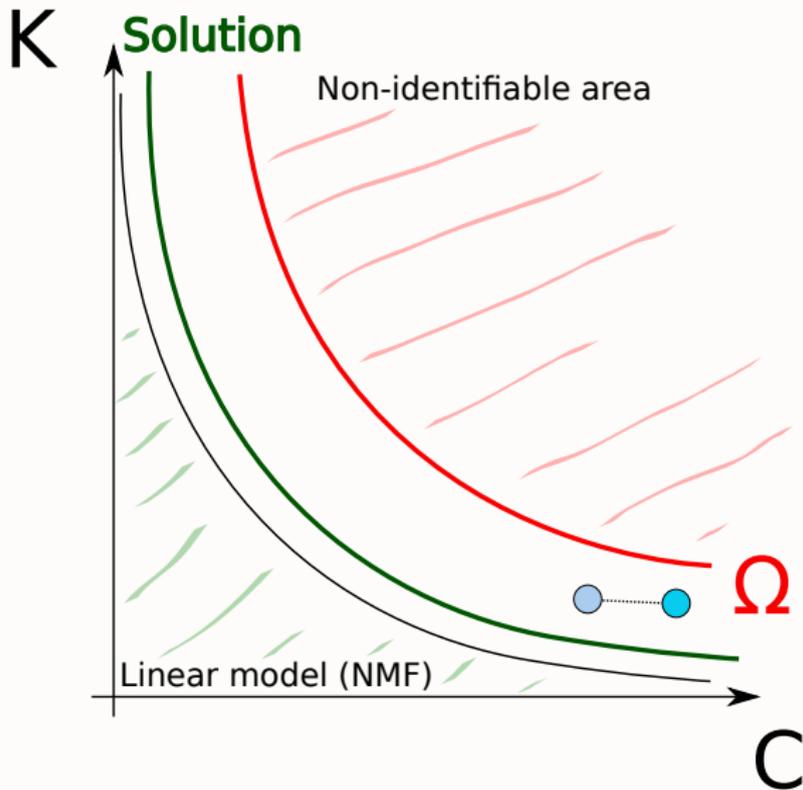
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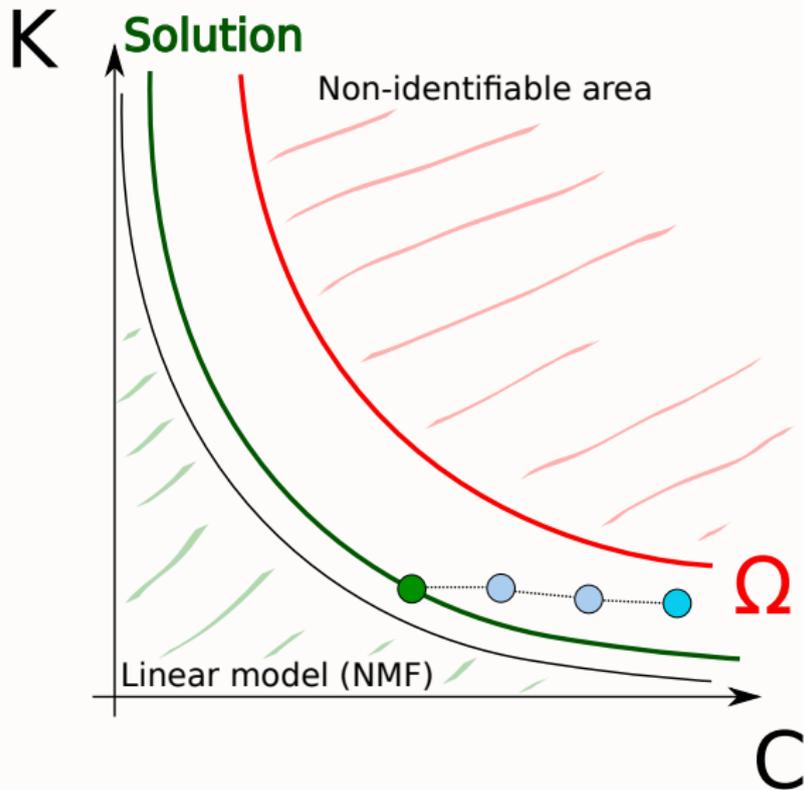
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### Definition of $\Omega$

To avoid the saturation area, we can constrain the product  $\mathbf{KC}$  by :

$$\|\mathbf{KC}^t\|_{\max} = R \sup_{ij} ((\mathbf{KC}^t)_{ij}) \leq \omega$$

This constraint is not so easy to implement, so we relax it by :

$$\|\mathbf{KC}^t\|_{\max} \leq \boxed{\|\mathbf{K}\|_{\max} \|\mathbf{C}^t\|_{\max} \leq \omega}$$



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Cost function:

$$\Upsilon(\mathbf{K}, \mathbf{C}) = \|\mathbf{Y} - \mathcal{L}(\mathbf{K}, \mathbf{C})\|_F$$

Alternating procedure

0 Initialize  $\mathbf{C}$

1 Estimate  $\mathbf{K}$  from the sub-problem:

$$\arg \min_{\mathbf{K} \geq 0, \|\mathbf{K}\|_{\max} \leq \frac{\omega}{\|\mathbf{C}\|_{\max}}} \Upsilon(\mathbf{K}, \mathbf{C})$$

2 Estimate  $\mathbf{C}$  from the sub-problem:

$$\arg \min_{\mathbf{C} \geq 0, \|\mathbf{C}\|_{\max} \leq \frac{\omega}{\|\mathbf{K}\|_{\max}}} \Upsilon(\mathbf{K}, \mathbf{C})$$

3 Repeat from step 1 until convergence

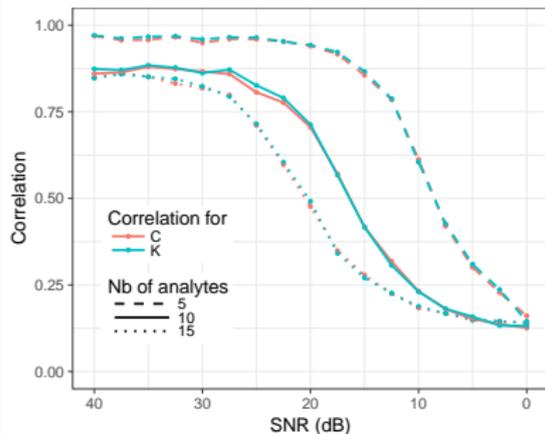
## Simulation settings

$R$ VOCs	5/10/15
$N$ experiments	100
$P$ sensors	100

Data are simulated with an additional gaussian noise:  
 $\mathbf{Y} = \mathcal{L}(\mathbf{K}, \mathbf{C}) + \boldsymbol{\epsilon}$  with  
 $\epsilon_{ij} \sim \mathcal{N}(0, \sigma_n)$

The noise is progressively intensified by decreasing the following Signal to Noise Ratio (SNR):

$$\text{SNR} = 20 \log\left(\frac{\sigma_s}{\sigma_n}\right) \text{ with } \sigma_s = \sqrt{\frac{\|\mathbf{Y}\|_2^2}{PN}}$$





We have

- ① formulated a non-linear mixture model for a type of chemical sensors used in an electronic nose.
- ② proposed an algorithm in order to estimate blindly the individual outputs and the concentrations.
- ③ assessed the performance of the algorithm in the presence of noise.

Further work will

- ① include experiments with real data.
- ② relax the assumption that we know the masses.
- ③ exploit time information.



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Thank you ! Questions ?