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Sine-based EB-ESPRIT for source localization

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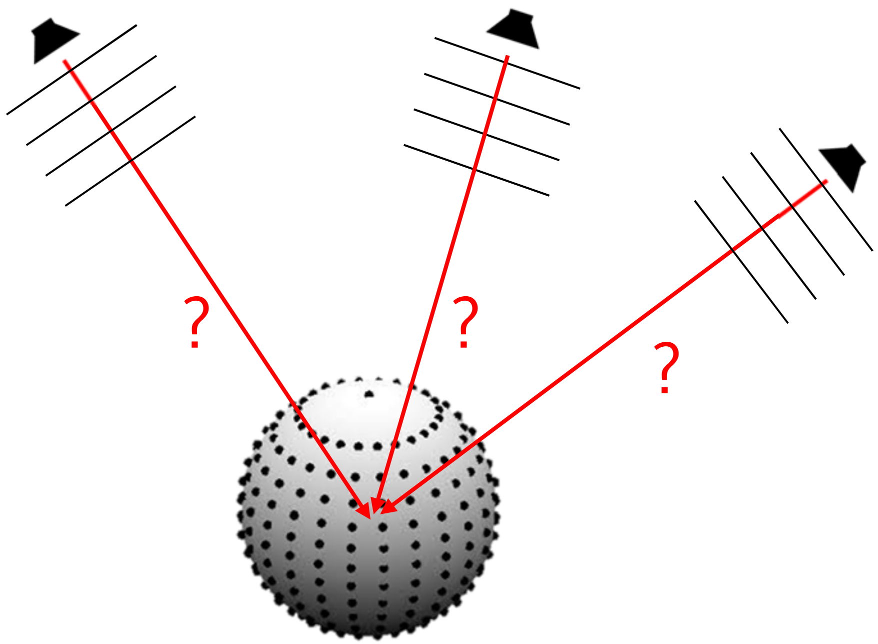
Objective

Direction-of-arrival estimation
using spherical microphone array

Objective

- **Direction-of-Arrival (DOA) estimation**
  - Measurement pressure data → incoming directions of sources

- **Spherical microphone array**
  - Measure 3-D sound field
  - Processing in spherical harmonic domain



Spherical microphone array

Objective

Subspace-based method

- Using orthogonality between signal and noise subspaces
- ESPRIT: Parametric estimation (without grid-search)

EB-ESPRIT (Eigenbeam – Estimation of Signal Parameters via Rotational Invariance Techniques)

- Processing in spherical harmonic domain
- Using a recurrence relation of spherical harmonics

Practical problem

- Singularity of tangent function
  (directional parameter of EB-ESPRIT)

Objective: solve the practical problem of EB-ESPRIT

Background

Signal processing
in spherical harmonic domain

Background | Spherical Fourier transform

Spherical Fourier transform (SFT)

Harmonic coefficients

$$p_{nm} = \int_{4\pi} p(\Omega) Y_n^m(\Omega)^* d\Omega$$

Sound field from D sources

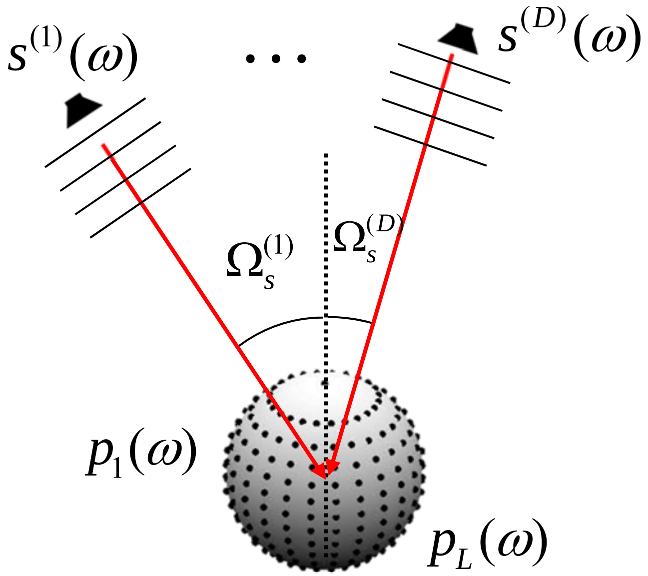
$$p(\Omega) = \sum_{n=0}^{\infty} \sum_{m=-n}^{n} \sum_{d=1}^{D} \underbrace{b_n(kr_0) Y_n^m(\Omega_s^{(d)})^* s^{(d)}}_{p_{nm}} Y_n^m(\Omega)$$

Sound source: $Y_n^m(\Omega_s^{(d)})^* s^{(d)}$

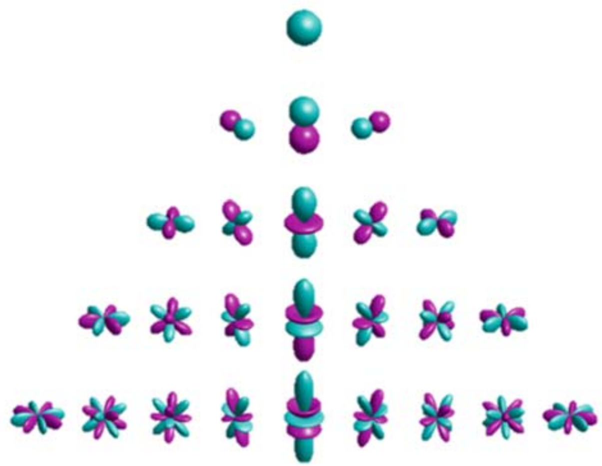
Spherical harmonics (basis function): $Y_n^m(\Omega)$

Radial dependency: $b_n(kr_0)$

$$Y_n^m(\theta,\phi) = \sqrt{\frac{2n+1}{4\pi}\frac{(n-m)!}{(n+m)!}} P_n^m(\cos\theta) e^{im\phi}$$



Measured pressure data (spatial domain)

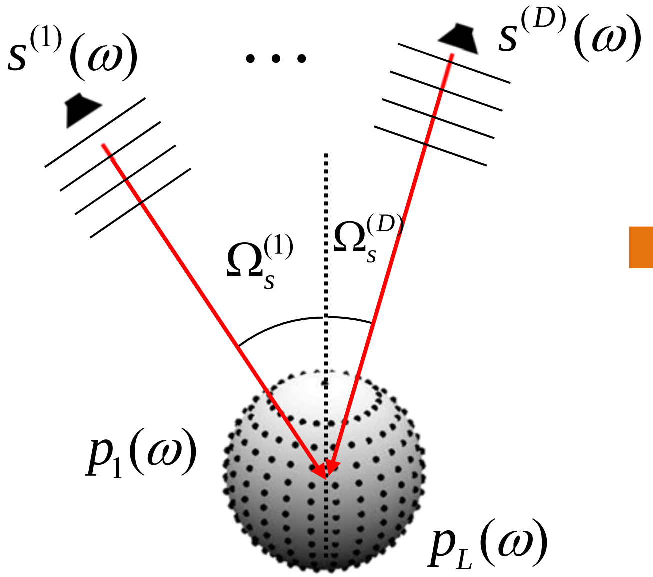


[Boaz, 2015]: B. Rafaely, *Fundamentals of Spherical Array Processing*. NewYork, NY, USA: Springer, 2015.

Spherical harmonics

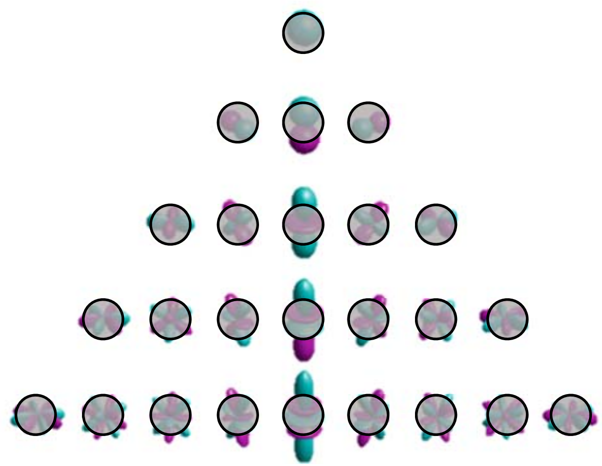
$$p_\ell(\Omega) = \sum_{d=1}^{D} p_\ell^{(d)}(\Omega) \xrightarrow[\times \frac{1}{b_n(kr_0)}]{\text{SFT}} a_{nm} = \sum_{d=1}^{D} Y_n^m(\Omega_s^{(d)})^* s^{(d)}$$

Vector form: $\mathbf{p}$ $\quad \mathbf{a} = \mathbf{Y}^H \mathbf{s}$



Measured pressure data
(spatial domain)

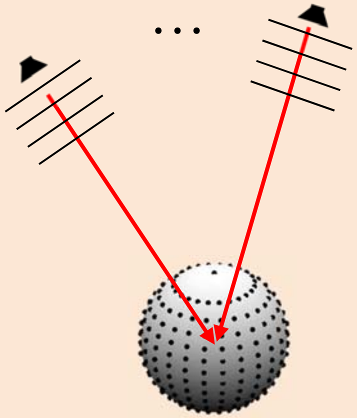
SFT $\times \frac{1}{b_n(kr_0)}$



Directional harmonic
coefficients
(spherical harmonic domain)

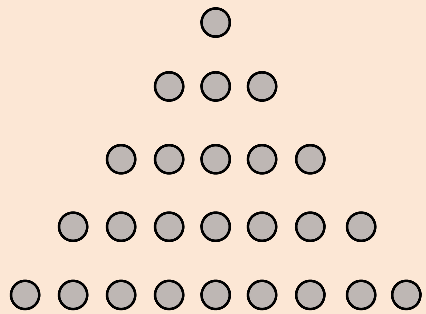
Signal processing in spherical harmonic domain

$$\mathbf{p} = \begin{bmatrix} p_1(\omega) \\ \vdots \\ p_L(\omega) \end{bmatrix}$$



1. Space domain

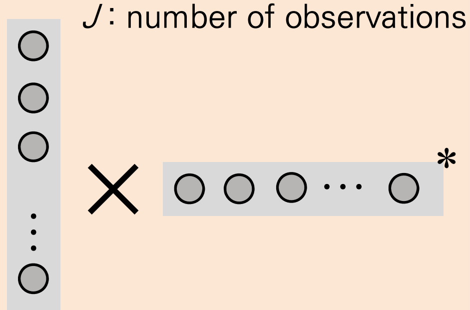
$$\mathbf{a} = \mathbf{Y}^H \mathbf{s} = \begin{bmatrix} a_{0,0}(k) \\ a_{1,-1}(k) \\ \vdots \\ a_{N,N}(k) \end{bmatrix}$$



2. Spherical harmonic domain

$$\mathbf{R} = E\left[\mathbf{a}\mathbf{a}^H\right] \approx \frac{1}{J}\sum_{j=1}^{J}\left(\mathbf{a}^{(j)}\mathbf{a}^{(j)H}\right)$$

J: number of observations

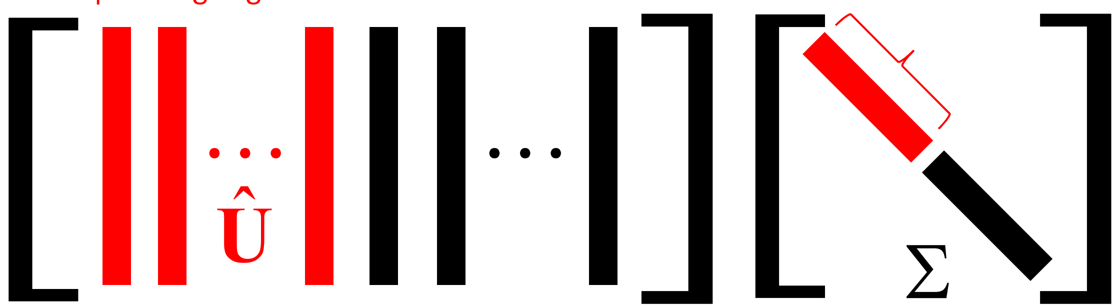


3. Construction of covariance matrix

(*: complex conjugate)

4. Eigenvalue Decomposition

$$\mathbf{R} = \mathbf{U}\mathbf{\Sigma}\mathbf{U}^H = \left[\hat{\mathbf{U}} \;\; \mathbf{U}_n\right]\mathbf{\Sigma}\left[\hat{\mathbf{U}} \;\; \mathbf{U}_n\right]^H$$



Signal processing in spherical harmonic domain

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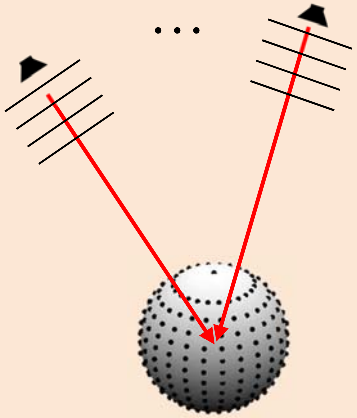

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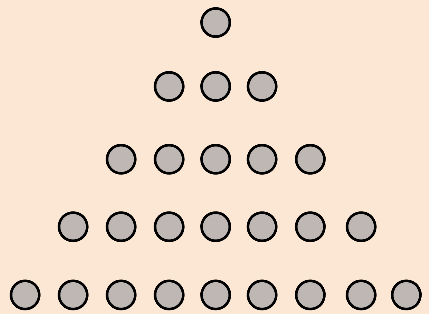

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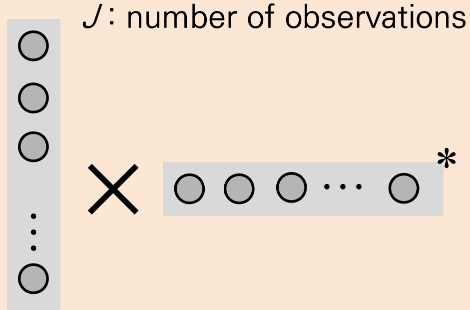

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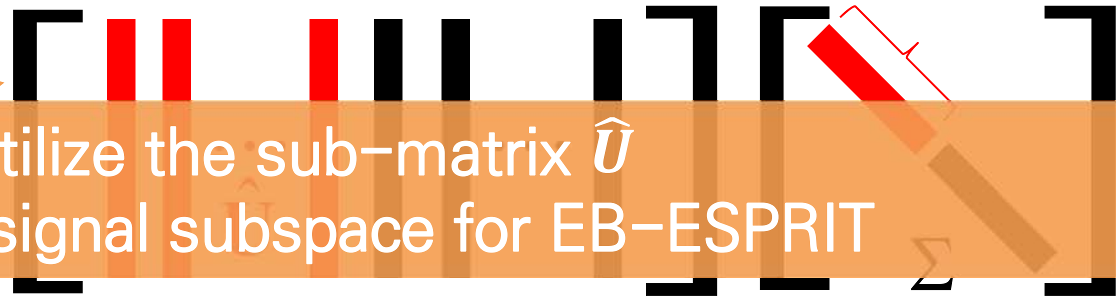
4. Eigenvalue Decomposition

$$\mathbf{R} = \mathbf{U}\Sigma\mathbf{U}^H$$

$$= \left[\hat{\mathbf{U}} \ \mathbf{U}_n\right]\Sigma\left[\hat{\mathbf{U}} \ \mathbf{U}_n\right]^H$$

Corresponding eigenvectors

D largest eigenvalues



Utilize the sub-matrix $\hat{U}$
spanning signal subspace for EB-ESPRIT

Problem Statement

Singularity problem

of EB-ESPRIT

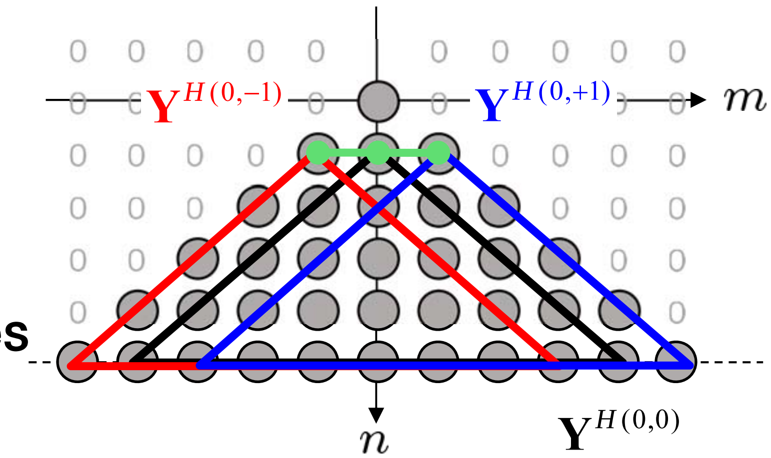
Recurrence relation of spherical harmonics

$$2mY_n^m(\Omega)^* + \Lambda^+ Y_n^{m+1}(\Omega)^* \tan\theta e^{i\phi} + \Lambda^- Y_n^{m-1}(\Omega)^* \tan\theta e^{-i\phi} = 0 \qquad \text{where } \Lambda^\pm = \sqrt{(n \mp m)(n \pm m + 1)}$$

DOA information (unknown)
➔ What we want to know!

- Matrix form

$$2\mathbf{M}\mathbf{Y}^{H(0,0)} + \mathbf{\Lambda}^+ \mathbf{Y}^{(0,+1)H} \mathbf{\Phi} + \mathbf{\Lambda}^- \mathbf{Y}^{(0,-1)H} \mathbf{\Phi}^* = 0$$



Relationship between signal subspaces and the directional coefficients

- Measurable data: $\mathbf{a} = \mathbf{Y}^H \mathbf{s}$

$$\mathbf{R} = E\left[\mathbf{a}\mathbf{a}^H\right] = \mathbf{Y}^H E\left[\mathbf{s}\mathbf{s}^H\right]\mathbf{Y} = \left[\hat{\mathbf{U}} \;\; \mathbf{U}_n\right]\mathbf{\Sigma}\left[\hat{\mathbf{U}} \;\; \mathbf{U}_n\right]^H$$

$$span\{\hat{\mathbf{U}}\} = span\{\mathbf{Y}^H\}$$

$$\hat{\mathbf{U}} = \mathbf{Y}^H \mathbf{T} \qquad \mathbf{T}: \text{ transformation matrix}$$

Recurrence relation for $\mathbf{Y}^H$

$$2\mathbf{M}\mathbf{Y}^{(0,0)H} + \boldsymbol{\Lambda}^{+}\mathbf{Y}^{(0,+1)H}\boldsymbol{\Phi} + \boldsymbol{\Lambda}^{-}\mathbf{Y}^{(0,-1)H}\boldsymbol{\Phi}^{*} = 0$$

$\times\mathbf{T}$

Recurrence relation for $\hat{\mathbf{U}}$

$$2\mathbf{M}\hat{\mathbf{U}}^{(0,0)} + \boldsymbol{\Lambda}^{+}\hat{\mathbf{U}}^{(0,+1)}\boldsymbol{\Psi} + \boldsymbol{\Lambda}^{-}\hat{\mathbf{U}}^{(0,-1)}\boldsymbol{\Psi}^{*} = 0$$

$\hat{\mathbf{U}} = \mathbf{Y}^H\mathbf{T}$

Solve the equation for $\boldsymbol{\Psi}$

Relationship between $\boldsymbol{\Psi}$ and $\boldsymbol{\Phi}$

$$\boldsymbol{\Psi} = \mathbf{T}^{-1}\boldsymbol{\Phi}\mathbf{T}$$

Eigen value decomposition of matrix Ψ

Matrix containing DOA information

$$\boldsymbol{\Phi}$$

$$\hat{\theta}_s^{(d)} = \tan^{-1}\left|\Phi_s^{(d)}\right|, \quad \hat{\phi}_s^{(d)} = \measuredangle\Phi_s^{(d)}$$

Recurrence relation for $\mathbf{Y}^H$

$$2\mathbf{M}\mathbf{Y}^{(0,0)H} + \boldsymbol{\Lambda}^{+}\mathbf{Y}^{(0,+1)H}\boldsymbol{\Phi} + \boldsymbol{\Lambda}^{-}\mathbf{Y}^{(0,-1)H}\boldsymbol{\Phi}^{*} = 0$$

$\times\mathbf{T}$

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$$\boldsymbol{\Phi}$$

EB-ESPRIT: DOA estimation in a parametric manner

Directional parameter for elevation: tangent function (singularity)

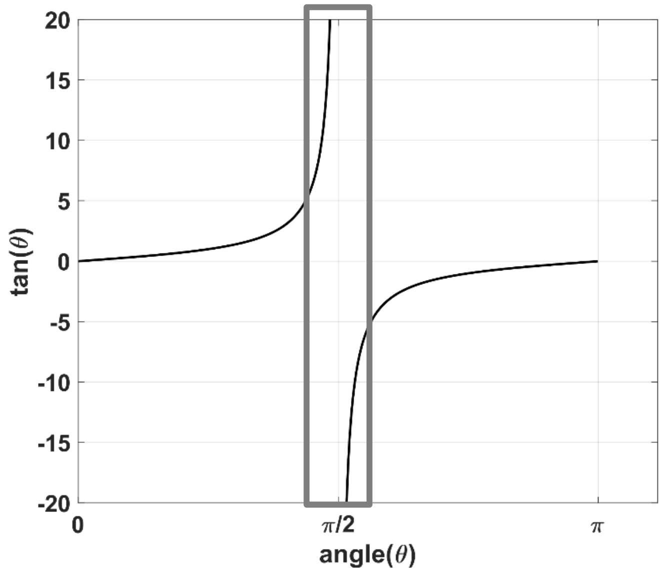
$$2mY_n^m(\Omega_s)^* + \Lambda^+ Y_n^{m+1}(\Omega_s)^* \tan\theta_s e^{i\phi_s} + \Lambda^- Y_n^{m-1}(\Omega_s)^* \tan\theta_s e^{-i\phi_s} = 0$$

Φ_s Φ_s^*

$$\theta = \tan^{-1}(|\Phi_s|)$$

Singularity problem

- Singularity of tangent function near $\theta \cong \pi/2$
- Cannot estimate the DOAs when the sources near the equator

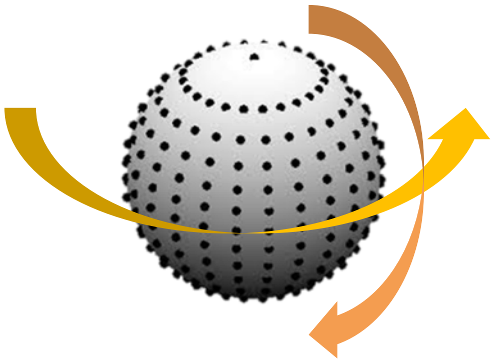


How do we overcome the singularity problem?

Prior works to overcome singularity problem

[Sun *et al*, ICASSP 2011]

- Rotation of the reference coordinate when the robustness measure is bad



→ Additional computations!

[Huang *et al*, IEEE T-ASLP 2017]

- Two-stage decoupled approach (TSDA) :

unitary spherical ESPRIT (U-SHESPRIT) → θ (cosine function)
unitary spherical root-MUSIC (U-SHRMUSIC) → ϕ

→ find elevations and azimuth angles separately
→ 2nd order-reduction of spherical harmonic coefficients (θ)

Prior works to overcome singularity problem

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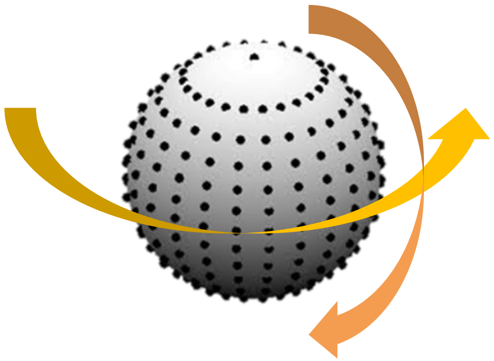

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unitary spherical root-MUSIC (U-SHRMUSIC) → ϕ

→ find elevations and azimuth angles separately
→ 2nd order reduction of spherical harmonic coefficients (θ)

How do we overcome the singularity problem without artifacts?

Proposed method

Sine-based EB-ESPRIT

- Sine-based recurrence relations of spherical harmonics

$$2mY_n^m(\Omega_s)^* + w_{nm}^+ Y_{n-1}^{m+1}(\Omega_s)^* \sin\theta_s e^{i\phi_s} + w_{nm}^- Y_{n-1}^{m-1}(\Omega_s)^* \sin\theta_s e^{-i\phi_s} = 0$$

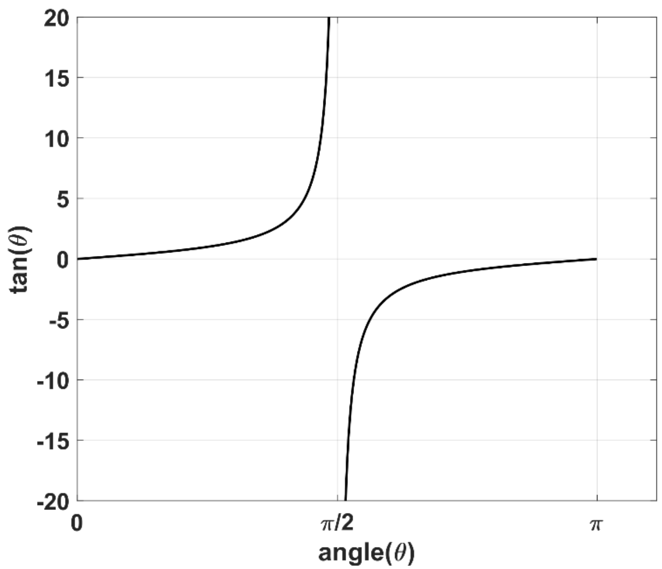
$$2mY_{n-1}^m(\Omega_s)^* + v_{nm}^+ Y_n^{m+1}(\Omega_s)^* \sin\theta_s e^{i\phi_s} + v_{nm}^- Y_n^{m-1}(\Omega_s)^* \sin\theta_s e^{-i\phi_s} = 0$$

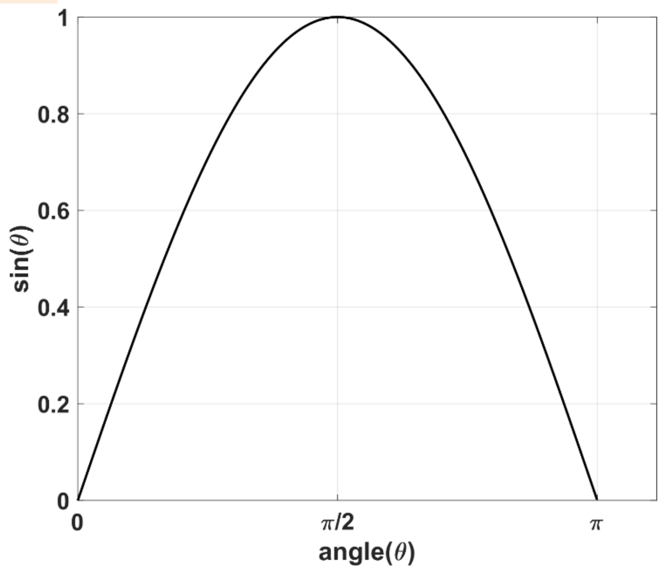
$$w_{nm}^{\pm} = \sqrt{(2n+1)(n\mp m)(n\mp m-1)/(2n-1)}$$

$$v_{nm}^{\pm} = \sqrt{(2n-1)(n\pm m)(n\pm m+1)/(2n+1)}$$

- Estimate the elevation using the arcsine function

$$\theta_s = \sin^{-1}|\Phi_s|$$





Sine-based recurrence relations of spherical harmonics

$$2mY_n^m(\Omega_s)^* + w_{nm}^+ Y_{n-1}^{m+1}(\Omega_s)^* \sin\theta_s e^{i\phi_s} + w_{nm}^- Y_{n-1}^{m-1}(\Omega_s)^* \sin\theta_s e^{-i\phi_s} = 0$$

$$2mY_{n-1}^m(\Omega_s)^* + v_{nm}^+ Y_n^{m+1}(\Omega_s)^* \sin\theta_s e^{i\phi_s} + v_{nm}^- Y_n^{m-1}(\Omega_s)^* \sin\theta_s e^{-i\phi_s} = 0$$

$$w_{nm}^{\pm} = \sqrt{(2n+1)(n\mp m)(n\mp m-1)/(2n-1)}$$

$$v_{nm}^{\pm} = \sqrt{(2n-1)(n\pm m)(n\pm m+1)/(2n+1)}$$

Estimate the elevation using the arcsine function

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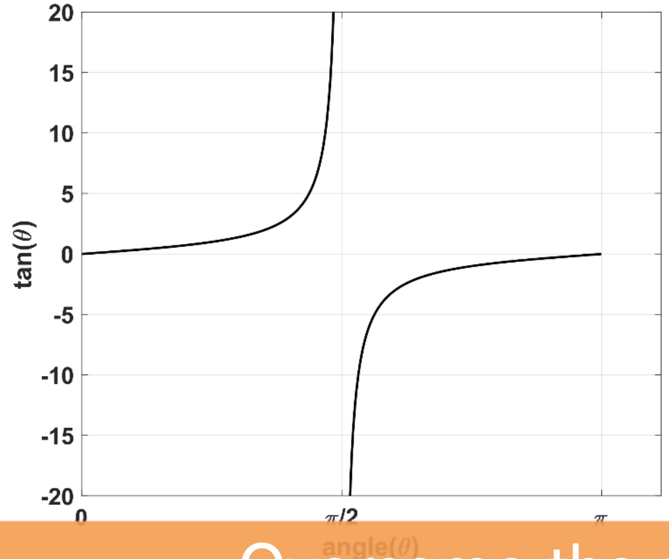
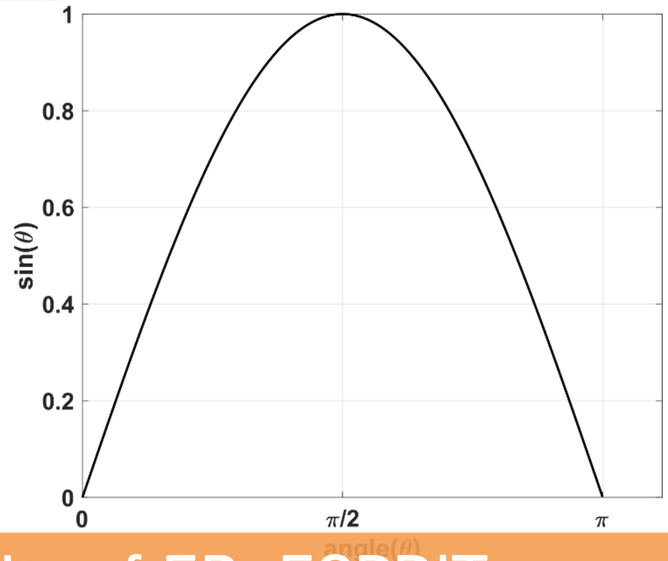




Overcome the singularity of EB-ESPRIT

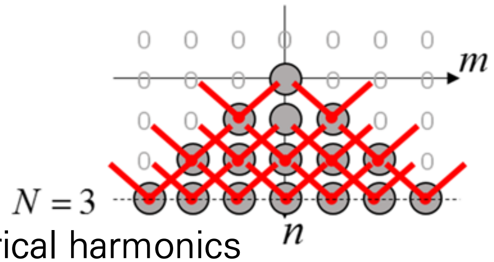
- Two types of recurrence relations of spherical harmonics

$$2mY_n^m(\Omega_s)^* + w_{nm}^+ Y_{n-1}^{m+1}(\Omega_s)^* \sin\theta_s e^{i\phi_s} + w_{nm}^- Y_{n-1}^{m-1}(\Omega_s)^* \sin\theta_s e^{-i\phi_s} = 0$$ → Type 1

- Type 1

# of independent equations:

$(N+1)^2 - 2 = 14$



Spherical harmonics

$|m| \le n \le N,$

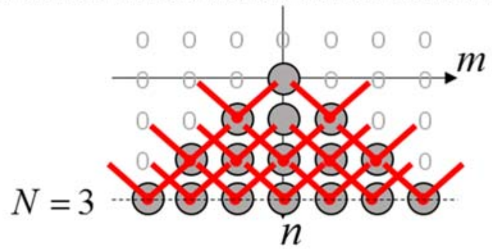
except $(n = m = 0)$ and $(n = 1,\ m = 0)$

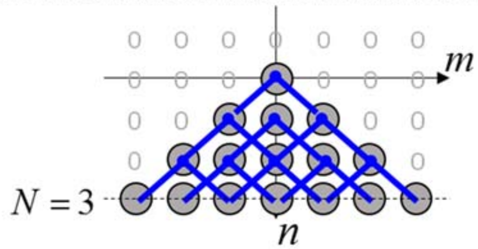
- Two types of recurrence relations of spherical harmonics

$$2mY_n^m\left(\Omega_s\right)^* + w_{nm}^+ Y_{n-1}^{m+1}(\Omega_s)^* \sin\theta_s e^{i\phi_s} + w_{nm}^- Y_{n-1}^{m-1}(\Omega_s)^* \sin\theta_s e^{-i\phi_s} = 0 \quad \rightarrow \text{Type 1}$$

$$2mY_{n-1}^m\left(\Omega_s\right)^* + v_{nm}^+ Y_n^{m+1}\left(\Omega_s\right)^* \sin\theta_s e^{i\phi_s} + v_{nm}^- Y_n^{m-1}\left(\Omega_s\right)^* \sin\theta_s e^{-i\phi_s} = 0 \quad \rightarrow \text{Type 2}$$

- Type 1	- Type 2
# of independent equations: $(N+1)^2 - 2 = 14$	# of independent equations: $N^2 = 9$





$$|m| \le n \le N$$

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- Two types of recurrence relations of spherical harmonics

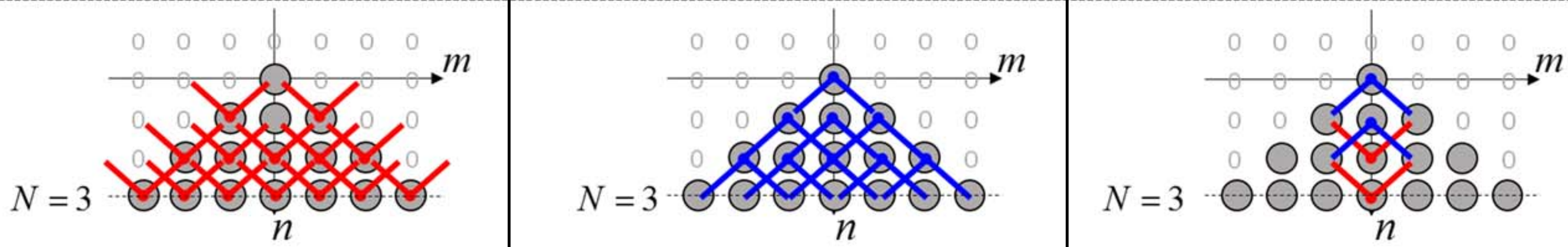
$$2mY_n^m(\Omega_s)^* + w_{nm}^+ Y_{n-1}^{m+1}(\Omega_s)^* \sin\theta_s e^{i\phi_s} + w_{nm}^- Y_{n-1}^{m-1}(\Omega_s)^* \sin\theta_s e^{-i\phi_s} = 0$$ → Type 1

$$2mY_{n-1}^m(\Omega_s)^* + v_{nm}^+ Y_n^{m+1}(\Omega_s)^* \sin\theta_s e^{i\phi_s} + v_{nm}^- Y_n^{m-1}(\Omega_s)^* \sin\theta_s e^{-i\phi_s} = 0$$ → Type 2

- Type 1
\# of independent equations:
$(N+1)^2 - 2 = 14$

- Type 2
\# of independent equations:
$N^2 = 9$

- Redundant case
\# of dependent equations
: $N - 1 = 2$



$m = 0$

Total number of independent equations:

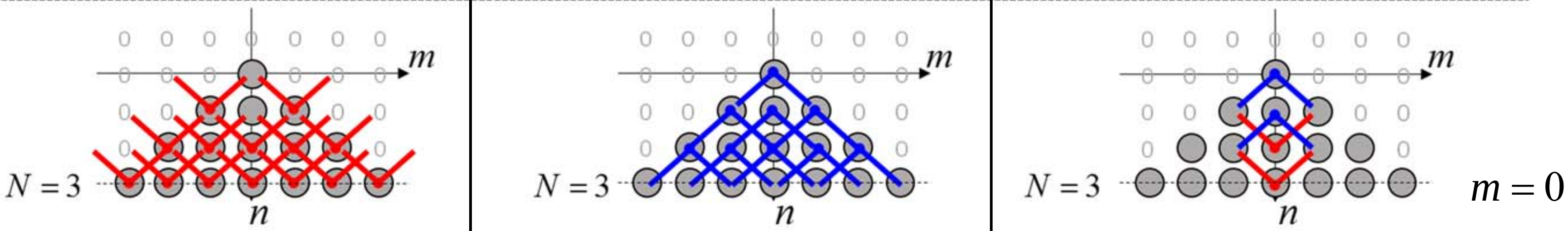
$2N^2 + N = 14 + 9 - 2 = 21$

- Two types of recurrence relations of spherical harmonics

$$2mY_n^m(\Omega_s)^* + w_{nm}^+ Y_{n-1}^{m+1}(\Omega_s)^* \sin\theta_s e^{i\phi_s} + w_{nm}^- Y_{n-1}^{m-1}(\Omega_s)^* \sin\theta_s e^{-i\phi_s} = 0 \quad \rightarrow \text{Type 1}$$

$$2mY_{n-1}^m(\Omega_s)^* + v_{nm}^+ Y_n^{m+1}(\Omega_s)^* \sin\theta_s e^{i\phi_s} + v_{nm}^- Y_n^{m-1}(\Omega_s)^* \sin\theta_s e^{-i\phi_s} = 0 \quad \rightarrow \text{Type 2}$$

- Type 1	- Type 2	- Redundant case
# of independent equations: $(N+1)^2 - 2 = 14$	# of independent equations: $N^2 = 9$	# of dependent equations : $N-1=2$



Total number of independent equations:

# of independent equations → # of detectable sources

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Procedure of DOA estimation

Recurrence relation

$$2mY_n^m(\Omega_s)^* + w_{nm}^+ Y_{n-1}^{m+1}(\Omega_s)^* \sin\theta_s e^{i\phi_s} + w_{nm}^- Y_{n-1}^{m-1}(\Omega_s)^* \sin\theta_s e^{-i\phi_s} = 0$$

$$2mY_{n-1}^m(\Omega_s)^* + v_{nm}^+ Y_n^{m+1}(\Omega_s)^* \sin\theta_s e^{i\phi_s} + v_{nm}^- Y_n^{m-1}(\Omega_s)^* \sin\theta_s e^{-i\phi_s} = 0$$

Matrix form

Recurrence relation for $\mathbf{Y}^H$

$$2\mathbf{M}\mathbf{Y}^{H(0,0)} + \mathbf{W}^+\mathbf{Y}^{H(-1,+1)}\mathbf{\Phi}_{\sin} + \mathbf{W}^-\mathbf{Y}^{H(-1,-1)}\mathbf{\Phi}_{\sin}^* = 0$$

$$2\mathbf{M}\mathbf{Y}^{H(-1,0)} + \mathbf{V}^+\mathbf{Y}^{H(0,+1)}\mathbf{\Phi}_{\sin} + \mathbf{V}^-\mathbf{Y}^{H(0,-1)}\mathbf{\Phi}_{\sin}^* = 0$$

$\times\mathbf{T}$

$\hat{\mathbf{U}} = \mathbf{Y}^H\mathbf{T}$

Recurrence relation for $\hat{\mathbf{U}}$

$$2\mathbf{M}\hat{\mathbf{U}}^{(0,0)} + \mathbf{W}^+\hat{\mathbf{U}}^{(-1,+1)}\mathbf{\Psi}_{\sin} + \mathbf{W}^-\hat{\mathbf{U}}^{(-1,-1)}\mathbf{\Psi}_{\sin}^* = 0$$

$$2\mathbf{M}\hat{\mathbf{U}}^{(-1,0)} + \mathbf{V}^+\hat{\mathbf{U}}^{(0,+1)}\mathbf{\Psi}_{\sin} + \mathbf{V}^-\hat{\mathbf{U}}^{(0,-1)}\mathbf{\Psi}_{\sin}^* = 0$$

Procedure of sine-based EB-ESPRIT

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Stacking equations

$$2[\mathbf{M} \quad \mathbf{M}]\begin{bmatrix}\hat{\mathbf{U}}^{(0,0)} \\ \hat{\mathbf{U}}^{(-1,0)}\end{bmatrix} + \begin{bmatrix}\mathbf{W}^{+}\hat{\mathbf{U}}^{(-1,+1)} & \mathbf{W}^{-}\hat{\mathbf{U}}^{(-1,-1)} \\ \mathbf{V}^{+}\hat{\mathbf{U}}^{(0,+1)} & \mathbf{V}^{-}\hat{\mathbf{U}}^{(0,-1)}\end{bmatrix}\begin{bmatrix}\mathbf{\Psi}_{\sin} \\ \mathbf{\Psi}^{*}_{\sin}\end{bmatrix} = 0 \quad \Rightarrow \quad 2\underline{\mathbf{M}}\underline{\mathbf{U}} + \mathbf{E}\underline{\mathbf{\Psi}}_{\sin} = 0$$

Solve the equation for $\underline{\mathbf{\Psi}}_{\sin}$

Least squared solution

$$\underline{\mathbf{\Psi}}_{\sin} = -2\mathbf{E}^{\dagger}\underline{\mathbf{M}}\underline{\mathbf{U}}$$

Relationship between $\mathbf{\Psi}_{\sin}$ and $\mathbf{\Phi}_{\sin}$

$$\mathbf{\Psi}_{\sin} = \mathbf{T}^{-1}\mathbf{\Phi}_{\sin}\mathbf{T}$$

Eigen value decomposition of matrix $\mathbf{\Psi}_{\sin}$

Matrix containing DOA information $\mathbf{\Phi}_{\sin}$

$$\hat{\theta}_s^{(d)} = \sin^{-1}\left|\Phi_{\sin}^{(d)}\right|, \quad \hat{\phi}_s^{(d)} = \measuredangle\Phi_{\sin}^{(d)}$$

Procedure of sine-based EB-ESPRIT

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Stacking equations

$$2[\mathbf{M}\quad \mathbf{M}]\begin{bmatrix}\hat{\mathbf{U}}^{(0,0)}\\ \hat{\mathbf{U}}^{(-1,0)}\end{bmatrix}+\begin{bmatrix}\mathbf{W}^{+}\hat{\mathbf{U}}^{(-1,+1)} & \mathbf{W}^{-}\hat{\mathbf{U}}^{(-1,-1)}\\ \mathbf{V}^{+}\hat{\mathbf{U}}^{(0,+1)} & \mathbf{V}^{-}\hat{\mathbf{U}}^{(0,-1)}\end{bmatrix}\begin{bmatrix}\mathbf{\Psi}_{\text{sin}}\\ \mathbf{\Psi}^{*}_{\text{sin}}\end{bmatrix}=0$$

$$2\underline{\mathbf{M}}\underline{\mathbf{U}}+\mathbf{E}\underline{\mathbf{\Psi}}_{\text{sin}}=0$$

Solve the equation for $\underline{\mathbf{\Psi}}_{\text{sin}}$

Least squared solution

$$\underline{\mathbf{\Psi}}_{\text{sin}}=-2\mathbf{E}^{\dagger}\underline{\mathbf{M}}\underline{\mathbf{U}}$$

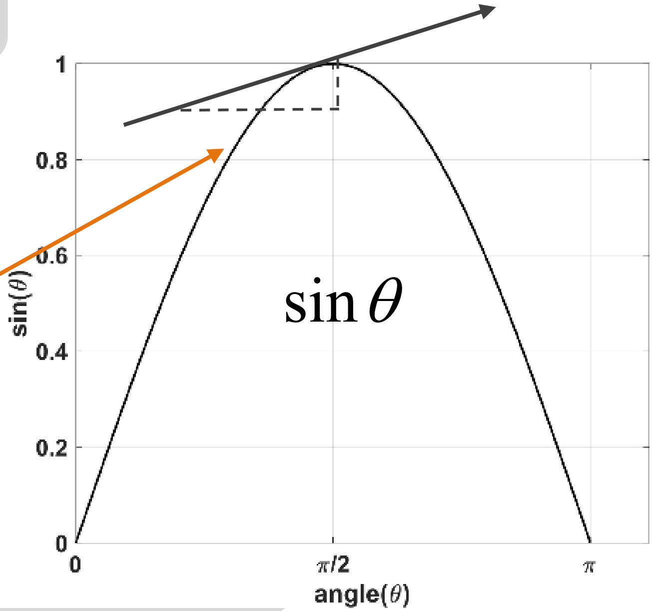
Relationship between $\mathbf{\Psi}_{\text{sin}}$ and $\mathbf{\Phi}_{\text{sin}}$

$$\mathbf{\Psi}_{\text{sin}}=\mathbf{T}^{-1}\mathbf{\Phi}_{\text{sin}}\mathbf{T}$$

Eigen v

Matrix containing DOA information $\mathbf{\Phi}_{\text{sin}}$

$$\hat{\theta}_s^{(d)}=\sin^{-1}\left|\Phi_{\text{sin}}^{(d)}\right|,\quad \hat{\phi}_s^{(d)}$$



Procedure of sine-based EB-ESPRIT

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Stacking equations

$$2[\mathbf{M} \quad \mathbf{M}]\begin{bmatrix}\hat{\mathbf{U}}^{(0,0)} \\ \hat{\mathbf{U}}^{(-1,0)}\end{bmatrix} + \begin{bmatrix}\mathbf{W}^{+}\hat{\mathbf{U}}^{(-1,+1)} & \mathbf{W}^{-}\hat{\mathbf{U}}^{(-1,-1)} \\ \mathbf{V}^{+}\hat{\mathbf{U}}^{(0,+1)} & \mathbf{V}^{-}\hat{\mathbf{U}}^{(0,-1)}\end{bmatrix}\begin{bmatrix}\mathbf{\Psi}_{\sin} \\ \mathbf{\Psi}^{*}_{\sin}\end{bmatrix} = 0$$

$$2\underline{\mathbf{M}}\underline{\mathbf{U}} + \mathbf{E}\underline{\mathbf{\Psi}}_{\sin} = 0$$

Solve the equation for $\underline{\mathbf{\Psi}}_{\sin}$

Least squared solution

$$\underline{\mathbf{\Psi}}_{\sin} = -2\mathbf{E}^{\dagger}\underline{\mathbf{M}}\underline{\mathbf{U}}$$

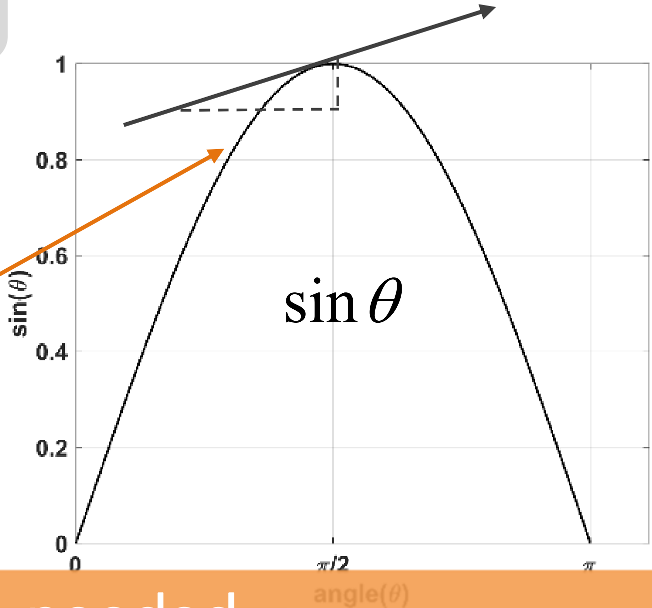
Relationship between $\mathbf{\Psi}_{\sin}$ and $\mathbf{\Phi}_{\sin}$

$$\mathbf{\Psi}_{\sin} = \mathbf{T}^{-1}\mathbf{\Phi}_{\sin}\mathbf{T}$$

Eigen v

Matrix containing DOA information $\mathbf{\Phi}_{\sin}$

$$\hat{\theta}_{s}^{(d)} = \sin^{-1}\left|\Phi_{\sin}^{(d)}\right|, \quad \hat{\phi}_{s}^{(d)}$$



Performance validation is needed

Performance validation

Simulation

Simulation for performance validation

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Simulation configuration

- Number of microphones : 32
  spherical t-design sampling, radius of sphere = 7.5 cm
- Maximum harmonics order: N = 3
- Target frequency: 1,456 Hz ($kr = 2$)
- Self-microphone noise (incoherent to signal)
- Compute covariance matrix: averaging 300 independent snapshots

Error measure of DOA estimation

- Estimation error

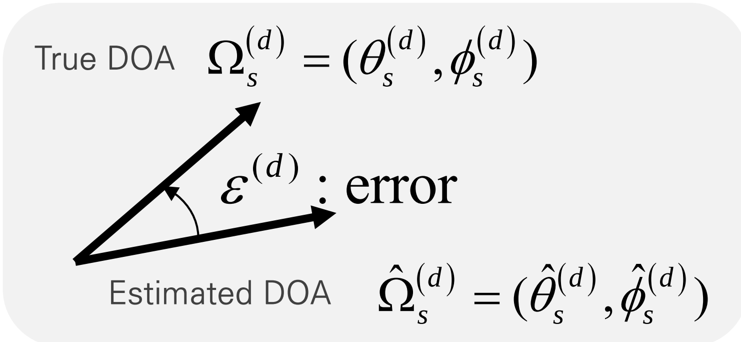
$$\varepsilon^{(d)} = \left|\Omega_s^{(d)} - \hat{\Omega}_s^{(d)}\right|, \quad \Delta\theta^{(d)} = \left|\theta_s^{(d)} - \hat{\theta}_s^{(d)}\right|, \ \Delta\phi^{(d)} = \left|\phi_s^{(d)} - \hat{\phi}_s^{(d)}\right|$$

- Signal-to-Noise Ratio (SNR)

$$\text{SNR (dB)} = 10\log_{10}\left(D\sigma_s^2 / L\sigma_n^2\right),$$

σ_s^2, σ_n^2: power of signals and noises

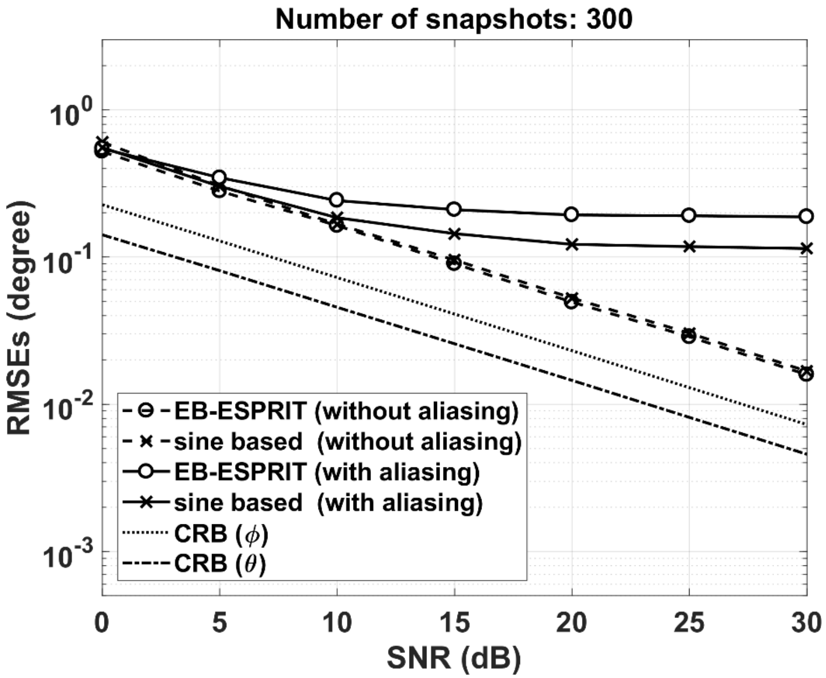
- Number of detectable sources



- Validation for two incoherent sources

$$\left(\theta_s^{(1)},\phi_s^{(1)}\right)=\left(20°,45°\right),\ \left(\theta_s^{(2)},\phi_s^{(2)}\right)=\left(46°,68°\right)$$

$$\text{RMSE}=\sqrt{\tfrac{1}{JD}\sum_{j=1}^{J}\sum_{d=1}^{D}\left|\varepsilon_j^{(d)}\right|^2}\ ,\ J=400\ \text{independent trials}$$



Single source simulation
Without spatial aliasing

Estimation performance is comparable to EB-ESPRIT

Simulation | RMSEs with various elevation angles

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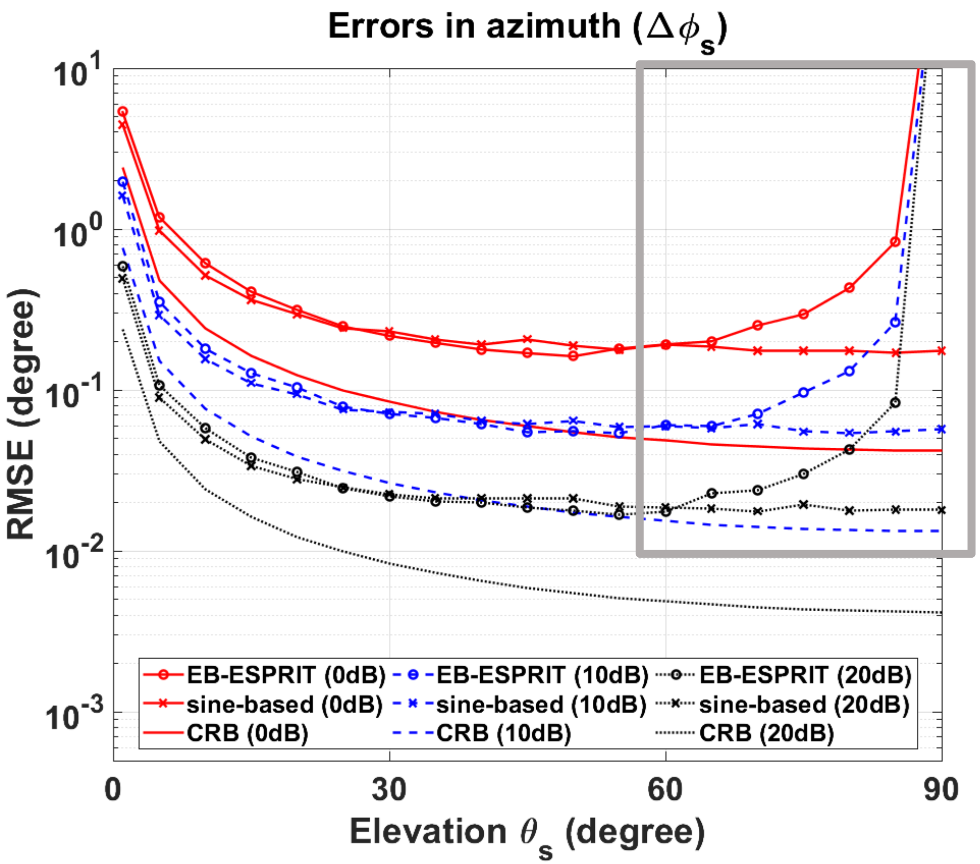
$$\mathrm{RMSE}(\theta)=\sqrt{\frac{1}{JD}\sum_{j=1}^{J}\sum_{d=1}^{D}\left|\Delta\theta^{(d)}\right|^2}$$

$$\mathrm{RMSE}(\phi)=\sqrt{\frac{1}{JD}\sum_{j=1}^{J}\sum_{d=1}^{D}\left|\Delta\phi^{(d)}\right|^2},\ J=400 \text{ independent trials}$$

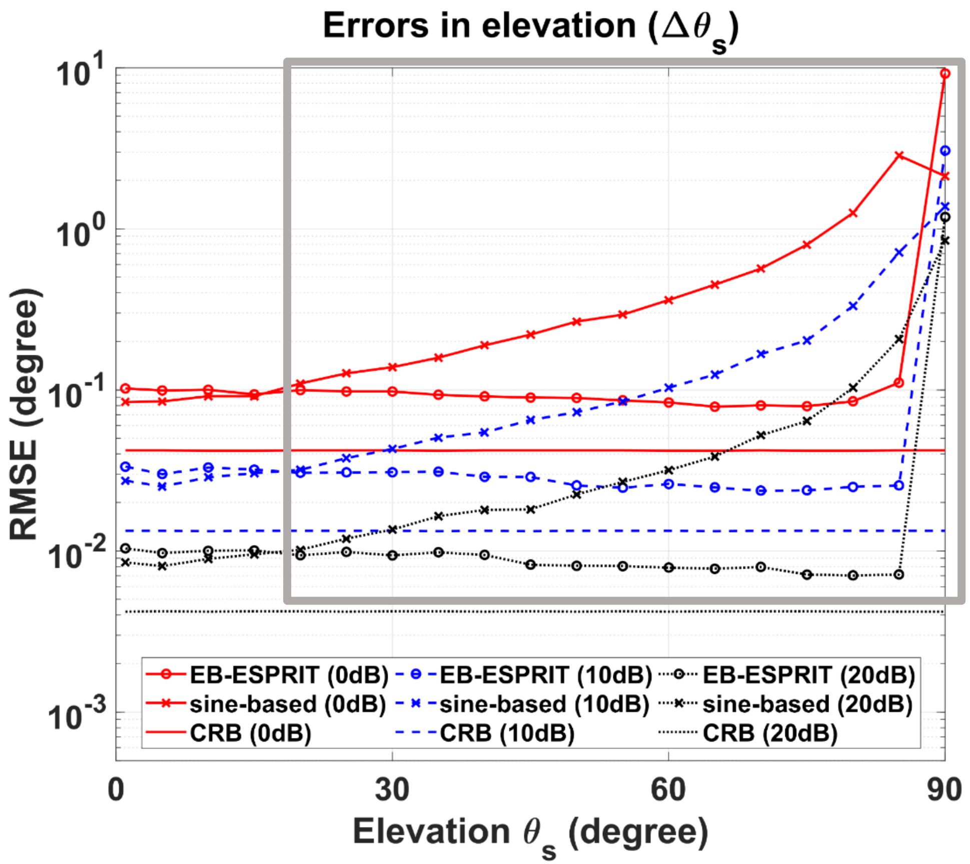
- Single source: $\phi_s = 60°,\ 0° \le \theta_s \le 90°$
- Validate the proposed method by comparing the RMSEs with various elevation angles and SNRs



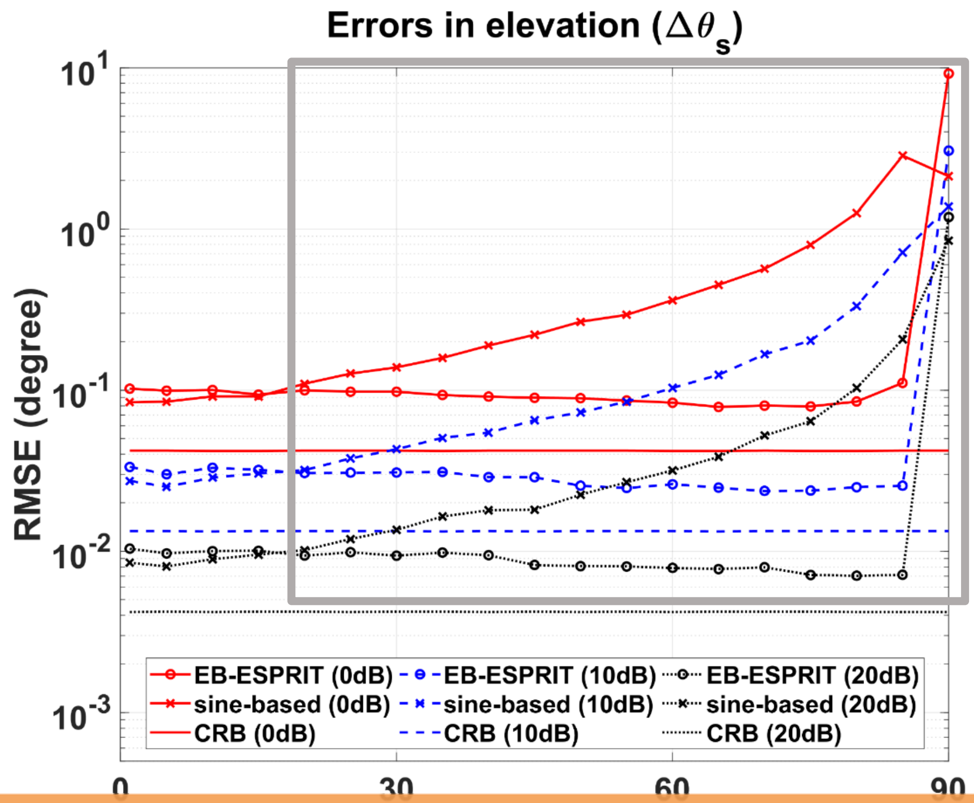
Single source simulation
Without spatial aliasing

Simulation | RMSEs with various elevation angles

- Performance degradation in elevation
- Sine function slowly changes near $\theta_s = 90°$



- Performance degradation in elevation
- Sine function slowly changes near $\theta_s = 90°$

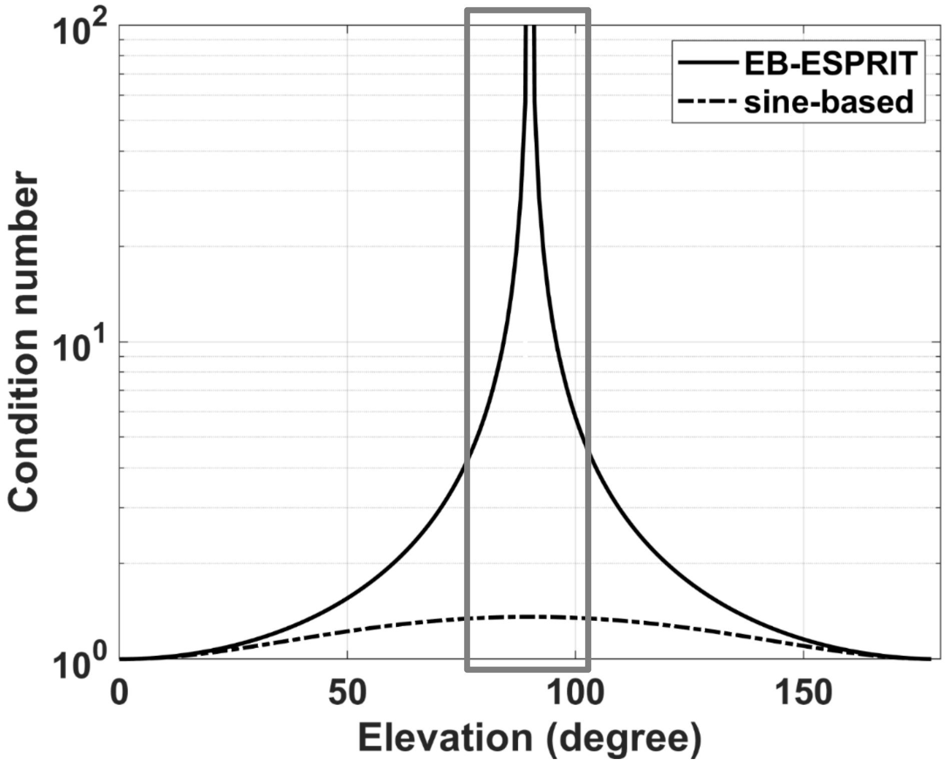


Despite performance degradation in elevation, a reasonable performance with error under 2°

Simulation | Robustness : 2-norm condition number

- Robustness measure: 2-norm condition number

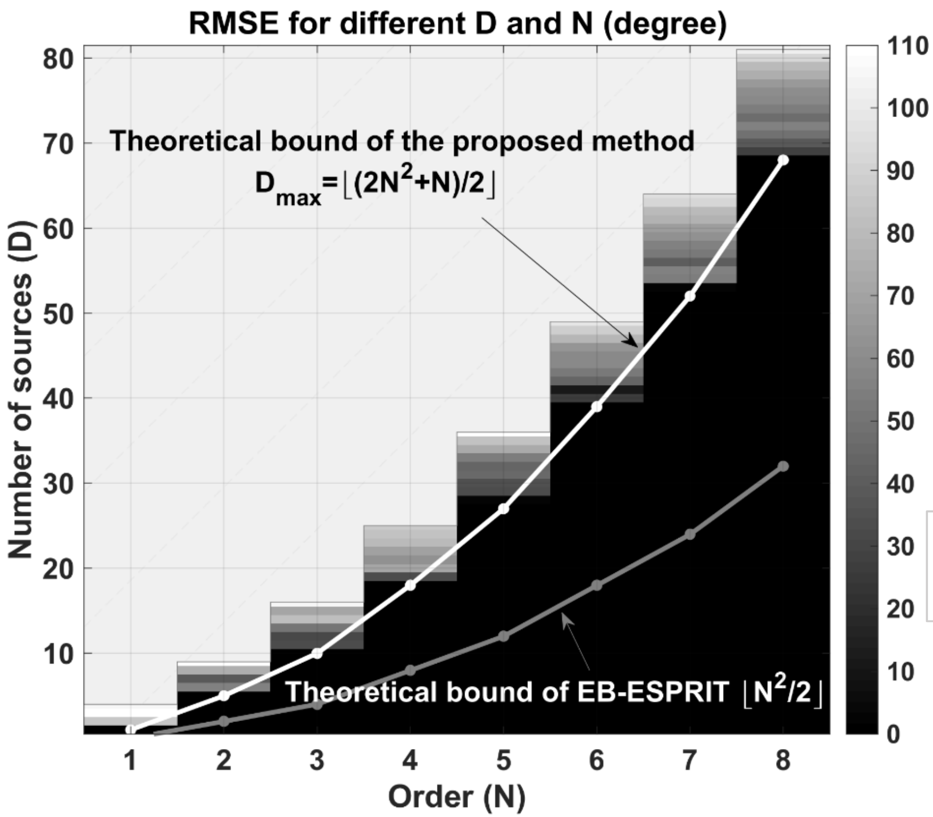
$$\underline{\boldsymbol{\Psi}} = -2\mathbf{E}^{\dagger}\underline{\mathbf{M}}\,\underline{\mathbf{U}} \rightarrow CN_2(\mathbf{E}) = \|\mathbf{E}\|_2 \cdot \left\|(\mathbf{E}^H\mathbf{E})^{-1}\mathbf{E}^H\right\|_2$$



- ✓ Single source simulation
- ✓ Without self-microphone noise
- ✓ condition number with different source elevation angles

Overcome the singularity problem

- RMSE variations with respect to the number of sources (D) and maximum order (N)



Random position
Without noise and aliasing

- Number of detectable sources

$$D_{\max}(\text{Proposed}) = \left\lfloor N^2 + N/2 \right\rfloor \quad > \quad D_{\max}(\text{EB-ESRPIT}) = \left\lfloor N^2/2 \right\rfloor$$

Conclusion

- **Sine-based EB-ESPRIT utilizes two recurrence relations of spherical harmonics which has sine-based directional parameters**
  - Can estimate the DOAs near the equator without singularity
  - No need for additional coordinate rotation
  - Can estimate elevation and azimuth at once

- **More number of detectable sources than conventional EB-ESPRIT**
  - Increase of the number of independent equations
    → can estimate more number of sources simultaneously

- **Limitation**
  - Performance degradation in elevation near the equator
    due to the slow rate of change of sine function

Thank you

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