Joint User Selection and Hybrid Analog-Digital Beamforming in Massive MIMO Systems

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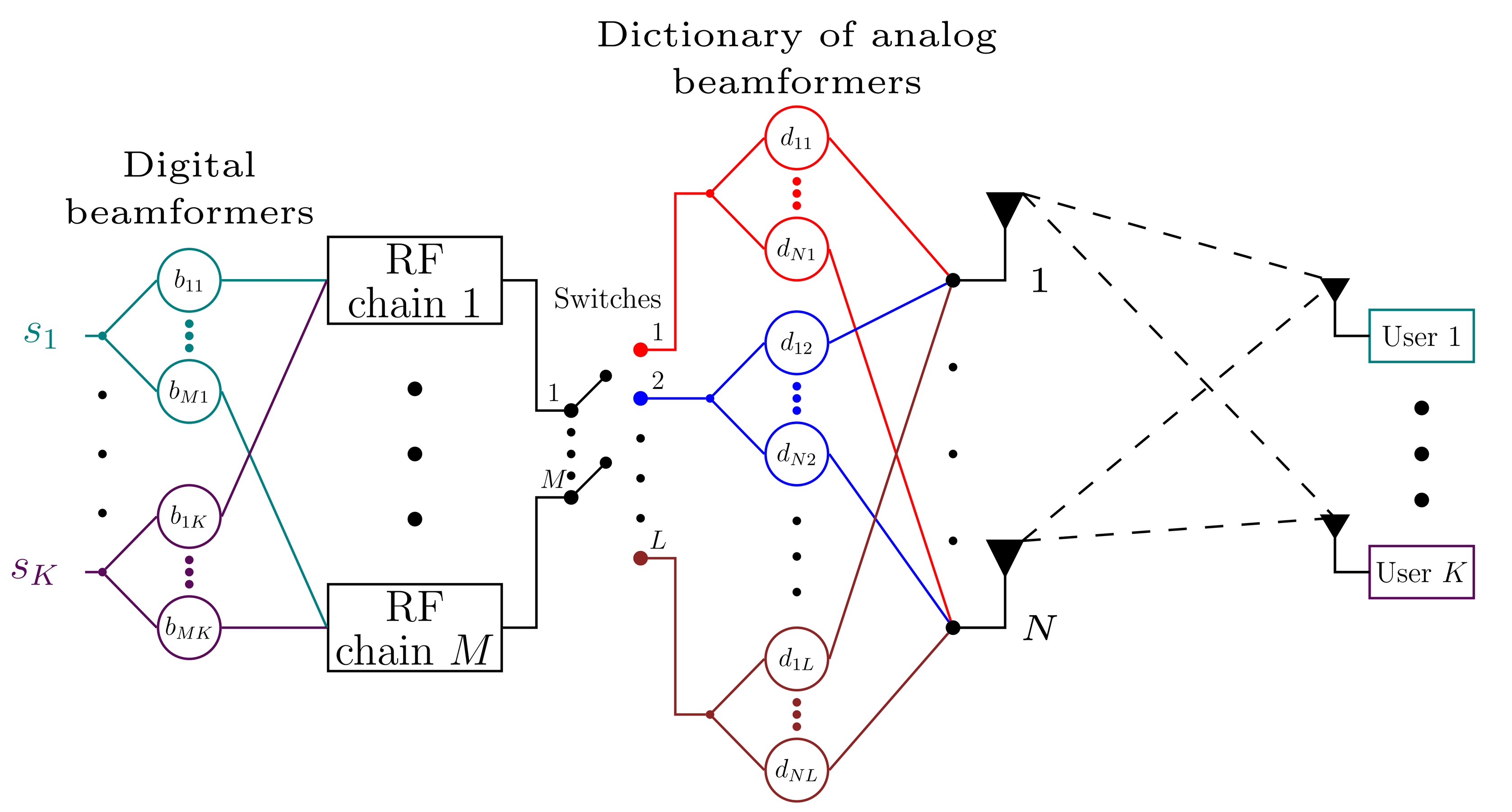
1 Introduction

- Massive MIMO with the conventional fully-digital beamforming:
  - Extremely high hardware cost
  - Unwieldy operational power
- Hybrid beamforming using a dictionary of fixed analog beamformers:
  - Significantly reduced hardware cost and operational power
  - Reduced control over interference ⇒ increased transmit power
- Solution: Exploit multi-user diversity while designing hybrid beamformers
- **Contribution:**
  - Formulated a problem to jointly perform hybrid beamforming and multi-user diversity exploitation
  - Devised a suboptimal algorithm to efficiently solve the formulated problem

2 System Model



- Co-channel downlink system
- Base station (BS) comprising N transmit antennas and M RF chains, where $M \leq N$
- Total transmit power budget at the BS $P_{\max}$
- K single antenna users. Only a subset of K' users are served in a resource block
- The transmit power of the kth symbol $p_k = |s_k|^2$
- Digital beamformer (DBF) $\mathbf{b}_k$ applied to s_k
- Analog beamformers (ABF) dictionary $\mathcal{D} = \{\mathbf{d}_1, \mathbf{d}_2, \ldots, \mathbf{d}_L\}$, where $L \geq M$
- ABF matrix $\mathbf{A} = [\mathbf{a}_1, \mathbf{a}_2, \ldots, \mathbf{a}_M]$
- The channel vector of the kth user $\mathbf{h}_k$ is known at the BS
- The received signal $y_k = \mathbf{h}_k^T \mathbf{A}\mathbf{b}_k s_k + \sum_{j \in \mathcal{K}' \setminus k} \mathbf{h}_k^T \mathbf{A}\mathbf{b}_j s_j + n_k$

3 Problem Formulation

- **Task:** Jointly select $\mathcal{K}' \subseteq \mathcal{K}$ users, choose M ABFs from the dictionary $\mathcal{D}$, and design optimal DBFs
- **Objective:** Maximize the minimum SINR among the selected users
- Optimization problem (MINLP) can be formulated as

$$\underset{C, \mathbf{S}, \{u_k, p_k, \mathbf{b}_k\}_{k\in\mathcal{K}}}{\text{maximize}} \quad C \tag{1a}$$

$$\text{subject to} \quad \frac{p_k \mathbf{b}_k^H \mathbf{A}^H \mathbf{R}_k \mathbf{A}\mathbf{b}_k}{\sum_{j\in\mathcal{K}\setminus k} p_j \mathbf{b}_j^H \mathbf{A}^H \mathbf{R}_k \mathbf{A}\mathbf{b}_j + 1} \geq u_k C, \quad \forall k \in \mathcal{K} \tag{1b}$$

$$\sum_{k\in\mathcal{K}} p_k \mathbf{b}_k^H \mathbf{A}^H \mathbf{A}\mathbf{b}_k \leq P_{\max}; \quad p_k \geq 0, \quad \forall k \in \mathcal{K} \tag{1c}$$

$$\mathbf{A} = \mathbf{D}\mathbf{S}; \quad \sum_{\ell=1}^{L} S_{\ell,m} = 1; \quad S_{\ell,m} \in \{0,1\}, \quad \forall \ell \in \mathcal{L}, \forall m \in \mathcal{M} \tag{1d}$$

$$\sum_{k=1}^{K} u_k = K'; \quad u_k \in \{0,1\}, \quad \forall k \in \mathcal{K} \tag{1e}$$

where $\mathbf{R}_k = \mathbf{h}_k \mathbf{h}_k^H / \sigma_k^2$, and $\mathbf{D} = [\mathbf{d}_1, \mathbf{d}_2, \ldots, \mathbf{d}_L]$

- Constraint (1b) - SINR constraint
- Constraint (1c) - power budget constraints
- Constraint (1d) administers M ABF selection from the dictionary
- Constraint (1e) enforces K' user selection from the set $\mathcal{K}$
- The problem is combinatorial and requires exhaustive search over the dictionary $\mathcal{D}$ and users in set $\mathcal{K}$, to obtain the optimal solution

4 Proposed Algorithm: HYBEEM

Hybrid beamforming with error minimization (HYBEEM) comprises the following three stages:

1. Fully-DBFs design: The continuous relaxation of discrete constraints results in fully-DBFs. The equivalent convex program is given by

$$\underset{C, \{p_k, \mathbf{f}_k\}_{k\in\mathcal{K}}}{\text{maximize}} \quad C \tag{2a}$$

$$\text{subject to} \quad \frac{p_k \mathbf{f}_k^H \mathbf{R}_k \mathbf{f}_k}{\sum_{j\in\mathcal{K}\setminus k} p_j \mathbf{f}_j^H \mathbf{R}_k \mathbf{f}_j + 1} \geq C, \quad \forall k \in \mathcal{K} \tag{2b}$$

$$\sum_{k\in\mathcal{K}} p_k \leq P_{\max}; \quad p_k \geq 0, \quad \forall k \in \mathcal{K} \tag{2c}$$

$$||\mathbf{f}_k||_2 = 1, \quad \forall k \in \mathcal{K} \tag{2d}$$

The optimal fully-DBFs matrix $\mathbf{G} = [\mathbf{g}_1, \mathbf{g}_2, \ldots, \mathbf{g}_K]$, where $\mathbf{g}_k = p_k^\star \mathbf{f}_k^\star$

2. Sparse regression: Employ sparse regression technique to select K' users and M ABFs

$$\underset{\mathbf{V}}{\text{minimize}} \quad \left|\left|(\mathbf{G} - \mathbf{D}\mathbf{V})^T\right|\right|_{2,1} + \epsilon\, ||\mathbf{V}||_{2,1} \tag{3a}$$

$$\text{subject to} \quad \text{trace}(\mathbf{V}^H \mathbf{D}^H \mathbf{D}\mathbf{V}) \leq P_{\max} \tag{3b}$$

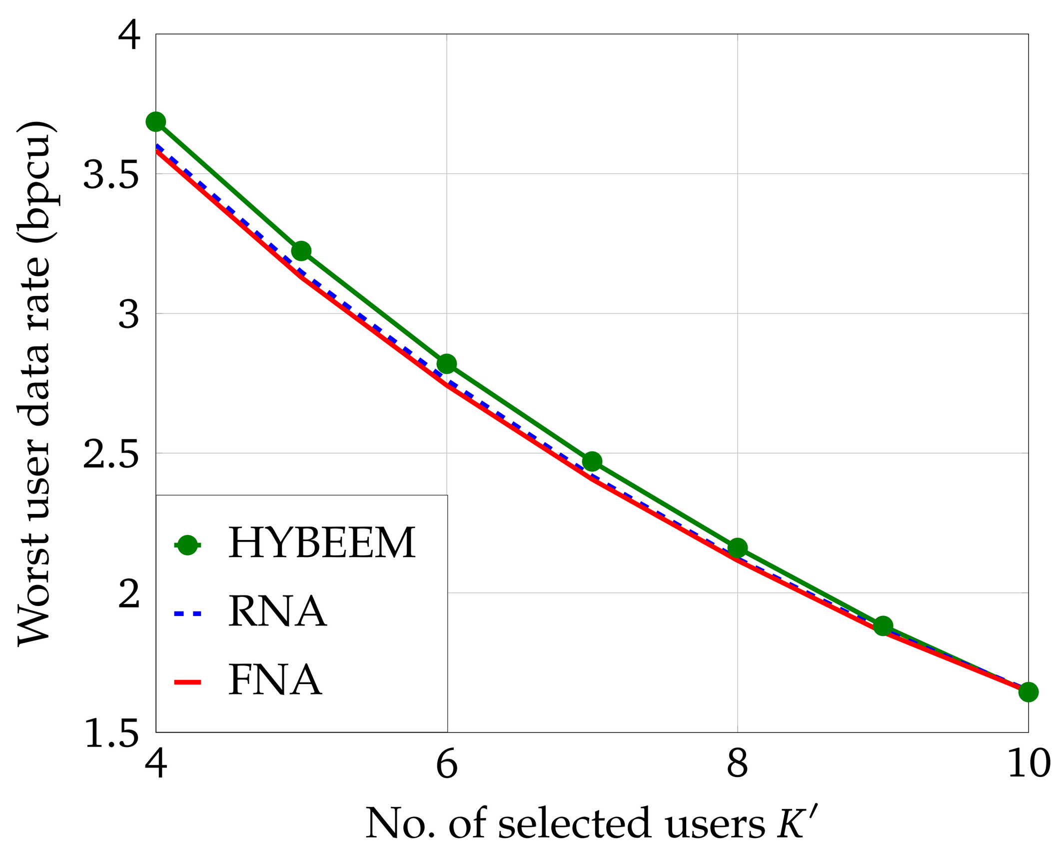
where $\mathbf{V} = [\mathbf{v}_1, \mathbf{v}_2, \ldots, \mathbf{v}_K] \in \mathbb{C}^{L\times K}$

- The $\ell_{2,1}$-norm term $\left|\left|(\mathbf{G} - \mathbf{D}\mathbf{V})^T\right|\right|_{2,1} = \sum_{k=1}^{K} ||\mathbf{g}_k - \mathbf{D}\mathbf{v}_k||_2$
- Let $e_k = ||\mathbf{g}_k - \mathbf{D}\mathbf{v}_k||_2$ and $\mathbf{e} = [e_1, e_2, \ldots, e_K]^T$. Then $\left|\left|(\mathbf{G} - \mathbf{D}\mathbf{V})^T\right|\right|_{2,1} = ||\mathbf{e}||_1$
- The ℓ_1-norm term $||\mathbf{e}||_1$ promotes sparsity on $\mathbf{e}$, i.e., zero error for many users
- The $\ell_{2,1}$-norm term $||\mathbf{V}||_{2,1}$ promotes row-sparsity, i.e., identifies a small subset of most suitable ABFs
- *User selection:* Select K' users with the smallest error values e_k
- *ABF selection:* Select the columns of $\mathbf{D}$ that correspond to the non-zero rows of $\mathbf{V}$

3. Optimal DBFs design: Compute optimal fully-DBFs for the selected users and ABFs

5 Numerical Results

- Competing methods:
  - Fully-DBF: Each transmit antenna has a dedicated RF chain, i.e., $M = N$
  - Exhaustive search method: Exhaustive search over dictionary $\mathcal{D}$ and set $\mathcal{K}$ to obtain optimal ABF matrix $\mathbf{A}$ and user set $\mathcal{K}'$ respectively
  - Row-norm based algorithm (RNA): Objective function (3a) $||\mathbf{G} - \mathbf{D}\mathbf{V}||_{2,1} + \epsilon\, ||\mathbf{V}||_{2,1}$
  - Frobenius-norm based algorithm (FNA): Objective function (3a) $||\mathbf{G} - \mathbf{D}\mathbf{V}||_F + \epsilon\, ||\mathbf{V}||_{2,1}$
- SINR balancing problem. Simulation settings: $N = L = 32$, $M = K = 10$, $P_{\max} = 46$ dBm



- Power minimization problem. Simulation settings: $N = L = 10$, $M = K = 6$, SINR = 5 dB