Fast direction of arrival estimation using a sensor-saving coprime array with enlarged inter-element spacing

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Abstract

Direction of arrival (DOA) estimation using coprime array is discussed, and a fast method using a sensor-saving coprime array with enlarged inter-element spacing is proposed. Original coprime array combines the results obtained from two coprime subarrays to uniquely determine the DOA estimation, and it is shown in this paper that one subarray actually only requires half of its original sensor number and can achieve enlarged inter-element spacing based on the array compensation from the cross correlation matrix. Thereafter, a fast DOA estimation method, which extracts the noise subspace without EVD, is proposed to obtain coprime DOA estimations based on one dimensional root finding technique. Simulation results verify that the proposed method can achieve almost the same estimation performance as conventional methods while requiring less sensors.

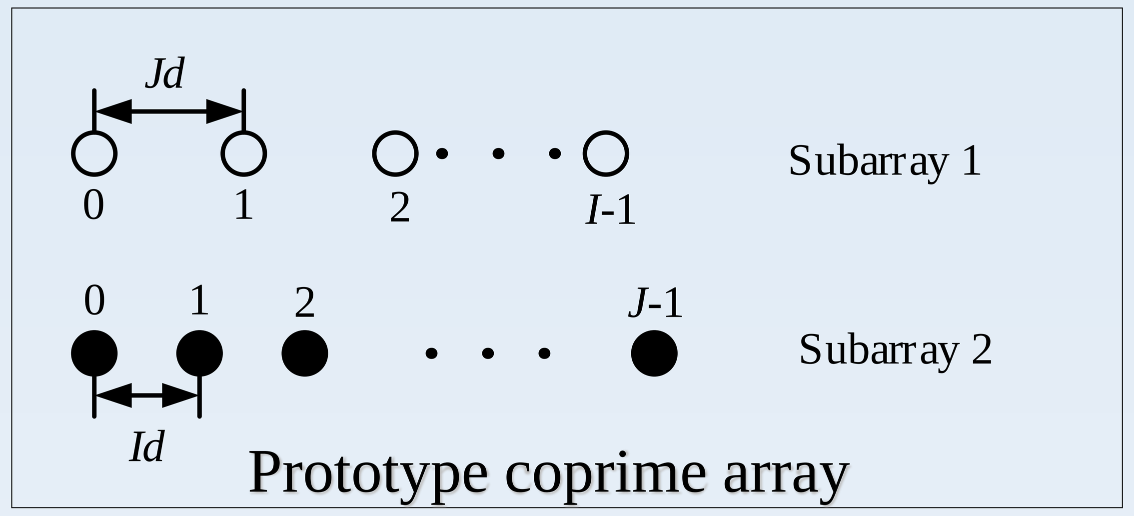
Background

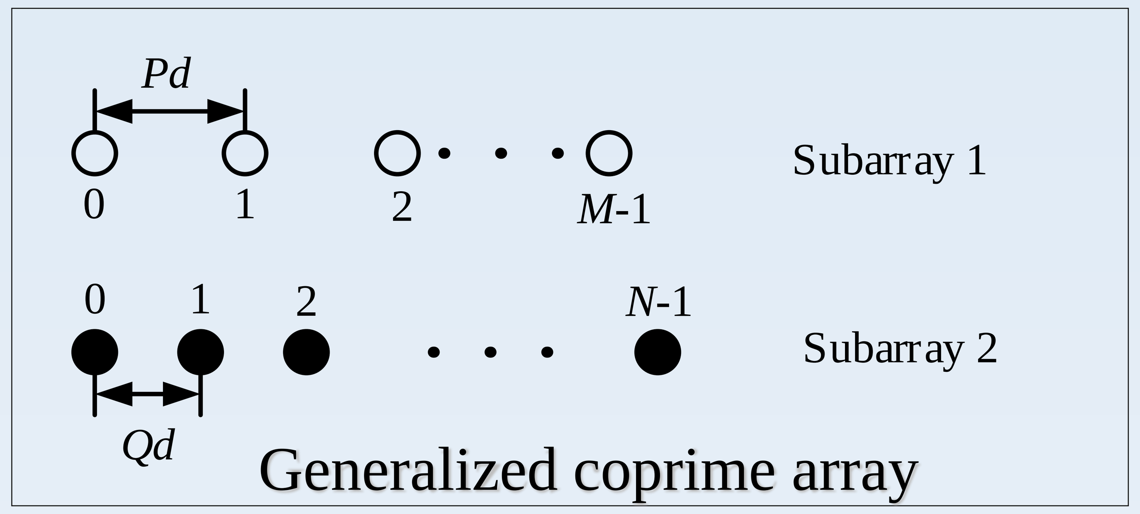
Two categories for coprime array

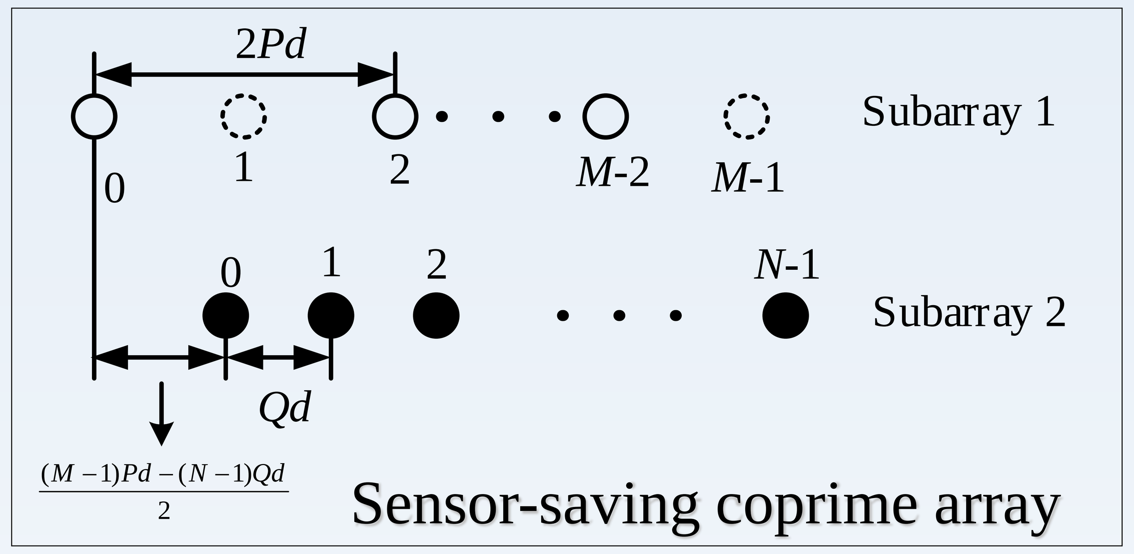
- Array geometry improvement
- treats the two subarrays separately

Our discussion

DOA estimation Method







$$\mathbf{a}_1(\alpha_k)=[1,e^{-j2\alpha_k},\ldots,e^{-j(M-2)\alpha_k}]^T\in\mathbb{C}^{\frac{M}{2}\times 1}$$

$$\mathbf{a}_2(\beta_k)=e^{-j(\frac{M-1}{2}\alpha_k-\frac{N-1}{2}\beta_k)}[1,e^{-j\beta_k},\ldots,e^{-j(N-1)\beta_k}]^T\in\mathbb{C}^{N\times 1}$$

1.Virtual array compensation

$$\mathbf{R}_z=\begin{bmatrix}\mathbf{R}\\ \mathbf{R}^*\mathbf{\Pi}_N\end{bmatrix}=\begin{bmatrix}\mathbf{A}_1\mathbf{R}_s\mathbf{A}_2^H\\ \mathbf{A}_1^*\mathbf{R}_s\mathbf{A}_2^T\mathbf{\Pi}_N\end{bmatrix}=\begin{bmatrix}\mathbf{A}_1\mathbf{R}_s\mathbf{A}_2^H\\ \mathbf{A}_1^*\mathbf{R}_s(\mathbf{\Pi}_N\mathbf{A}_2^*)^H\end{bmatrix}$$

$$=\begin{bmatrix}\mathbf{A}_1\mathbf{R}_s\mathbf{A}_2^H\\ \mathbf{A}_1^*\mathbf{R}_s\mathbf{\Delta}^H\mathbf{A}_2^H\end{bmatrix}=\begin{bmatrix}\mathbf{A}_1\\ \mathbf{A}_1^*\mathbf{\Delta}^H\end{bmatrix}\mathbf{R}_s\mathbf{A}_2^H$$

lost elements in subarray 1 are exactly compensated

$$[\mathbf{a}_1^T(\alpha_k),e^{-j(M-1)\alpha_K}\mathbf{a}_1^H(\alpha_k)]=$$
$$[1,e^{-j2\alpha_k},\ldots,e^{-j(M-2)\alpha_k},e^{-j(M-1)\alpha_k},e^{-j(M-3)\alpha_k},\ldots,e^{-j\alpha_k}]^T\in\mathbb{C}^{M\times 1}$$
$$k=1,\ldots,K$$

2. Subspace construction

$$\mathbf{R}_E=\begin{bmatrix}\mathbf{R}_{fa}^H\\ \mathbf{R}_{fb}^H\end{bmatrix}=\begin{bmatrix}\mathbf{A}_2\mathbf{R}_s\mathbf{A}_{E1a}^H\\ \mathbf{A}_2\mathbf{R}_s\mathbf{A}_{E1b}^H\end{bmatrix}=\begin{bmatrix}\mathbf{A}_2\mathbf{R}_s\mathbf{A}_{E1a}^H\\ \mathbf{A}_2\mathbf{R}_s\mathbf{\Phi}_1^H\mathbf{A}_{E1a}^H\end{bmatrix}$$

$$=\begin{bmatrix}\mathbf{A}_2\\ \mathbf{A}_2\mathbf{\Phi}_1^H\end{bmatrix}\mathbf{R}_s\mathbf{A}_{E1a}^H$$

Span the same column subspace

$$\mathbf{P}_s=\mathbf{R}_E\mathbf{R}_{E1}^H$$

3. 2D coprime angle estimation

$$\min_{z,\beta}\mathbf{c}^H(\alpha)\underbrace{\begin{bmatrix}\mathbf{a}_2^T(1/z)\mathbf{Q}_1\mathbf{a}_2(z) & \mathbf{a}_2^T(1/z)\mathbf{Q}_2\mathbf{a}_2(z)\\ \mathbf{a}_2^T(1/z)\mathbf{Q}_3\mathbf{a}_2(z) & \mathbf{a}_2^T(1/z)\mathbf{Q}_4\mathbf{a}_2(z)\end{bmatrix}}_{\mathbf{F}(z)}\mathbf{c}(\alpha)$$

$$=\mathbf{c}^H(\alpha)\mathbf{F}(z)\mathbf{c}(\alpha)$$

$$\mathbf{Q}_n=\mathbf{I}_{2N}-\mathbf{P}_s(\mathbf{P}_s^H\mathbf{P}_s)^{-1}\mathbf{P}_s^H$$

$$\det(\mathbf{F}(z))=0$$

$$\mathrm{z}_k$$

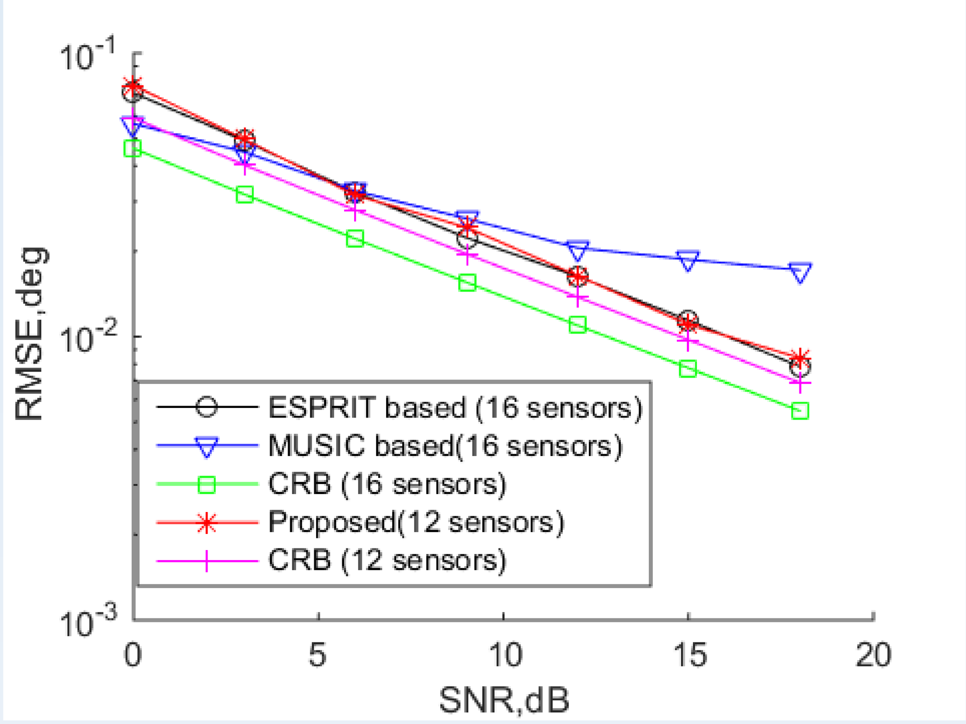
$$\sin\widehat{\theta}_{k,1}=-angle(\mathrm{z}_k)/(Q\pi),k=1,\ldots,K$$

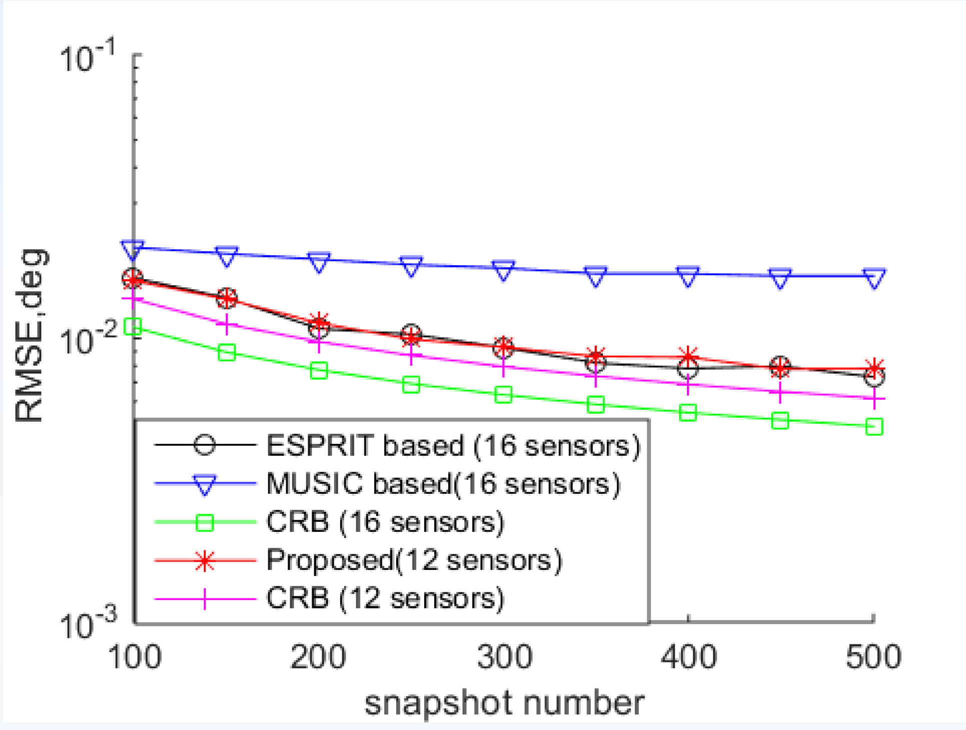
$$\mathbf{c}(\alpha_k)=\begin{bmatrix}1\\ e^{j\alpha_k}\end{bmatrix}=\begin{bmatrix}1\\ -\dfrac{\mathbf{a}_2^T(1/z_k)\mathbf{Q}_3\mathbf{a}_2(z_k)}{\mathbf{a}_2^T(1/z_k)\mathbf{Q}_1\mathbf{a}_2(z_k)}\end{bmatrix}$$

Automatically pairing

$$\sin\breve{\theta}_{k,1}=angle(-\frac{\mathbf{a}_2^T(1/z_k)\mathbf{Q}_3\mathbf{a}_2(z_k)}{\mathbf{a}_2^T(1/z_k)\mathbf{Q}_1\mathbf{a}_2(z_k)})/(P\pi)$$
$$k=1,\ldots,K$$

Results





Conclusion

Advantages :

1. Sensor-saving;
2. Enlarged inter-element spacing;
3. Low computation complexity.

Further consideration :

For some specific *M*,*N*, *P* and *Q*, the two subarrays have overlapped or very closed elements, which will enhance the mutual coupling.