Subspace-Based Imaging Using Only Power Measurements

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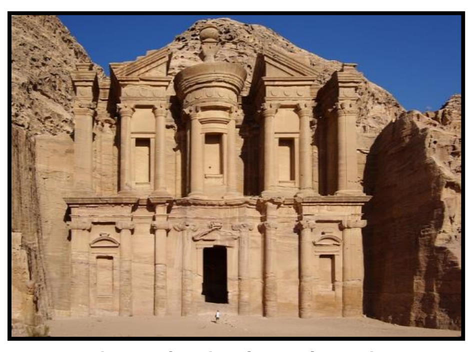
Imaging Using RF Signals

- Important problem of considerable interest
- Applications
  - Search and rescue
  - Archaeology
  - Mine detection

Through wall imaging



Archaeological exploration

Mine detection

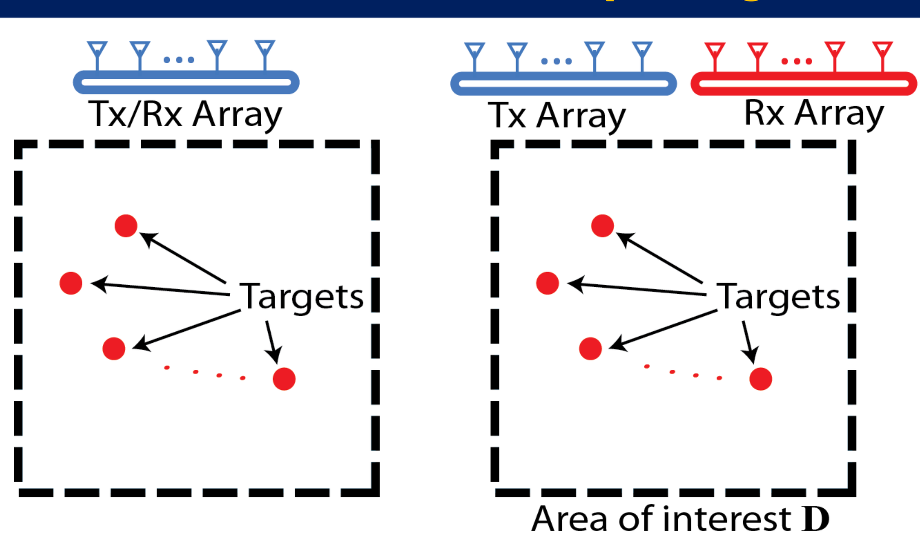
Subspace-Based Imaging

- Subspace-based imaging methods
  - Low computational complexity
  - High resolution capability
- Conventional subspace-based methods require phase (e.g., Time-Reversal (TR) MUSIC)
  - Not available in many wireless devices
  - Needs synchronization across antennas

Power-Only Subspace-Based Imaging

- Requires large number of antennas: square of number of objects (Marengo et. al, 2007)
- Requires control of phase of transmitted signals (Novikov et. al, 2015)
- **Objective: Power-only** subspace-based imaging algorithm with **much less number of antennas ($2M+1$ vs. M^2+1)**

Conventional TR-MUSIC (Using Phase)



- The scattered electric field:
  - Assuming first order scattering (Born approximation) (Devaney, 2000)

$$E_{sc}(\mathbf{x}_i^r, \mathbf{x}_j^t) = \iiint_{\mathbf{D}} G(\mathbf{x}_i^r, \mathbf{x}')\tau(\mathbf{x}')G(\mathbf{x}', \mathbf{x}_j^t)d\mathbf{x}'$$

Scattered field; i[th] Rx position; j[th] Tx position; Green's function; Scattering amplitude

  - Assuming M point-like objects

$$E_{sc}(\mathbf{x}_i^r, \mathbf{x}_j^t) = \sum_{m=1}^{M} \tau_m G(\mathbf{x}_i^r, \mathbf{x}_m)G(\mathbf{x}_m, \mathbf{x}_j^t)$$

Conventional TR-MUSIC (cont'd)

- $N_r \times N_t$ measurement matrix

$$K = \sum_{m=1}^{M} \tau_m \mathbf{g}_m^r \mathbf{g}_m^{t\,T}$$

where

$$\mathbf{g}_m^r = [G(\mathbf{x}_1^r, \mathbf{x}_m), G(\mathbf{x}_2^r, \mathbf{x}_m), \ldots, G(\mathbf{x}_{N_r}^r, \mathbf{x}_m)]^T$$

$$\mathbf{g}_m^t = [G(\mathbf{x}_m, \mathbf{x}_1^t), G(\mathbf{x}_m, \mathbf{x}_2^t), \ldots, G(\mathbf{x}_m, \mathbf{x}_{N_t}^t)]^T$$

- Rank$\{K\} = M$ (sum of M rank-1 matrices)
- Using SVD

$$K = U\Sigma V^H$$

$$U = [U_s|U_n] \qquad V = [V_s|V_n]$$

$$\Sigma = \text{diag}(\sigma_1, \sigma_2, \ldots, \sigma_M, 0, 0, \ldots, 0)$$

- Bases of signal subspaces are $\mathbf{g}_m^r, \mathbf{g}_m^t$
- Pseudospectrum $I(\mathbf{x}_p)$ peaks at objects locations

$$I(\mathbf{x}_p) = \frac{1}{\sum_{i=M+1}^{N_r} |\mathbf{g}_p^r \cdot \mathbf{u}_i|} \times \frac{1}{\sum_{j=M+1}^{N_t} |\mathbf{g}_p^t \cdot \mathbf{v}_j|}$$

Proposed Power-Only TR-MUSIC

- Our approach for power-only TR-MUSIC with few antennas has two main components:
  - Rytov approximation for the total received electric field
  - Smart antenna placement scheme

Rytov Approximation

- The total received electric field

$$E(\mathbf{x}_i^r, \mathbf{x}_j^t) = E_{\text{inc}}(\mathbf{x}_i^r, \mathbf{x}_j^t) \quad \text{(Incident field)}$$
$$+ \sum_{m=1}^{M} \tau_m G(\mathbf{x}_i^r, \mathbf{x}_m)E(\mathbf{x}_m, \mathbf{x}_j^t)$$

- Non-linear function of τ
- Rytov approximation

$$E(\mathbf{x}_i^r, \mathbf{x}_j^t) \approx E_{\text{inc}}(\mathbf{x}_i^r, \mathbf{x}_j^t) \times \exp\left\{\frac{\sum_{m=1}^{M} \tau_m G(\mathbf{x}_i^r, \mathbf{x}_m)G(\mathbf{x}_m, \mathbf{x}_j^t)}{G(\mathbf{x}_i^r, \mathbf{x}_j^t)}\right\}$$

Power Model

- The power equation (in dB)

$$P(\mathbf{x}_i^r, \mathbf{x}_j^t) = P_{\text{inc}}(\mathbf{x}_i^r, \mathbf{x}_j^t) + c\,\Re\left\{\frac{\sum_{m=1}^{M} \tau_m G(\mathbf{x}_i^r, \mathbf{x}_m)G(\mathbf{x}_m, \mathbf{x}_j^t)}{G(\mathbf{x}_i^r, \mathbf{x}_j^t)}\right\}$$

Constant

- The power model – linear in τ

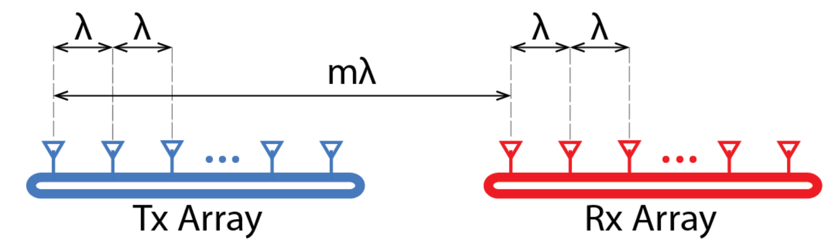
$$P_{\mathbf{D}}(\mathbf{x}_i^r, \mathbf{x}_j^t) = \Re\left\{\frac{\sum_{m=1}^{M} \tau_m G(\mathbf{x}_i^r, \mathbf{x}_m)G(\mathbf{x}_m, \mathbf{x}_j^t)}{G(\mathbf{x}_i^r, \mathbf{x}_j^t)}\right\}$$

- Linearity of power model allows for clutter removal

Smart Antenna Placement Scheme

- Place the antennas such that

$$|\mathbf{x}_i^r - \mathbf{x}_j^t| = m\lambda, \quad \forall i, j$$



- Power model then becomes

$$P_{\mathbf{D}}(\mathbf{x}_i^r, \mathbf{x}_j^t) = \Re\left\{\frac{\sum_{m=1}^{M} \tau_m G(\mathbf{x}_i^r, \mathbf{x}_m)G(\mathbf{x}_m, \mathbf{x}_j^t)}{\frac{e^{j\frac{2\pi}{\lambda}|\mathbf{x}_i^r - \mathbf{x}_j^t|}}{4\pi|\mathbf{x}_i^r - \mathbf{x}_j^t|}}\right\} \quad (e^{j\frac{2\pi}{\lambda}|\mathbf{x}_i^r - \mathbf{x}_j^t|} \to 1)$$

- New power equation

$$p(\mathbf{x}_i^r, \mathbf{x}_j^t) = \frac{P_{\mathbf{D}}(\mathbf{x}_i^r, \mathbf{x}_j^t)}{4\pi|\mathbf{x}_i^r - \mathbf{x}_j^t|}$$
$$= \Re\left\{\sum_{m=1}^{M} \tau_m G(\mathbf{x}_i^r, \mathbf{x}_m)G(\mathbf{x}_m, \mathbf{x}_j^t)\right\}$$

TR-MUSIC Using Power Measurements

- The new measurement matrix

$$P = \Re\left\{\sum_{m=1}^{M} \tau_m \mathbf{g}_m^r \mathbf{g}_m^{t\,T}\right\}$$
$$= \sum_{m=1}^{M} \tau_m \mathbf{g}_{m\Re}^r \mathbf{g}_{m\Re}^{t\,T} - \sum_{m=1}^{M} \tau_m \mathbf{g}_{m\Im}^r \mathbf{g}_{m\Im}^{t\,T}$$

- Rank$\{P\} = 2M$
- Using SVD, $P = U\Sigma V^H$ where

$$\Sigma = \text{diag}(\sigma_1, \sigma_2, \ldots, \sigma_{2M}, 0, \ldots, 0)$$

- **Requires only $2M+1$ antennas**
- Bases of signal subspaces are

$$\mathbf{g}_{m\Re}^r, \mathbf{g}_{m\Im}^r, \mathbf{g}_{m\Re}^t, \mathbf{g}_{m\Im}^t$$

- Pseudospectrum $I(\mathbf{x}_p)$ peaks at objects locations

$$I(\mathbf{x}_p) = \frac{1}{\sum_{i=2M+1}^{N_r} |\mathbf{g}_p^r \cdot \mathbf{u}_i|} \times \frac{1}{\sum_{j=2M+1}^{N_t} |\mathbf{g}_p^t \cdot \mathbf{v}_j|}$$

Conclusions

- New framework for power-only high-resolution TR-MUSIC imaging
  - Rytov approximation for wave modeling
  - Smart antenna placement scheme
  - **Requires only $2M+1$ antennas, as compared to state of the art of M^2+1**
- Simulations demonstrate ability of proposed framework to resolve closely-spaced targets

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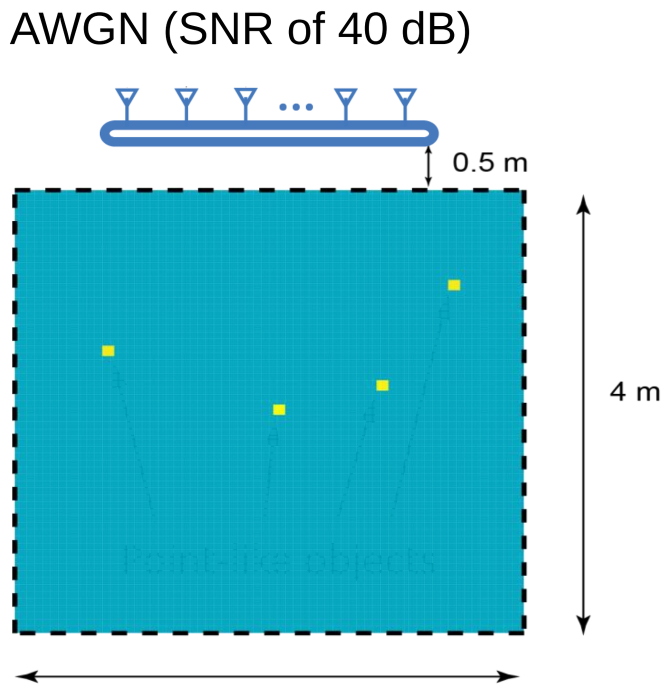
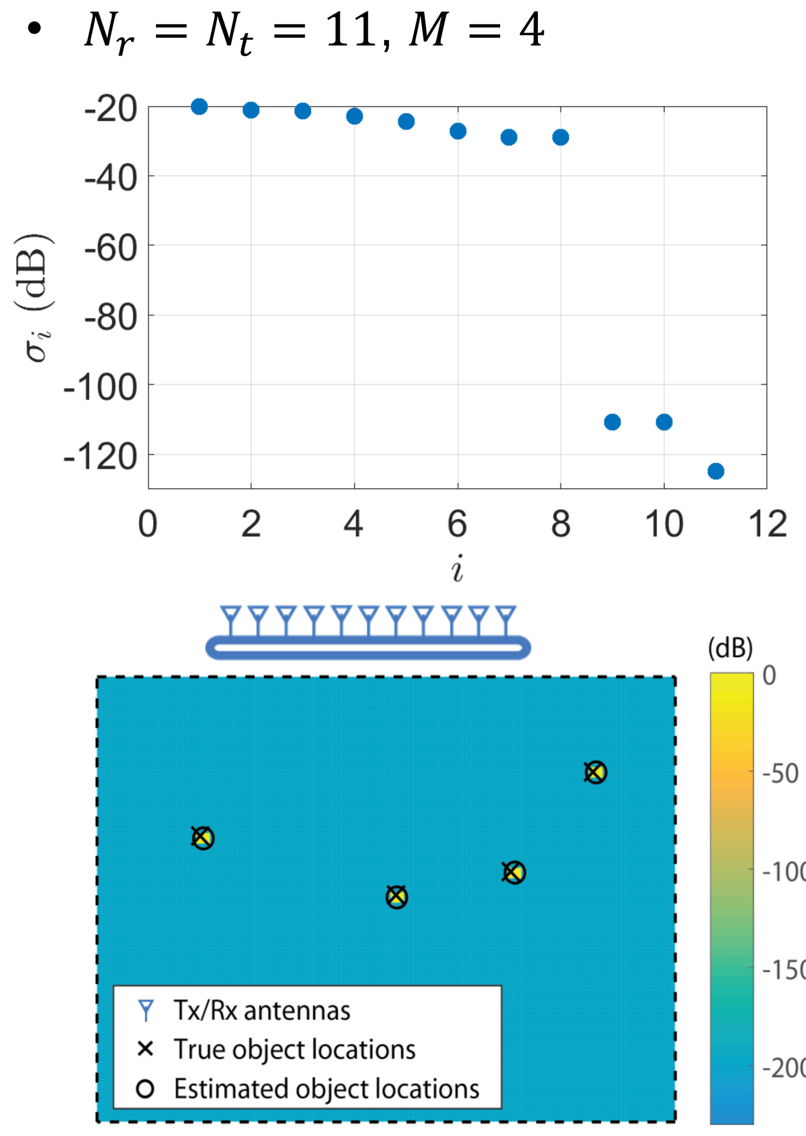
Results

Simulation settings

- Point objects in 4 m x 4 m area
- Noise-free cases:
  - Rytov forward model
  - No AWGN
- Noisy cases:
  - Exact forward model
  - AWGN (SNR of 40 dB)

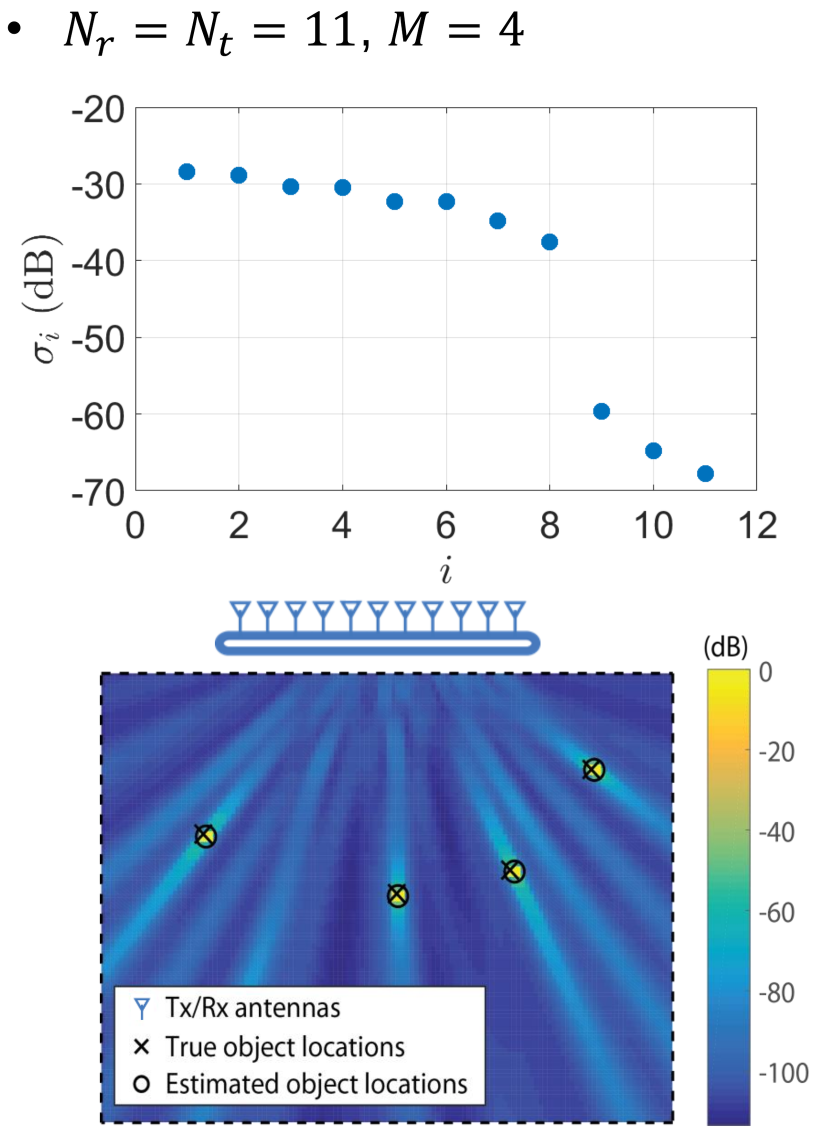


- Noise-free, single Tx/Rx array
- $N_r = N_t = 11$, $M = 4$



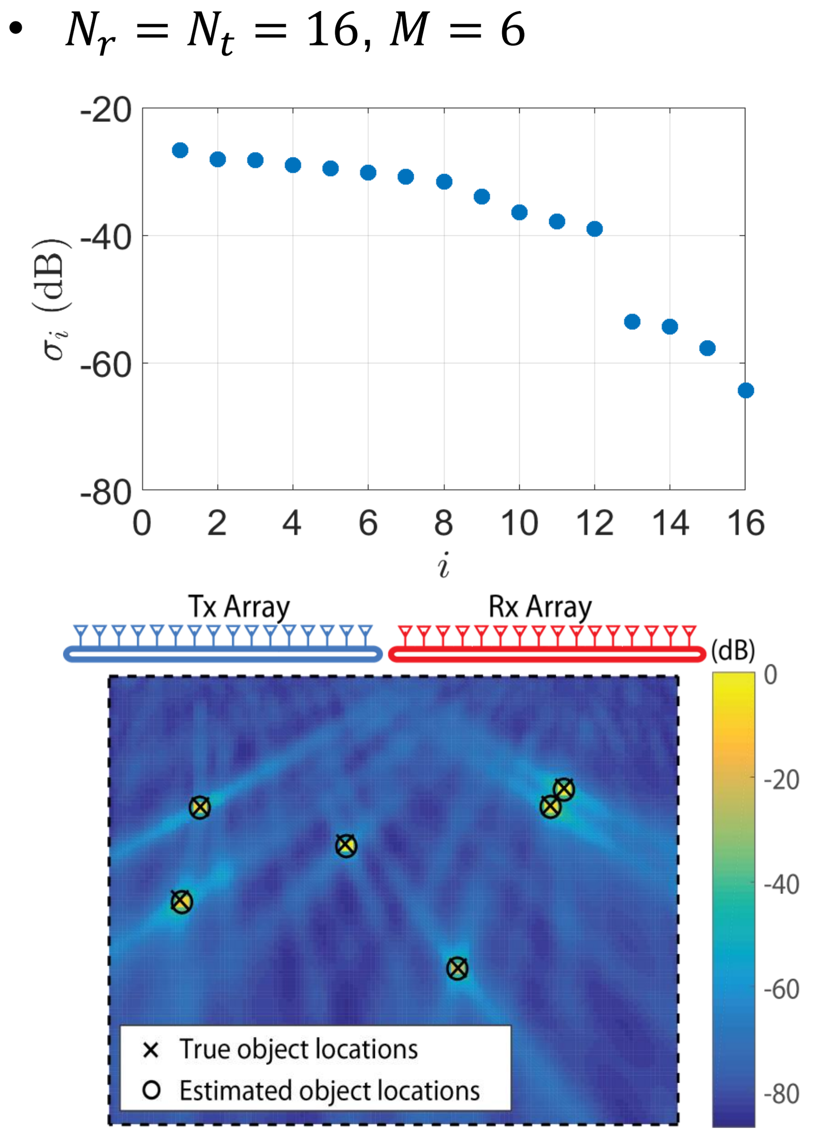
Magnitude of the pseudospectrum

- Noisy, single Tx/Rx array
- $N_r = N_t = 11$, $M = 4$



Magnitude of the pseudospectrum

- Noisy, double array
- $N_r = N_t = 16$, $M = 6$



Magnitude of the pseudospectrum