# Antenna Array Based Positional Modulation with a Two-Ray Multi-Path Model

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## Introduction

- Directional modulation (DM) is a physical layer security technique to keep known constellation mappings in a desired direction or directions, while scrambling them for the remaining ones.
- However, eavesdroppers aligned with or very close to the desired direction/directions will be a problem for secure signal transmission, as their received modulation patterns are similar to the given one.
- To make sure that a given modulation pattern can only be received at certain desired positions, one solution is adopting a multi-path model, where signals via both line of sight (LOS) and reflected paths are combined at the receiver side.
- In this work, the typical two-ray multi-path model is studied based on an antenna array.
- The antenna location optimisation problem is investigated in the context of positional modulation and a compressive-sensing based design is proposed.

## Review of Two-Path Model

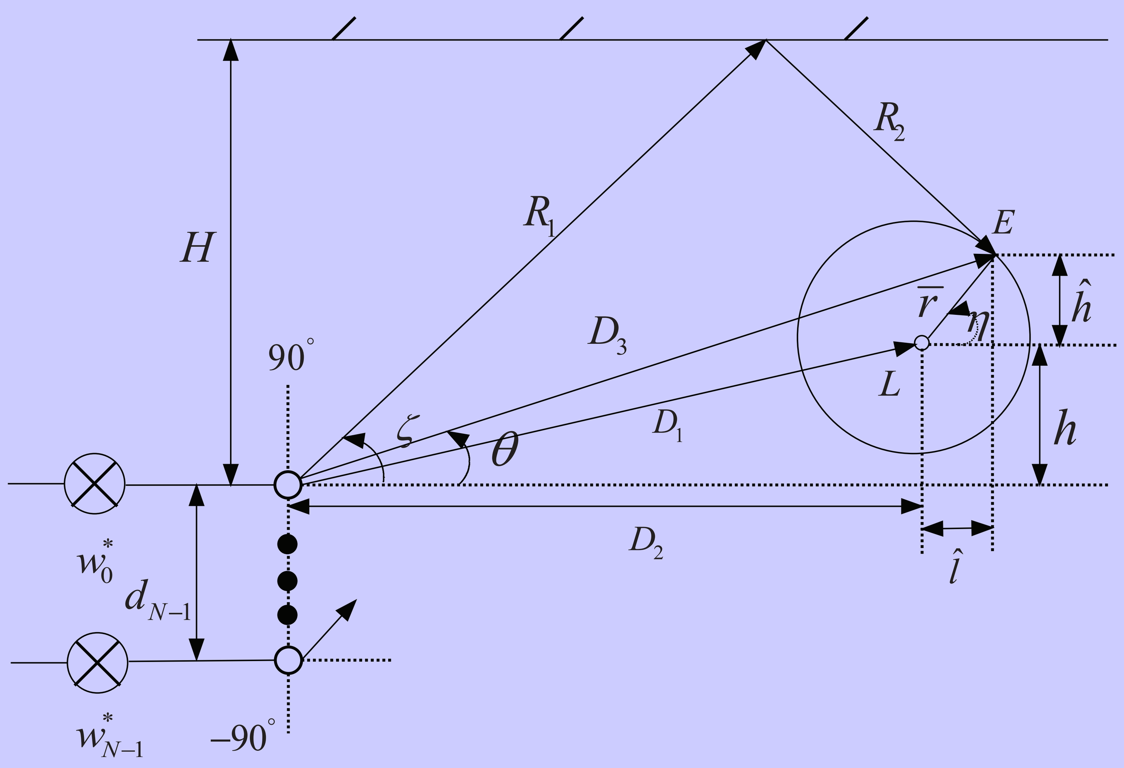


**Figure 1:** Multi-path signal transmission to the desired receiver $L$ and eavesdroppers $E$

- The desired position $L$ with a distance $D_1$ to the transmit array and a vertical distance $h$ to the broadside direction.
- The positions of eavesdroppers $E$ are on the circumference of the circle with the radius $\bar{r}$ and angle $\eta$.
- To produce the required reflected path $R_1 + R_2$, a reflecting surface with a distance $H$ above and perpendicular to the antenna array is created to form the two-ray model.
- In the two-ray model, the beam response of the array is a combination of signals through the LOS path and the reflected path.

## Proposed Design for Positional Modulation

- The objective of positional modulation design is to find a set of weight coefficients creating signals with a given modulation pattern to desired locations, while the modulations of the signals received around them are distorted.
- As each antenna element corresponds to $M$ weight coefficients and these $M$ coefficients correspond to $M$ symbols, to remove the $n$-th antenna, we need all coefficients in the following vector $\tilde{\mathbf{w}}_n$ to be zero-valued or $||\tilde{\mathbf{w}}_n||_2 = 0$,

$$\tilde{\mathbf{w}}_n = [w_{n,0}, \ldots, w_{n,M-1}], \tag{1}$$

where $w_{n,m}$ represents the coefficients on the $n$-th antenna for the $m$-th symbol.
- To calculate the minimum number of antenna elements, we gather all $||\tilde{\mathbf{w}}_n||_2$ for $n = 0, \ldots, N-1$ to form a new vector $\hat{\mathbf{w}}$,

$$\hat{\mathbf{w}} = [||\tilde{\mathbf{w}}_0||_2, ||\tilde{\mathbf{w}}_1||_2, \ldots, ||\tilde{\mathbf{w}}_{N-1}||_2]^T. \tag{2}$$

- Reweighted $l_1$ norm minimisation is used for sparse array design,

$$\begin{aligned} &\min_{\mathbf{W}} \sum_{n=0}^{N-1} \delta_n^u ||\tilde{\mathbf{w}}_n^u||_2 \\ &\text{subject to } ||\mathbf{P}_E - (\tilde{\boldsymbol{\nu}}_E \cdot e^{j\tilde{\boldsymbol{\psi}}_E} \cdot ((\mathbf{W}^u)^H \mathbf{S}_E) + \tilde{\boldsymbol{\xi}}_E \cdot e^{j\tilde{\boldsymbol{\phi}}_E} \cdot ((\mathbf{W}^u)^H \hat{\mathbf{S}}_E))||_2 \leq \alpha \\ &\tilde{\boldsymbol{\nu}}_L \cdot e^{j\tilde{\boldsymbol{\psi}}_L} \cdot ((\mathbf{W}^u)^H \mathbf{S}_L) + \tilde{\boldsymbol{\xi}}_L \cdot e^{j\tilde{\boldsymbol{\phi}}_L} \cdot ((\mathbf{W}^u)^H \hat{\mathbf{S}}_L) = \mathbf{P}_L. \end{aligned} \tag{3}$$

- $\mathbf{S}_L$ and $\mathbf{S}_E$ are the sets of steering vectors for the LOS path to desired receivers and eavesdroppers. $\hat{\mathbf{S}}_L$ and $\hat{\mathbf{S}}_E$ are the sets of steering vectors for the reflected path to desired receivers and eavesdroppers.
- $\mathbf{P}_L$ and $\mathbf{P}_E$ are desired responses for the desired locations and the eavesdroppers.
- $\tilde{\psi}_L$ and $\tilde{\phi}_L$ are the phase shifts for the LOS and reflected paths to desired receivers. $\tilde{\boldsymbol{\psi}}_E$ and $\tilde{\boldsymbol{\phi}}_E$ are the phase shifts for the LOS and reflected paths to eavesdroppers.
- $\tilde{\nu}_L$, $\tilde{\boldsymbol{\xi}}_L$, $\tilde{\boldsymbol{\nu}}_E$ and $\tilde{\boldsymbol{\xi}}_E$ are their corresponding attenuation ratios.
- $\alpha$ is the allowed difference between desired and designed responses.

## Design Examples

- One desired location at the circle centre with $\theta = 0°$, and $H = 500\lambda$, $D_1 = D = 1000\lambda$. Eavesdroppers are located at the circumference of the circle with $\bar{r} = 8.4\lambda$ and $\eta \in [0°, 360°)$, sampled every $1°$.
- The desired response is a value of one magnitude (the gain is $0$dB) with $90°$ phase shift at the desired location (QPSK), i.e. symbols '00', '01', '11', '10' correspond to $45°$, $135°$, $-135°$ and $-45°$, respectively, and a value of $0.1$ (magnitude) with random phase shifts at eavesdroppers.
- Here the signal to noise ratio (SNR) is set at $12$ dB at the desired location, and we assume the additive white Gaussian noise (AWGN) level is at the same level for all eavesdroppers.
- The number of antenna elements for the ULA design is $N = 30$, while for the sparse array design, the maximum aperture of the array is set to $20\lambda$ with $401$ equally spaced potential antennas.

## Results

- The resultant beam pattern for the eavesdroppers is shown in Fig. 2. The response level at all locations of the eavesdroppers ($\eta \in [0°, 360°)$) is lower than $0$dB which is the beam response for the desired locations.
- The resultant phase patterns for the eavesdroppers is shown in Fig. 3. The phase of signal at these eavesdroppers are random while the desired phase for these four symbols should be QPSK modulation.
- Considering the imperfect knowledge of the geometry, e.g. the locations of eavesdroppers are not exactly the same as the locations we thought, and weight coefficients are are designed for $\bar{r} = 8.4\lambda$.
- Then Fig. 4 shows the BERs based on the ULA design in the multi-path model, where BERs at these eavesdroppers in these cases are still much higher than the rate in the desired location ($10^{-5}$).
- While in LOS model, as shown in Fig. 5, BERs based on $\bar{r} = 8.4\lambda$ at some positions of the eavesdroppers are close to $10^{-3}$, lower than the counterpart ($10^{-1}$) in the multi-path model, indicated by dash line in Fig. 4, demonstrating the effectiveness of the multi-path scheme.
- For eavesdroppers close to the desired direction and also integer wavelengths away from the desired location, e.g. $\bar{r} = 8\lambda$, $\eta = 0°$ and $\eta = 180°$, the BERs reach $10^{-5}$, same as in desired locations, much lower than the BERs at these positions in the multi-path model, further demonstrating the effectiveness of the proposed positional modulation designs.

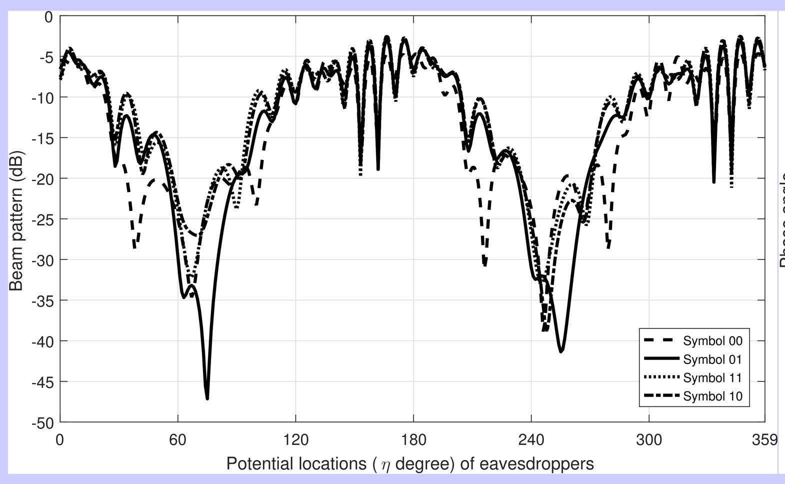


**Figure 2:** Resultant beam pattern based on the ULA design

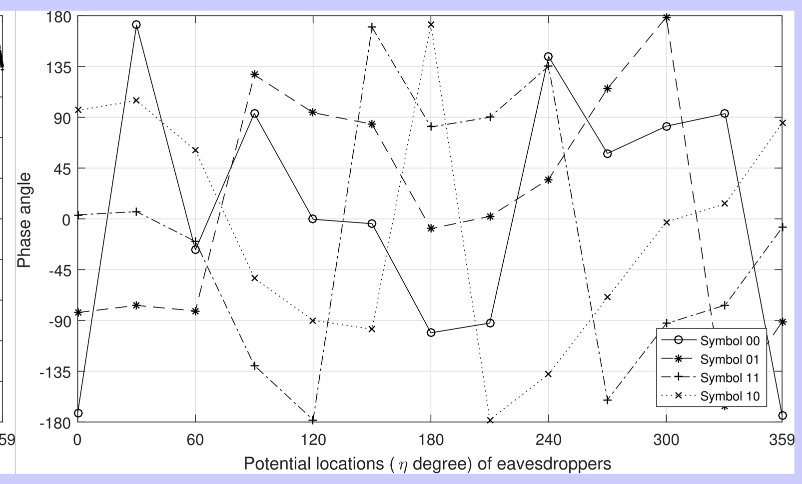


**Figure 3:** Resultant phase pattern based on the ULA design

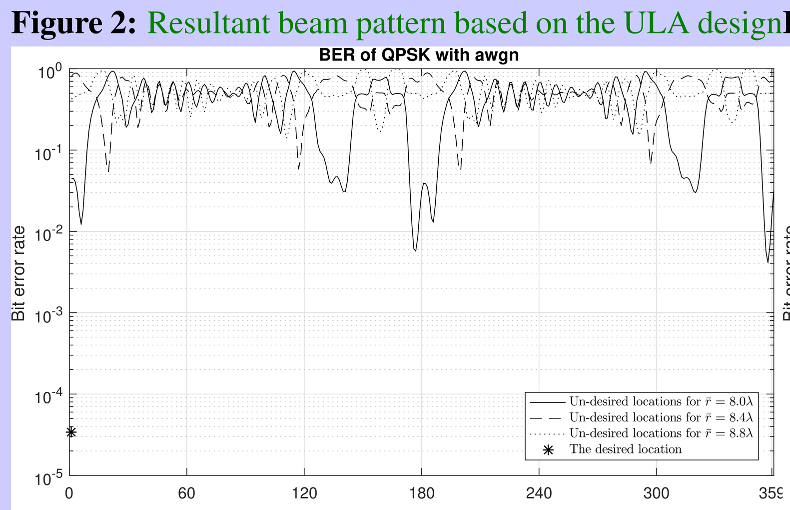


**Figure 4:** BERs patterns for the eavesdroppers and desired receiver based on ULA designs in multi-path model

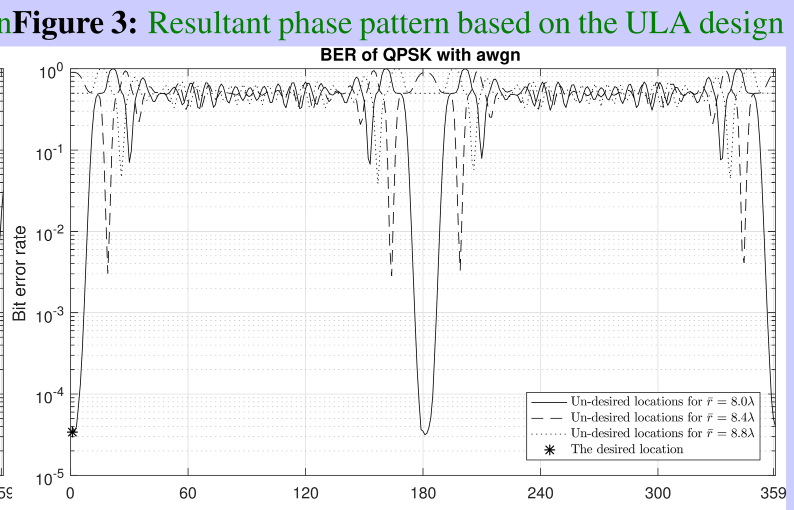


**Figure 5:** BERs patterns for the eavesdroppers and desired receiver based on ULA designs in LOS model

| | ULA | Usua $l_1$ | Reweighted $l_1$ norm |
|---|---|---|---|
| Antenna number | 30 | 117 | 8 |
| Aperture/$\lambda$ | 14.5 | 20 | 19.8 |
| Average spacing/$\lambda$ | 0.5 | 0.1724 | 2.8286 |
| Error norm of array responses | 14.0707 | 13.1773 | 13.9028 |

**Table 1:** Summary of the design results.

## Conclusions

- A two-ray transmission model has been studied for positional modulation, where signals via LOS and reflected paths are combined at the receiver side.
- With the positional modulation technique, signals with a given modulation pattern can only be received at desired locations, but scrambled for positions around them.
- By the proposed designs, the multi-path effect is exploited to overcome the drawback of traditional DM design when eavesdroppers are aligned with or very close to the desired users.
- Examples for a given array geometry and an optimised sparse array have been provided to verify the effectiveness of the proposed designs.