Online Mutual Coupling Calibration Using a Signal Source at Unknown Location

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Abstract

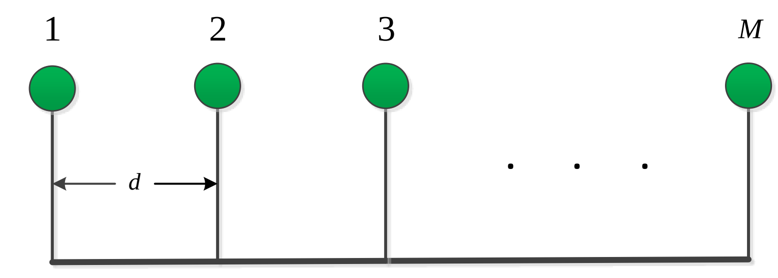
The effect of mutual coupling among the antenna elements will degrade the performance of array signal processing methods. However, existing approaches to deal with this problem either require the angular information of the calibration source(s) or require computationally expensive procedures to determine the mutual coupling coefficients.

In this paper, we derive an efficient mutual coupling calibration method by using a signal source at unknown location. In particular, by exploiting the fact that the mutual coupling matrix (MCM) of uniform linear array (ULA) can be approximated as banded symmetric matrix, it is shown that both the signal direction-of-arrival (DOA) and mutual coupling coefficients can be estimated in closed-form without any spectral grid search.

Numerical simulations are carried out to show the effectiveness and superiority of the proposed algorithm for mutual coupling coefficients and DOA estimation.

Signal Model

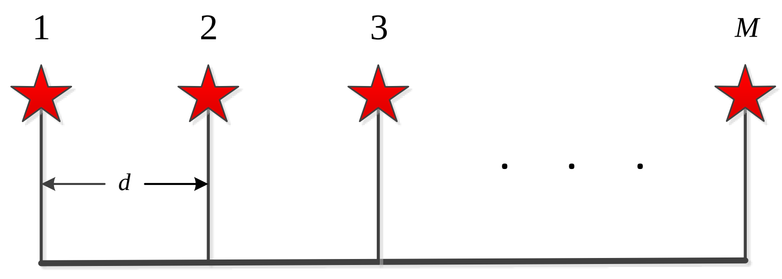
Consider a ULA with M elements, if the antennas are same and there are no other uncertainties, the ideal received signal can be written as



$$\mathbf{x}(t) = \mathbf{a}(\theta)s(t) + \mathbf{n}(t).$$

Figure 1. Ideal array model

However, in the presence of mutual coupling, the steering vector will be distorted. Specifically, the true received signal model be modified as follows [1]



$$\mathbf{x}(t) = \mathbf{C}\mathbf{a}(\theta)s(t) + \mathbf{n}(t)$$

Figure 2. Array with mutual coupling

where $\mathbf{C}$ is a symmetric Toeplitz mutual coupling matrix.

The array covariance matrix is thus given by

$$\mathbf{R} = E[\mathbf{x}(t)\mathbf{x}^H(t)] = \sigma_s^2\mathbf{C}\mathbf{a}(\theta)\mathbf{a}^H(\theta)\mathbf{C}^H + \sigma_n^2\mathbf{I}$$

where $\mathbf{a}(\theta), s(t), \mathbf{n}(t)$ denote steering vector, signal, and noise vector, respectively

Proposed Algorithm

In this work, we tackle the mutual coupling calibration problem with a signal source at unknown location. To begin with, we divide the ULA that exists coupling among the antenna elements into two same subarrays. In addition, the DOA of calibration source could be estimated by solving the rotational invariance equation. Since the mutual coupling coefficient matrix is related to the angular information [2], the matrix can be obtained to compensate the mutual coupling of ULA.

DOA Estimation and Mutual Coupling Calibration

Obtain the covariance matrix — $\mathbf{x}(t) = \mathbf{a}(\theta)s(t) + \mathbf{n}(t)$, $\mathbf{R}_X = E\{\mathbf{x}(t)\mathbf{x}^H(t)\}$

Derive the rotational invariance equation — $\Gamma(\bar{\gamma})\mathbf{e}_2 = \mathbf{e}_1\beta(\theta)$

Solve the equation to get DOA of calibration source — $\hat{\beta}(\theta) = \mathbf{e}_1^{-1}\Gamma(\bar{\gamma})\mathbf{e}_2$

Formulate the relation between mutual coupling coefficient and DOA — $\mathbf{H}\mathbf{c} = \mathbf{g}$

Estimate the vector of mutual coupling coefficient — $\hat{\mathbf{c}} = (\hat{\mathbf{H}}^H\hat{\mathbf{H}})^{-1}\hat{\mathbf{H}}^H\hat{\mathbf{g}}$

More specifically，the special relation between mutual coupling coefficient and DOA of calibration source can be clearly described as [3]

$$\bar{\gamma}_k = \frac{1+\sum_{i=1}^{P-1} c_i\beta(\theta)^{-i} + \sum_{i=1}^{M-k} c_i\beta(\theta)^{i}}{1+\sum_{i=1}^{P-1} c_i\beta(\theta)^{-i} + \sum_{i=1}^{M-k-1} c_i\beta(\theta)^{i}},$$

Simulation Results A

Example 1: Estimated Amplitude and Phase Error Against SNR

- ULA with 10 sensors
- A narrow-band signal with $25.5°$ and 500 snapshots
- Mutual coupling coefficient $c_1 = 0.85e^{(-j\pi/3)}, c_2 = 0.55e^{(-j\pi/5)}$ $c_3 = 0.35e^{(-j\pi/7)}$

Table 1. Estimated amplitude and phase of c_1 against SNR

	Proposed Method		Method in [3]	
SNR	$\lvert\hat{\rho}_1-\rho_1\rvert$	$\lvert\hat{\phi}_1-\phi_1\rvert$	$\lvert\hat{\rho}_1-\rho_1\rvert$	$\lvert\hat{\phi}_1-\phi_1\rvert$
0dB	0.0611166	0.0828583	0.1428299	0.1329692
2dB	0.0365616	0.0778389	0.1006141	0.1271440
4dB	0.0396614	0.0487395	0.0863126	0.0824641
6dB	0.0253214	0.0487513	0.0557826	0.0780522
8dB	0.0239933	0.0316352	0.0557209	0.0464422
10dB	0.0153815	0.0267313	0.0378565	0.0420040

Table 2. Estimated amplitude and phase of c_2 against SNR

	Proposed Method		Method in [3]	
SNR	$\lvert\hat{\rho}_2-\rho_2\rvert$	$\lvert\hat{\phi}_2-\phi_2\rvert$	$\lvert\hat{\rho}_2-\rho_2\rvert$	$\lvert\hat{\phi}_2-\phi_2\rvert$
0dB	0.0610328	0.0246590	0.0888815	0.0571711
2dB	0.0600697	0.0170532	0.0786391	0.0371618
4dB	0.0443455	0.0135230	0.0730502	0.0382958
6dB	0.0282450	0.0113780	0.0428841	0.0238552
8dB	0.0250168	0.0079013	0.0332778	0.0215816
10dB	0.0158789	0.0053462	0.0267459	0.0180756

Simulation Results B

Example 2: Root Mean Square Error (RMSE) Comparison

- All the parameters is same as example 1

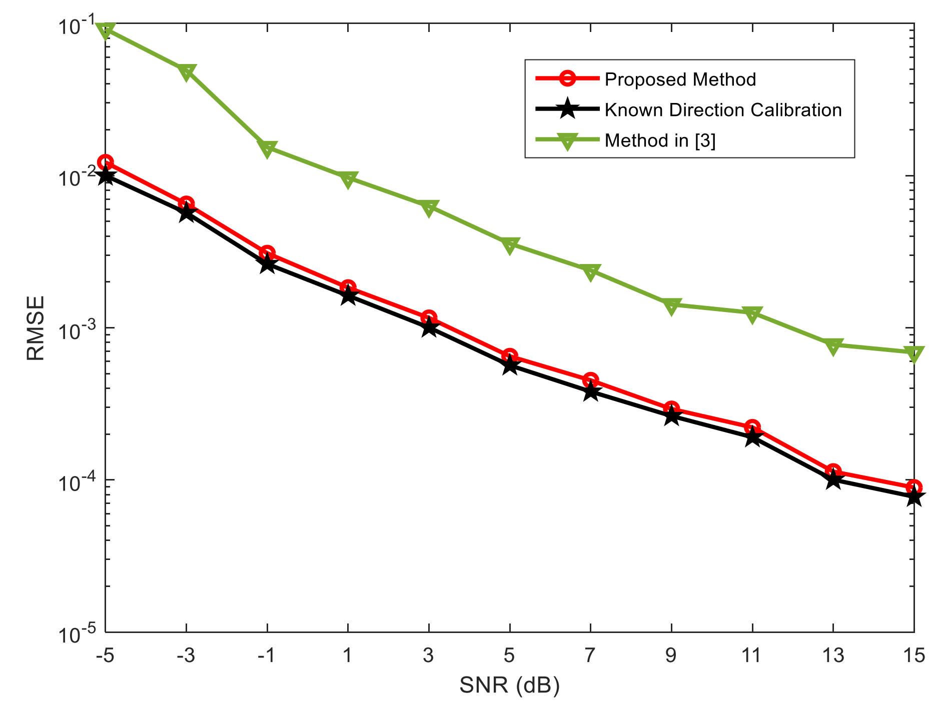


Figure 3. RMSE of Coefficient Amplitude versus SNR.

Simulation Results C

Example 3: Root Mean Square Error (RMSE) Comparison

- ULA with 10 sensors
- A narrow-band signal with $25.5°$ and 200 times Monte Carlo

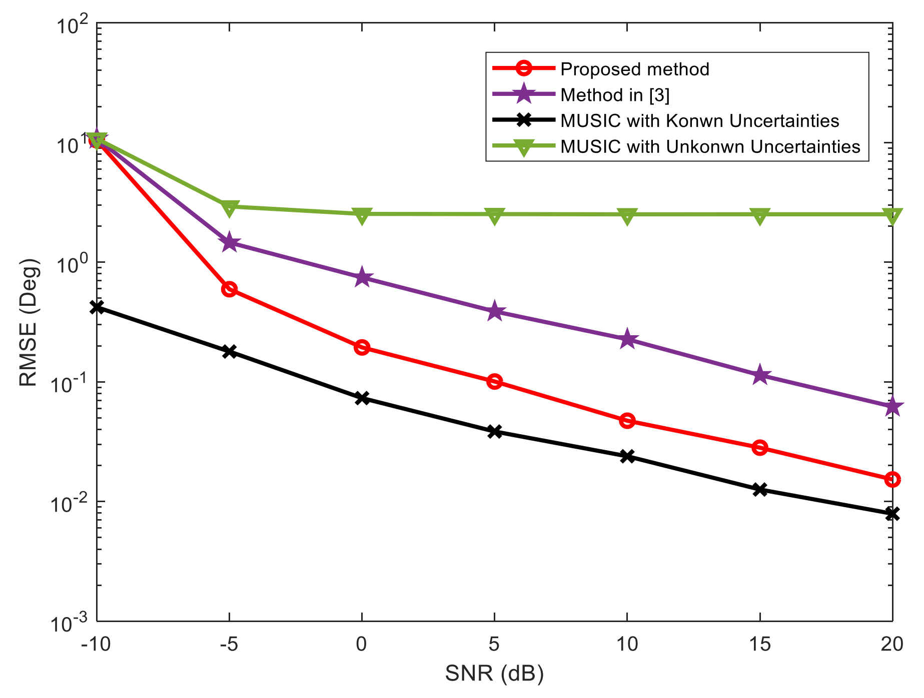


Figure 4. RMSE of DOA Estimation versus SNR.

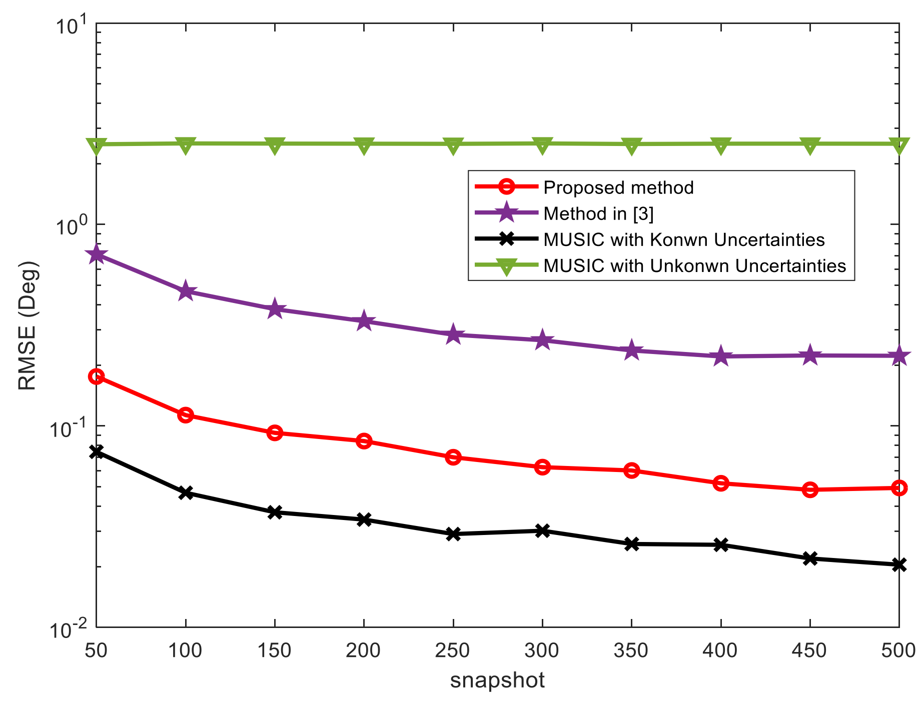


Figure 5. RMSE of DOA estimation versus number of snapshots.

References

[1] B. Liao, Z. G. Zhang, and S. C. Chan, “DOA estimation and tracking of ULAs with mutual coupling,” IEEE Transactions on Aerospace and Electronic Systems, vol. 48, no. 1, pp. 891–905, Jan 2012.
[2] Z. Ye, J. Dai, X. Xu, and X. Wu, “Doa estimation for uniform linear array with mutual coupling,” IEEE Transactions on Aerospace and Electronic Systems, vol. 45, no. 1, pp. 280–288, Jan 2009.
[3] B. Liao, Z. G. Zhang, and S. C. Chan, “A subspace-based method for doa estimation of uniform linear array in the presence of mutual coupling,” in Proceedings of 2010 IEEE International Symposium on Circuits and Systems, May 2010, pp. 1879–1882.