BAYESIAN APPROACH FOR AUTOMATIC JOINT PARAMETER ESTIMATION IN 3D IMAGE RECONSTRUCTION FROM MULTI-FOCUS MICROSCOPE

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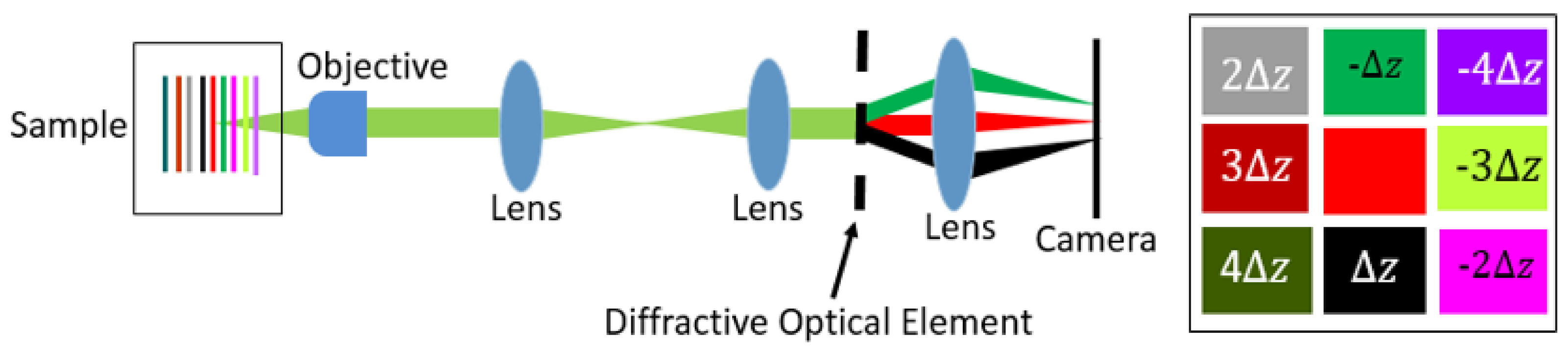
Introduction to MFM

Multi-focus microscopy (MFM) is microscopy that captures multiple focal planes with a single shot [1]. A diffractive grating splits the light from different focal depth to form an array of *K* x *K* image tiles on a camera. It is able to capture dynamic scenes in biological samples, such as movement of a cell or a molecule.



(a) Schematics of MFM (b) On camera

Fig 1. Multi-focus microscopy

Huang et al. developed a 3D MFM image reconstruction method using total variation (TV) regularization [2], but its performance heavily depends on the regularization parameter that has to be chosen through an exhaustive search.

Bayesian Modeling

The image acquisition process in MFM microscopy can be modeled as

$$\mathbf{g}_i = \mathbf{D}\mathbf{H}_i\mathbf{f} + \boldsymbol{\epsilon}_i \qquad \boldsymbol{\epsilon}_i \sim \mathcal{N}(\mathbf{0}, \beta_i^{-1}\mathbf{I}) \qquad i = 1, \ldots, K^2$$

$$\mathbf{g}_i \in \mathbb{R}^{M\times 1} \quad \mathbf{f} \in \mathbb{R}^{N\times 1} \quad \mathbf{H}_i \in \mathbb{R}^{N\times N} \quad \mathbf{D} \in \mathbb{R}^{M\times N} \quad \boldsymbol{\epsilon}_i \in \mathbb{R}^{M\times 1}$$

and it leads to the likelihood function as

$$\mathrm{p}(\mathbf{g}|\mathbf{f},\boldsymbol{\beta}) = \prod_{i=1}^{K^2} \frac{\beta_i^{M/2}}{(2\pi)^{M/2}} \exp\left\{-\frac{\beta_i}{2}\|\mathbf{g}_i - \mathbf{D}\mathbf{H}_i\mathbf{f}\|_2^2\right\}$$

where $\mathbf{g} = [\mathbf{g}_1^T, \ldots, \mathbf{g}_{K^2}^T]^T$ and $\boldsymbol{\beta} = [\beta_1, \ldots, \beta_{K^2}]^T$.

The prior is as

$$\mathrm{p}(\mathbf{f}|\alpha) = \frac{1}{Z_\alpha}\exp\{-\alpha\mathrm{TV}(\mathbf{f})\} \propto \alpha^{N/2}\exp\{-\alpha\mathrm{TV}(\mathbf{f})\}$$

where $\mathrm{TV}(\mathbf{f}) = \sum_{j=1}^{N}\sqrt{(\Delta_j^x\mathbf{f})^2 + (\Delta_j^y\mathbf{f})^2 + (\Delta_j^z\mathbf{f})^2}$.

Finally, the joint distribution is as

$$\mathrm{p}(\mathbf{g},\mathbf{f},\boldsymbol{\beta},\alpha) = \mathrm{p}(\mathbf{g}|\mathbf{f},\boldsymbol{\beta})\mathrm{p}(\mathbf{f}|\alpha)\mathrm{p}(\boldsymbol{\beta})\mathrm{p}(\alpha)$$

where $\mathrm{p}(\alpha)$ and $\mathrm{p}(\boldsymbol{\beta})$ are non-informative flat priors.

Maximum a Posteriori (MAP) Inference

We use MAP inference to estimate all the unknowns of the model with the positivity constraint of pixel values.

$$\{\hat{\mathbf{f}}, \hat{\boldsymbol{\beta}}, \hat{\alpha}\} = \arg\max_{\mathbf{f}\geq\mathbf{0},\boldsymbol{\beta},\alpha} \mathrm{p}(\mathbf{f},\boldsymbol{\beta},\alpha|\mathbf{g}) = \arg\min_{\mathbf{f}\geq\mathbf{0},\boldsymbol{\beta},\alpha} -\log\mathrm{p}(\mathbf{g},\mathbf{f},\boldsymbol{\beta},\alpha)$$

Alternating minimization scheme is used to optimize as

$$\mathbf{f}^{(n+1)} = \arg\min_{\mathbf{f}\geq\mathbf{0}} \frac{1}{2}\sum_{i=1}^{K^2}\beta_i^{(n)}\|\mathbf{g}_i - \mathbf{D}\mathbf{H}_i\mathbf{f}\|^2 + \alpha^{(n)}\mathrm{TV}(\mathbf{f})$$

$$\beta_i^{(n+1)} = \frac{M}{\|\mathbf{g}_i - \mathbf{D}\mathbf{H}_i\mathbf{f}^{(n+1)}\|^2}$$

$$\alpha^{(n+1)} = \frac{N}{2\mathrm{TV}(\mathbf{f}^{(n+1)})}$$

Experimental Results

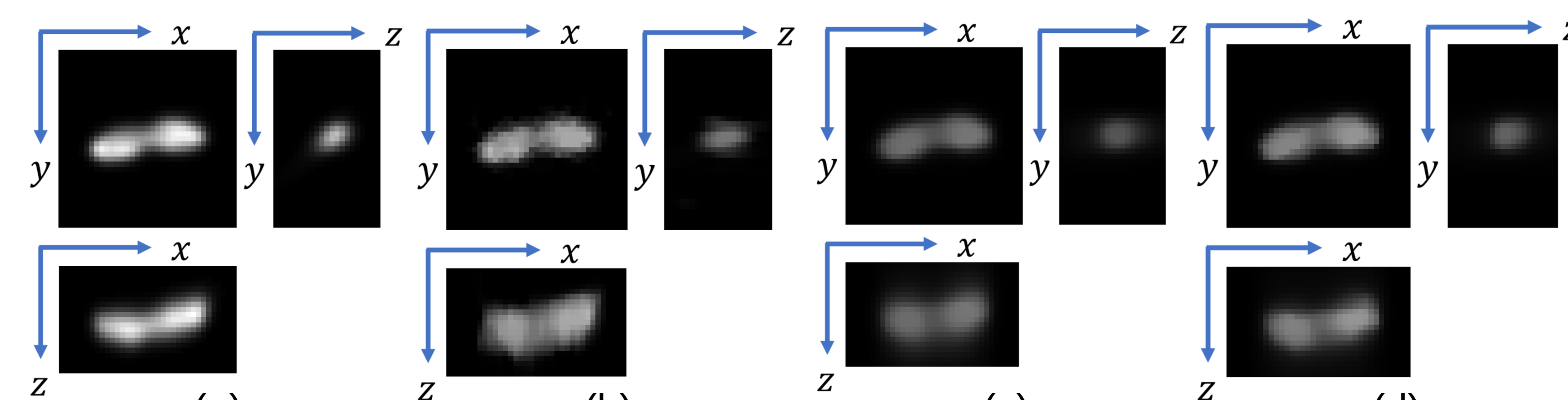


Fig 2. MFM reconstruction results. (a) Ground truth, (b) RLS-TV, (c) MAP-TV1, and (d) MAP-TV2

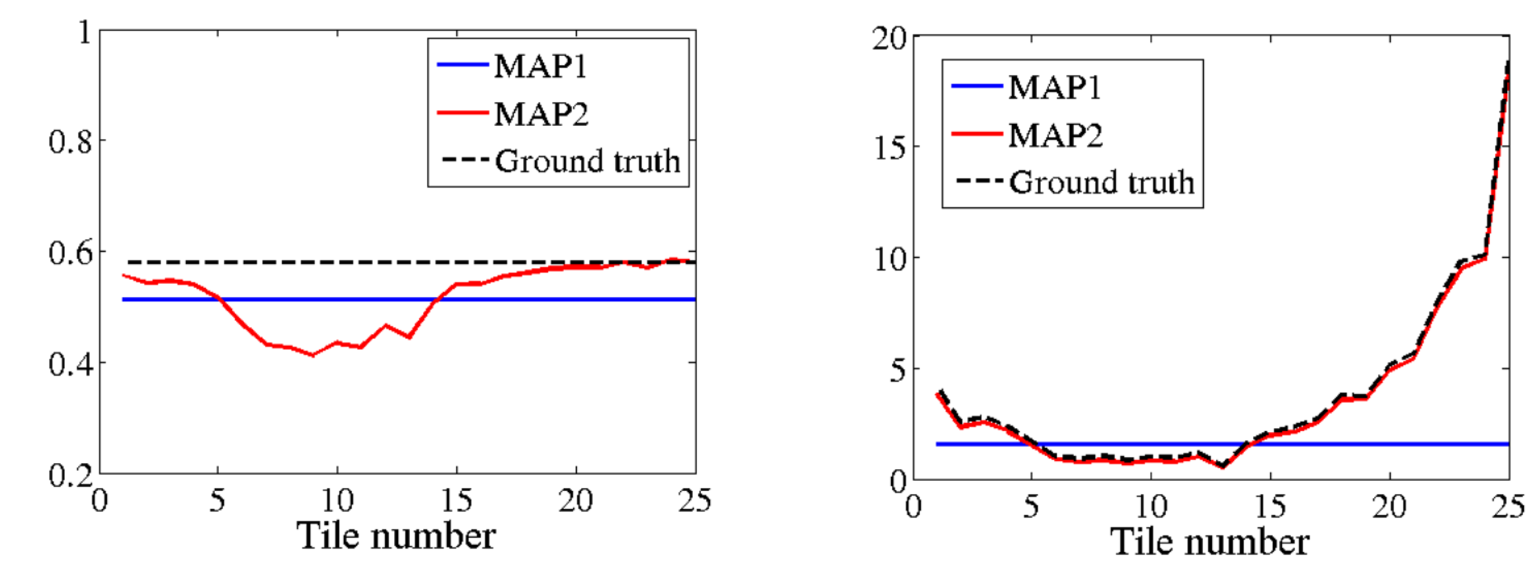


Fig 3. Estimation of β_i with different types of noise

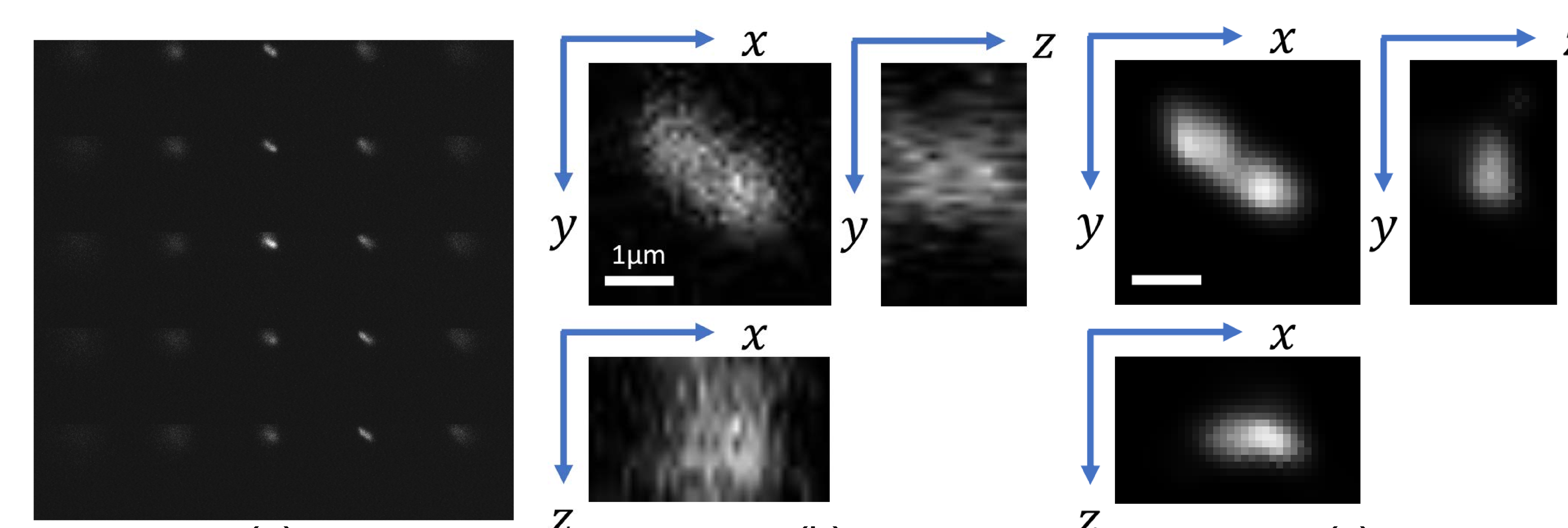


Fig 4. Real data experiment. (a) MFM image, (b) tile stacking, (c) MAP-TV2

References

[1] S. Abrahamsson et al., "Fast multicolor 3D imaging using aberration-corrected multifocus microscopy," *Nature Methods*, vol. 10, no. 1, pp. 60–63, 2012.
[2] X. Huang, et al, "3D snapshot microscopy of extended objects," arXiv:1802.01565, 2018.