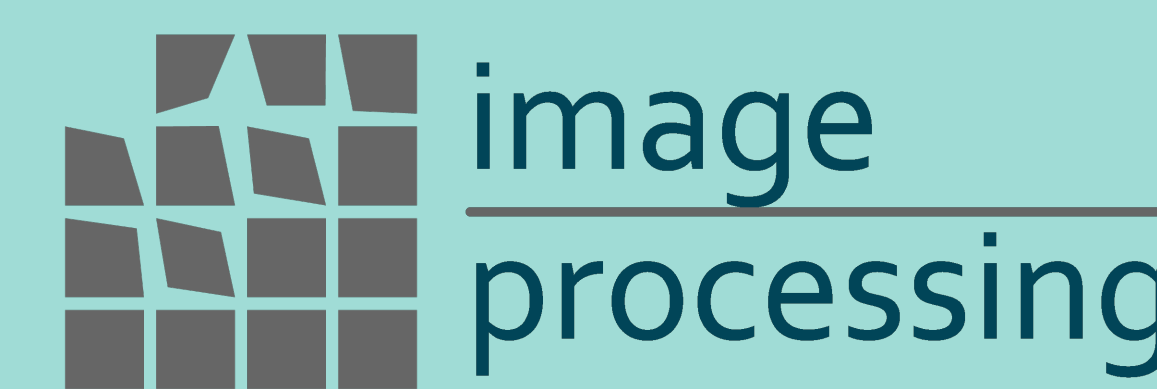# AFFINE INVARIANTS OF VECTOR FIELDS

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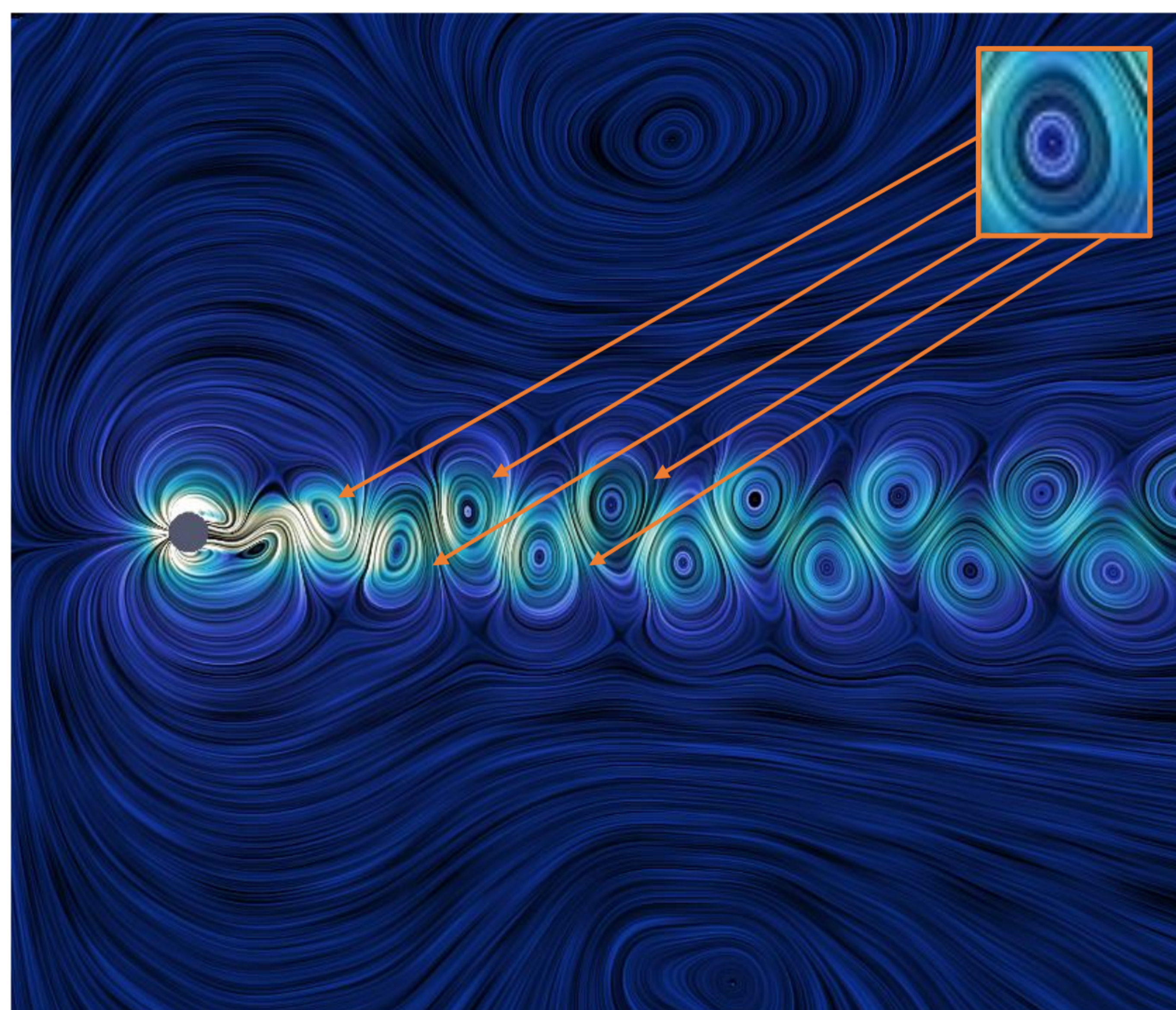
ŪTIA

image processing

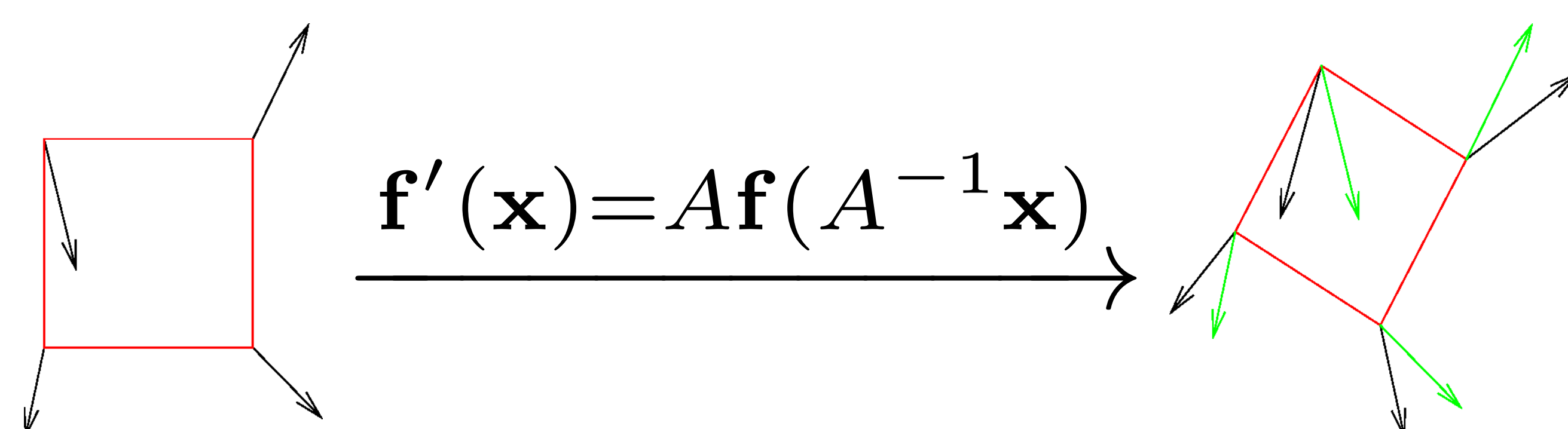## Method

Vector field (VF)

$$\mathbf{f}(\mathbf{x}) = (f_1(\mathbf{x}), f_2(\mathbf{x})) = f_1(x, y) + i f_2(x, y)$$

Patterns in VFs

• Detection of singularities and specific patterns

Total affine transformation

$$\mathbf{f}'(\mathbf{x}) = A\mathbf{f}(A^{-1}\mathbf{x})$$

Geometric moments

$$m_{pq}^{(i)} = \iint x^p y^q f_i(x, y) \, \mathrm{d}x \, \mathrm{d}y$$

Cross products

$$C_{kj} = x_k y_j - x_j y_k$$
$$F_{kj} = f_1(x_k, y_k) f_2(x_j, y_j)$$
$$\qquad - f_1(x_j, y_j) f_2(x_k, y_k)$$
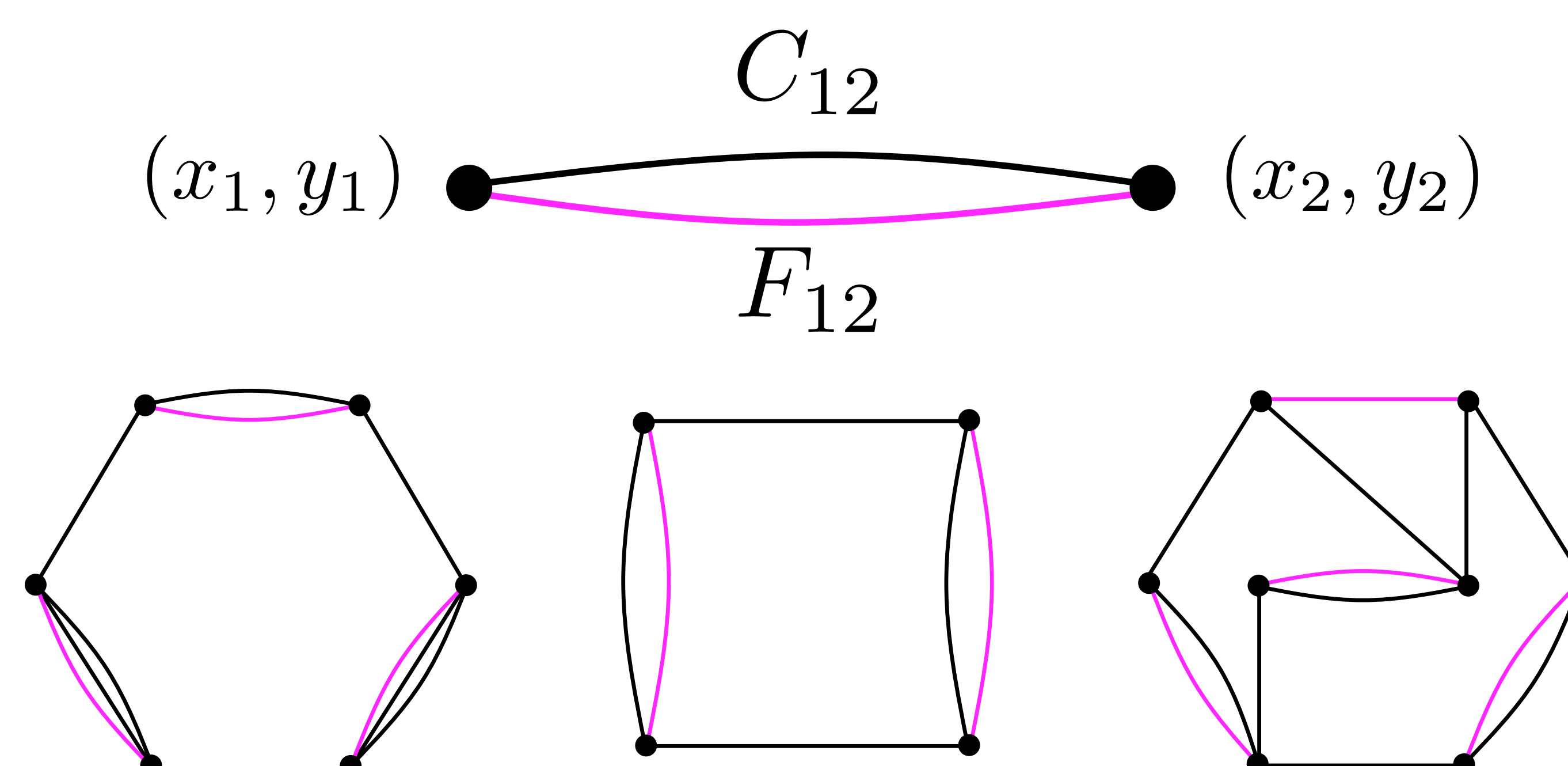
Relative invariants

• VF affine moment invariants (VFAMI)

$$V(\mathbf{f}) = \int_{-\infty}^{+\infty} \cdots \int_{-\infty}^{+\infty} \prod_{k,j=1}^{r} C_{kj}^{n_{kj}} F_{kj}^{v_{kj}} \prod_{i=1}^{r} \mathrm{d}x_i \, \mathrm{d}y_i$$

$$V(\mathbf{f}') = |A|^{v+w+r} V(\mathbf{f})$$

• Absolute invariants via normalization

VFAMI + bi-layer graphs
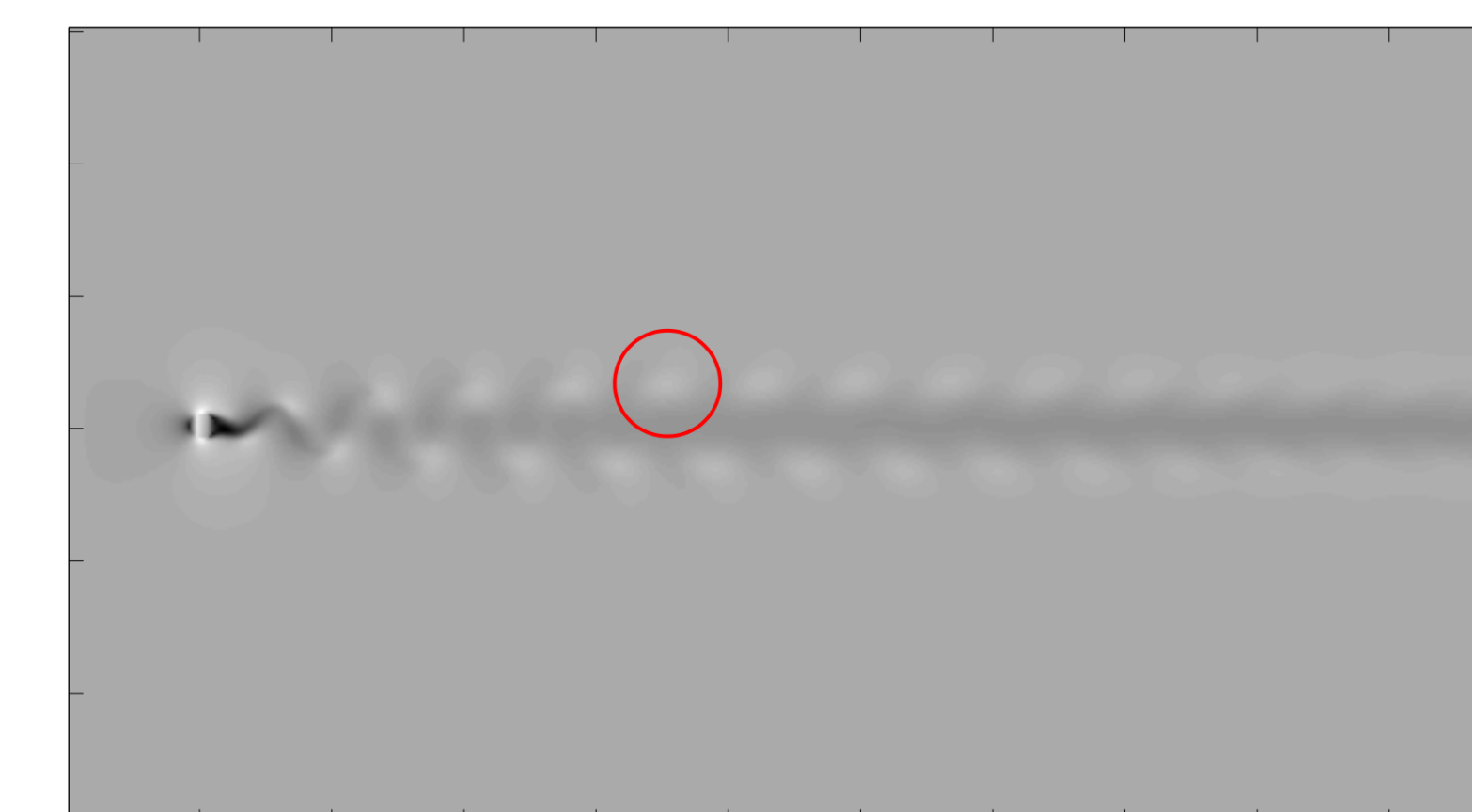

$$C_{12}$$
$$(x_1, y_1) \qquad\qquad (x_2, y_2)$$
$$F_{12}$$

Experiment

Template matching

• Vortex detection in a fluid flow VF

Selected template from Kármán vortex street

• Find all patches of similar shape modulo total affine transformation

Templates found in the transformed field