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An Adaptive Sequential Competition Test for Beam Selection in Massive MIMO Systems

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Outline

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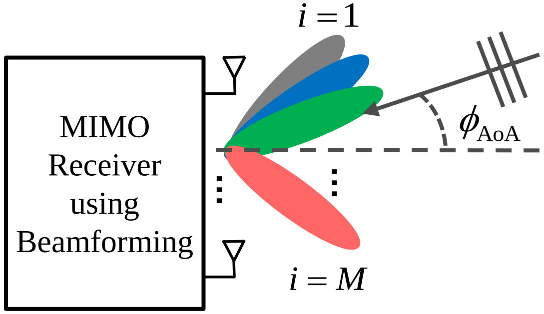
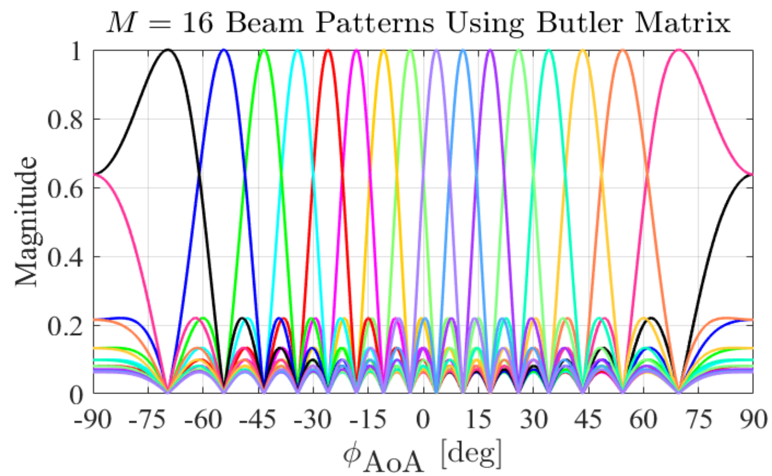
1. Introduction: Beam Selection Problem

- Why mm-Wave?
  - higher carrier frequencies frees up higher bandwidth for data transmission
- Why beamforming?
  - Directional transmission can counter significant path loss ($\propto f^2$) due to high carrier freq.
- Beam selection problem:





- Which beam should be selected in order to capture the highest signal power?
- How to do the beam selection in the most efficient way in terms of training time?

2. System Model

Received sequence under beam i :

$$r_i[n] = A_i s[n] + w_i[n]$$

Training Sequence $s[n]$ with length N:

$$P\{s[n] = +1\} = P\{s[n] = -1\} = 1/2$$

$$\mathrm{E}[s[n]s[n-k]] \simeq \delta[k]$$

Correlated received sequence under Beam i :

$$y_i[n] = s[n]r_i[n]$$

$i \in \{1, \ldots, M\}$

$n = 0, \ldots, N-1$

$w_i[n] \propto \mathcal{N}(0, \sigma^2)$

Unknown variance σ^2

Unknown magnitude A_i

Assuming perfect sync.

Since the discrimination between two beams, considering a **single** plane wave signal (one AoA) shows most essential features of our problem, we restrict the number of candidate beams to $\boldsymbol{M} = \mathbf{2}$ and consider a **single** user scenario that uses correlated observations.

3. Fixed Length Test

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Difference magnitude:

$$D = A_1 - A_2$$

Sufficient statistic and Minimum Variance Unbiased Estimator (MVUE) for D:

$$\hat{D} = \frac{1}{N} \sum_{n=0}^{N-1} y_1[n] - y_2[n] = \bar{\mathbf{y}}_1 - \bar{\mathbf{y}}_2 \quad (1)$$

$$\hat{D} \propto \mathcal{N}(A_1 - A_2, 2\sigma^2/N)$$

Deflection coefficient of $\hat{D}$:

$$d^2 = N(A_1 - A_2)^2/2\sigma^2 \quad (2)$$

Which beam should be selected in order to capture the highest signal power?

⇨

Hypothesis Test:

$$\begin{cases} \text{if } \hat{D} > 0 & \Rightarrow \text{Select Beam 1} \\ \text{if } \hat{D} < 0 & \Rightarrow \text{Select Beam 2} \end{cases}$$

Fixed Length Test: Performance Criterion

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How to specify the performance?

$$A_{\max} \equiv \max(A_1, A_2)$$

$$r \equiv \frac{\min(A_1, A_2)}{A_{\max}} \in [0,1]$$

$$\hat{D} \propto \mathcal{N}(A_1 - A_2, 2\sigma^2/N)$$

$$d^2 = N(A_1 - A_2)^2/2\sigma^2$$

Normalized average loss of signal magnitude:

$$\bar{a} = \mathrm{P}\{\hat{D} > 0\}\frac{A_1}{A_{\max}} + \mathrm{P}\{\hat{D} < 0\}\frac{A_2}{A_{\max}}$$

$$\bar{l} = 1 - \bar{a} = (1-r)Q(d)$$

Required deflection coeff. to achieve target performance $\bar{l}_{\text{target}}$:

$$d_{\text{req}}^2 = \left(Q^{-1}\left(\frac{\bar{l}_{\text{target}}}{1-r}\right)\right)^2 \qquad (3)$$

$$N_{\text{req}} = 2\sigma^2 d_{\text{req}}^2/(A_1 - A_2)^2$$

Design Problem:

The receiver doesn't know the values corresponding to r and $\frac{(A_1-A_2)^2}{\sigma^2}$.

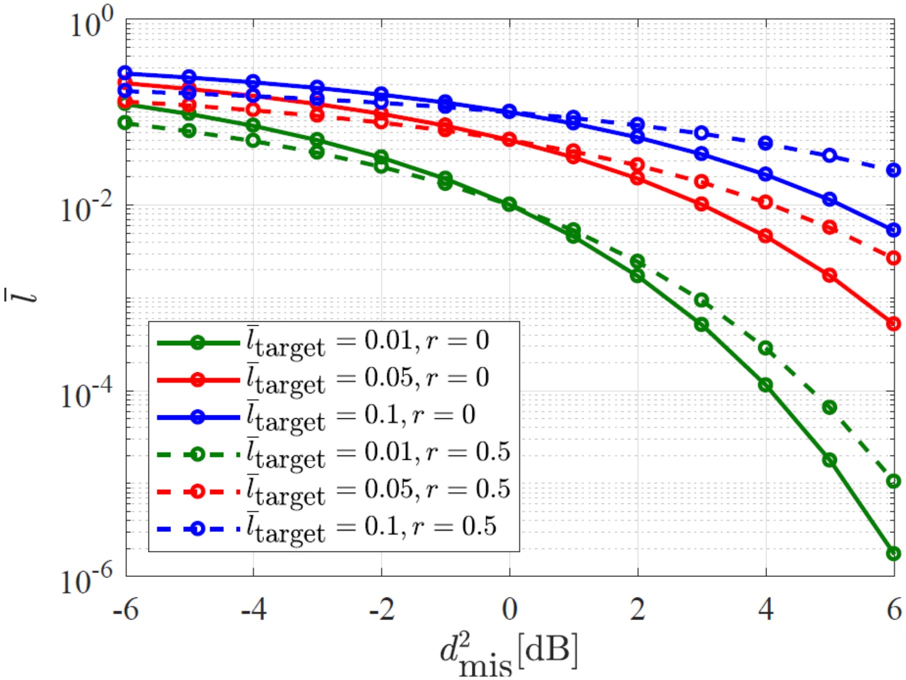
Fixed Length Test: Problems

Effectively achieved deflection coef. by the design based on N_{fix}:

$$d_{\text{eff}}^2 = N_{\text{fix}}(A_1 - A_2)^2/2\sigma^2$$

This can cause a mismatch:

$$d_{\text{mis}}^2 = d_{\text{eff}}^2 - d_{\text{req}}^2$$



- Naively fixing the test length to some value N_{fix} based on a certain assumed operating point can result in a strongly variable performance in practical scenarios.

- Additionally, if N_{fix} is conservatively set to a high value based on the worst still acceptable operating point, a lot of time spent will be wasted for detection of the best beam if the channel quality is better than expected.

4. Detection of a Unknown DC Level in WGN with Unknown Variance

Consider the following hypothesis testing problem:

$$\mathcal{H}_0 \quad : y[n] = w[n]$$
$$\mathcal{H}_1 \quad : y[n] = A + w[n]$$

$n = 0, \ldots, N-1$

$w[n] \propto \mathcal{N}(0, \sigma^2)$

Unknown variance σ^2

Unknown amplitude A

Question:
How can we detect the presence or absence of a nonzero DC level in zero mean WGN where both the DC level and the noise variance are unknown?

- Noise variance under null hypothesis is not known.
- Neyman-Pearson approach can not find the proper threshold on the likelihood ratio to bound the probability of false alarm.
- Possible solution: replacing the unknown parameters in likelihood functions with their Maximum Likelihood Estimates (MLE) .

Generalized Likelihood Ratio

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Generalized Likelihood Ratio (GLR):

$$L_G(\mathbf{y}) = \frac{p(\mathbf{y}; \hat{A}, \hat{\sigma}^2_{\mathcal{H}_1}, \mathcal{H}_1)}{p(\mathbf{y}; \hat{\sigma}^2_{\mathcal{H}_0}, \mathcal{H}_0)} = \left(\frac{\hat{\sigma}^2_{\mathcal{H}_0}}{\hat{\sigma}^2_{\mathcal{H}_1}}\right)^{\frac{N}{2}}$$

Sufficient Statistics or Generalized log Likelihood Ratio (GLLR):

$$\gamma \equiv 2 \ln L_G(\mathbf{y})$$

For large N it has the following chi-squared distributions:

$$\gamma = N \ln(1 + \frac{\bar{\mathbf{y}}^2}{\hat{\sigma}^2_{\mathcal{H}_1}}) \sim \begin{cases} \chi_1^2, & \text{under } \mathcal{H}_0 \\ \chi_1^2(\lambda), & \text{under } \mathcal{H}_1 \end{cases} \quad (4)$$

With non-centrality parameter (deflection coef.):

$$\lambda = N \frac{A^2}{\sigma^2}$$

MLE of A and σ^2 under $\mathcal{H}_1$:

$$\hat{A} = \bar{\mathbf{y}}$$

$$\hat{\sigma}^2_{\mathcal{H}_1} = (1/N) \sum_n (y[n] - \bar{\mathbf{y}})^2$$

MLE of σ^2 under $\mathcal{H}_0$:

$$\hat{\sigma}^2_{\mathcal{H}_0} = (1/N) \sum_n y[n]^2$$

$$\hat{\sigma}^2_{\mathcal{H}_0} = \hat{\sigma}^2_{\mathcal{H}_1} + \bar{\mathbf{y}}^2$$

GLR Test

Probability of false alarm:

$$P_{\mathrm{FA}} = \Pr\{\gamma > \gamma_{\mathrm{th}}; \mathcal{H}_0\}$$

$$\gamma = x^2 \quad , \quad x \sim \mathcal{N}(0,1)$$

$$P_{\mathrm{FA}} = \Pr\{x > \sqrt{\gamma_{\mathrm{th}}}\} + \Pr\{x < -\sqrt{\gamma_{\mathrm{th}}}\} = 2Q(\sqrt{\gamma_{\mathrm{th}}})$$

Since the GLLR PDF under null hypothesis is fully known we can bound the false alarm probability by finding the proper threshold.

Threshold on the GLLR:

$$\gamma_{\mathrm{th}} = \left[Q^{-1}\left(\frac{P_{\mathrm{FA}}}{2}\right)\right]^2 \tag{5}$$

Probability of misdetection:

$$P_{\mathrm{MD}} = 1 - \Pr\{\gamma > \gamma_{\mathrm{th}}; \mathcal{H}_1\}$$

$$P_{\mathrm{MD}} = Q(\sqrt{\lambda} - \sqrt{\gamma_{\mathrm{th}}}) - Q(\sqrt{\lambda} + \sqrt{\gamma_{\mathrm{th}}})$$

P_{MD} is dependent on:

- number of observations
- SNR per observation

Probability of Misdetection using GLRT

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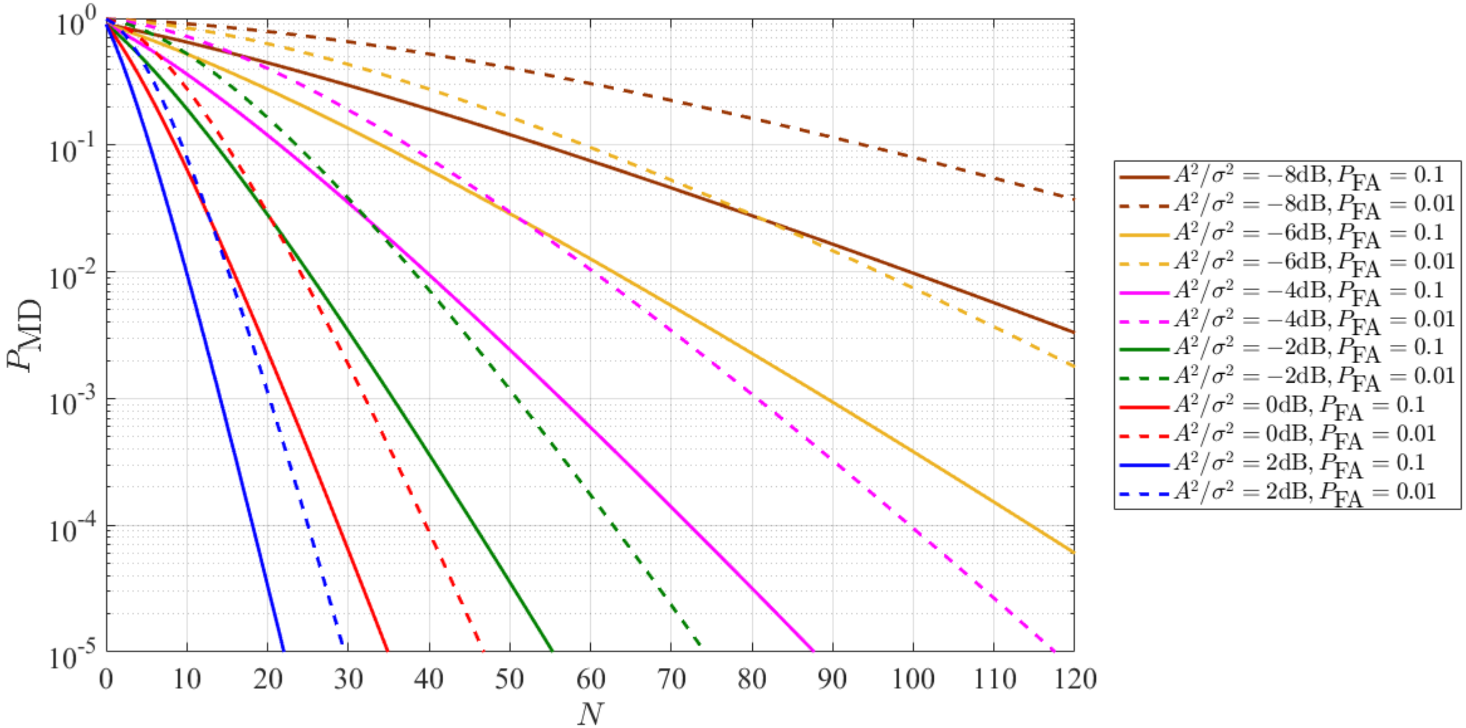
$$P_{\mathrm{MD}} = Q(\sqrt{\lambda} - \sqrt{\gamma_{\mathrm{th}}}) - Q(\sqrt{\lambda} + \sqrt{\gamma_{\mathrm{th}}})$$



1. As N increases, the P_{MD} decreases exponentially
2. The rate of decrease depends strongly on A^2/σ^2

- How can we exploit this property for our beam selection problem?

Variable Length Parallel Binary Tests

- Consider the initial M-ary beam selection problem.
- Separate observation sequences are available under each beam.
- This time instead of comparing the estimates of the values $\{A_1, \ldots, A_M\}$ against each other, let us rather compare them separately to their absence.

Parallel binary tests:

$$\text{beam}_i \quad : \begin{cases} \mathcal{H}_0 : & y_i[n] = w_i[n] \\ \mathcal{H}_i : & y_i[n] = A_i + w_i[n] \end{cases}$$

Obviously, $\mathcal{H}_0$ is the wrong hypothesis under each beam.

Now if at each n we compare $\gamma_i(n)$ to γ_{th}:

$$\text{if} \quad |A_i| > |A_j| \implies P_{\mathrm{D},i}(n) > P_{\mathrm{D},j}(n)$$

$i \in \{1, \ldots, M\}$

Variable n

$w_i[n] \propto \mathcal{N}(0, \sigma^2)$

Unknown variance: σ^2

Unknown magnitude: A_i

Probability of correct binary decision under beam i : $P_{\mathrm{D},i}(n)$

Same decision threshold under all beams: γ_{th}

The beam that observes the stronger signal will cross the threshold **earlier** on average!

5. Sequential Competition Test for Beam Selection

Sequential competition test:

$$\begin{cases} \text{beam}_1 : & \gamma_1(n) = n \ln\left(1 + \frac{\bar{\mathbf{y}}_1^2}{\hat{\sigma}^2_{\mathcal{H}_1}}\right) \underset{\text{undecided}}{\overset{\mathcal{H}_1}{\gtrless}} \gamma_{\text{th}} \\ \vdots & \\ \text{beam}_M : & \gamma_M(n) = n \ln\left(1 + \frac{\bar{\mathbf{y}}_M^2}{\hat{\sigma}^2_{\mathcal{H}_M}}\right) \underset{\text{undecided}}{\overset{\mathcal{H}_M}{\gtrless}} \gamma_{\text{th}} \end{cases} \quad (6)$$

- The test terminates as soon as one of the stochastic trajectories $\gamma_i(n)$ for $i = 1, \cdots, M$ crosses the threshold, while the index of this trajectory indicates the selected beam.
- Otherwise we continue by taking the next observation into account.
- The interpretation is that we let the beams compete to distinguish themselves from pure zero mean WGN noise with unknown variance, and the one which does it faster is the winning beam in the competition.

Sequential Competition Test: Comparison

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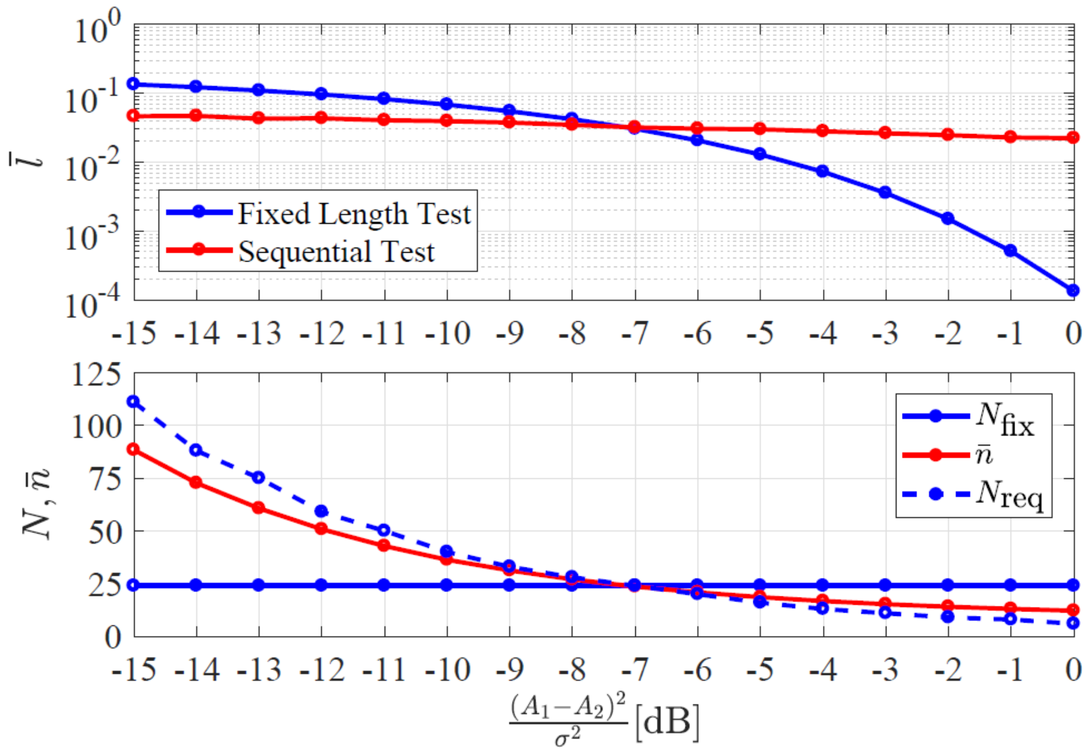
$$N_{\text{fix}} = 24 \qquad \frac{(A_1 - A_2)^2}{\sigma^2} = -7\text{dB} \qquad \bar{l}_{\text{target}} = 0.03 \qquad P_{\text{FA}} = 10^{-3}$$

$\bar{n}$ indicates the average number of observations used by sequential test

- Sequential test adaptively changes the number of observations with respect to operating point
- This makes the performance in terms of $\bar{l}$ invariant to the operating point
- Furthermore, sequential test outperforms the ideally tuned fixed length test in lower SNR in terms of required number of observations to achieve the same performance



6. Simulation: Beam Selection in Massive MIMO Systems

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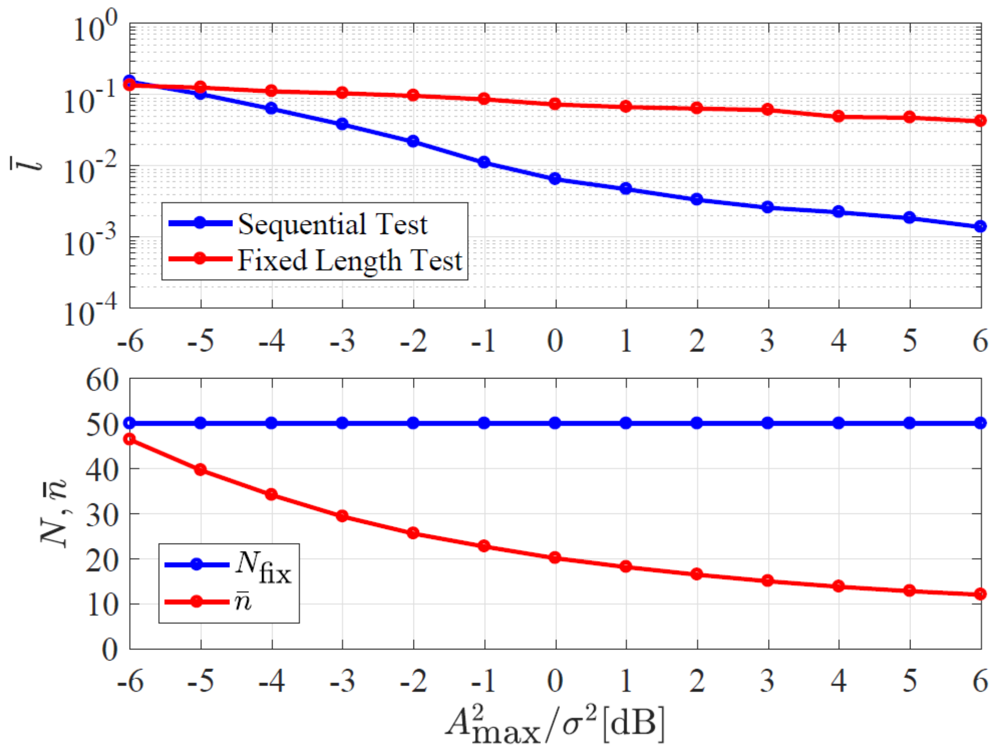

M=16 uniform linear array using Butler Matrix

AoA distributed uniformly in $[-90^0, 90^0]$

Maximum available $\mathrm{SNR} = A_{\max}^2/\sigma^2$

$N_{fix} = 50$

γ_{th} based on $P_{\text{FA}} = 10^{-3}$

Total simulation runs: 10^4

- Sequential test adaptively reduces average test length as SNR grows
- This results in reduced delay due to training.
- The achievable speed up becomes even more interesting in systems using analogue or hybrid beamforming, where exhaustive search for beam selection can cause huge delays

7. Concluding Remarks

- We proposed a novel sequential competition test based on GLR statistics to solve the composite beam selection problem.
- Our sequential competition test shows adaptively w.r.t. the SNR operating point.
- It speeds up beam selection even when compared to an optimally designed fixed length test at lower SNR regime where it matters the most
- These properties can be of interest in massive MIMO systems using hybrid beamforming as well as under conditions where the training time is limited due to small channel coherence time.

9. References

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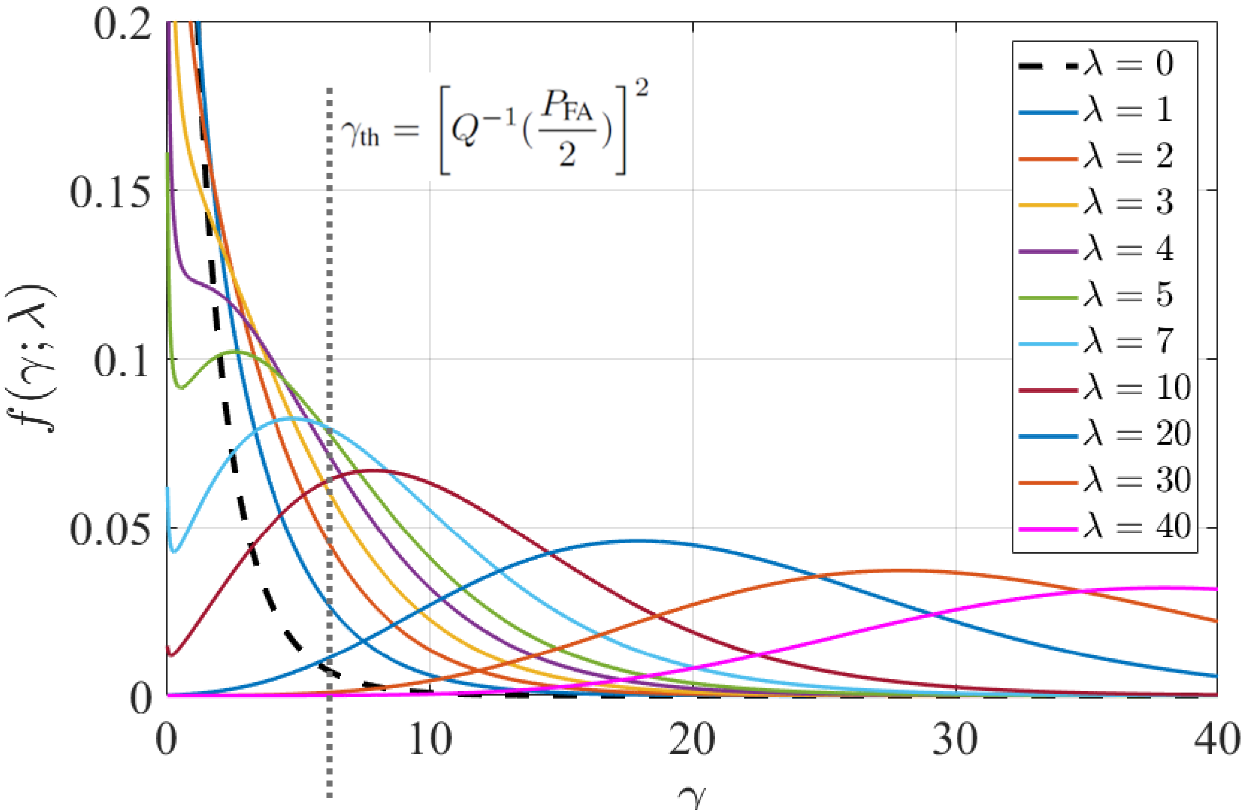
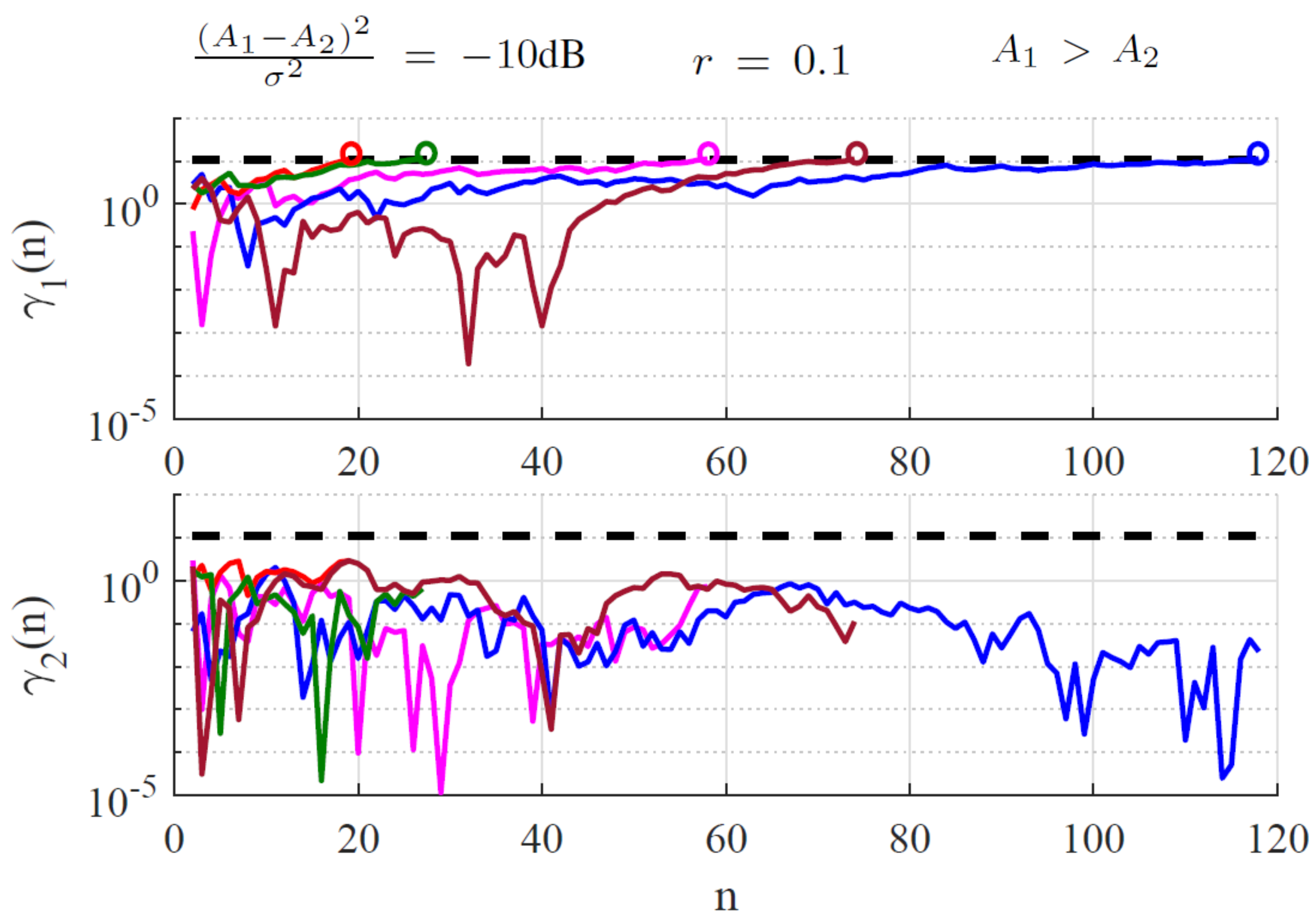
Thank You!

Chi-squared Distributions

$$\gamma = N\ln(1+\frac{\bar{\mathbf{y}}^2}{\hat{\sigma}^2_{\mathcal{H}_1}}) \sim \begin{cases} \chi_1^2, & \text{under } \mathcal{H}_0 \\ \chi_1^2(\lambda), & \text{under } \mathcal{H}_1 \end{cases} \qquad \lambda = N\frac{A^2}{\sigma^2}$$



Sequential Competition Test: Visualized



Sequential Competition Test: Performance over r