Spectral Graph Wavelet Based Nonrigid Image Registration

Nhung Pham, David Helbert, Pascal Bourdon, Philippe Carré

XLIM/ASALI, UMR CNRS 7252, Université de Poitiers

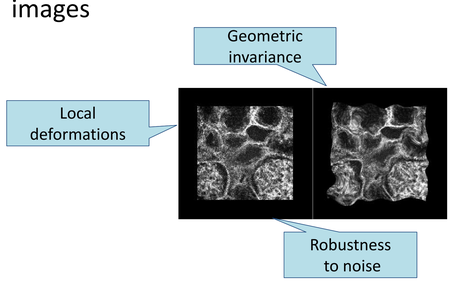
Bât. SP2MI, Téléport 2, 11 Bd Marie et Pierre Curie, BP 30179, 86962 Futuroscope Chasseneuil Cedex, France

hong.nhung.pham@univ-poitiers.fr

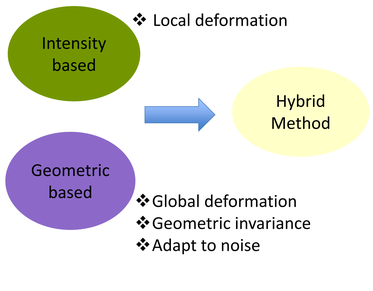
Introduction

Motivation

❖ Nonrigid image registration for medical images



Objective



Spectral Graph Wavelets Transform

❖ Modulated by generating kernel g

❖ SGW of f at point i and scale t:

$$W_f(t,i) = \sum_{\ell=1}^{N} g(t\lambda_\ell)\hat{f}(\ell)\mathrm{v}_\ell(i)$$

(λ_ℓ ,v_ℓ: ℓ^{th} eigenvalue and eigenvector of L
$\hat{f}(\ell)$: Fourier transform of graph signal f)

❖ Low frequency encoded by h:
$h(0) > 0$, $\lim_{x\to\infty} h(x) = 0$

SGWT: - Fast and easy to implement
- Represent image spectra hierarchically

❖ Scaling function

$$S_f(i) = \sum_{\ell=1}^{N} h(\lambda_l)\,\hat{f}(\ell)\mathrm{v}_\ell(i)$$

Proposed Method

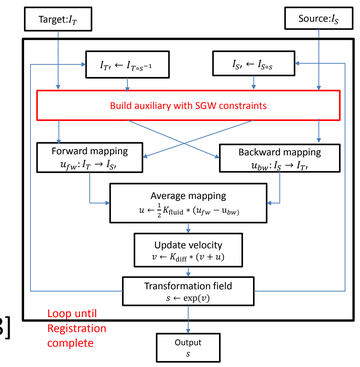
❖Objective function:

$$E(I_T, I_S, s) = \alpha_i E_{sim}(I_T, I_{S\circ s}) + \alpha_r E_{reg}(s) + \alpha_w ||W_T - W_{S\circ s}||^2$$

(α_w: geometric weight)

❖ Optimization: Log-Demons framework. [Ver2008]

Registration with SGW constraint



Theory and Method

Non-parametric Image Registration

❖ Objective function:

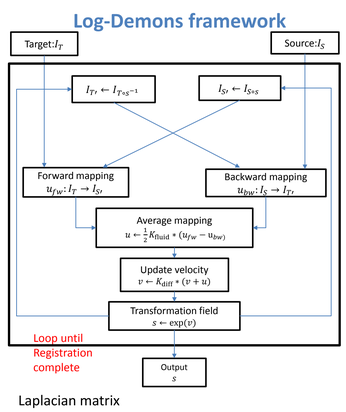
$$E(I_T, I_S, s) = \alpha_i E_{sim}(I_T, I_{S\circ s}) + \alpha_r E_{reg}(s),$$

$E_{sim}(I_T, I_{S\circ s}) = ||I_T - I_{S\circ s}||^2$,
$E_{reg}(s) = ||X_T - X_{S\circ s}||^2$
α_i: intensity weight
α_r: spatial weight

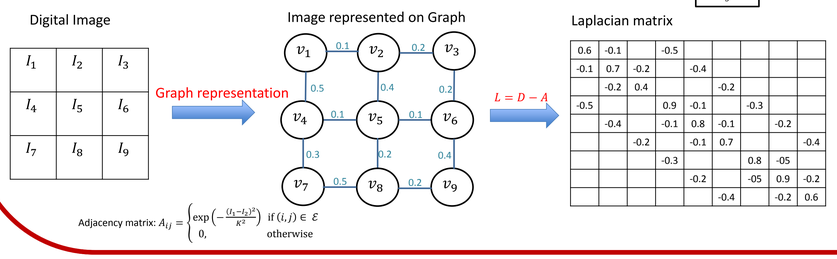
Advantages:
- Fast and easy to use
- Adapt well with medical images

Disadvantages:
- Local minima
- Sensitive to noise
- Lack of isometric invariance

Log-Demons framework



Data Representation on Graph

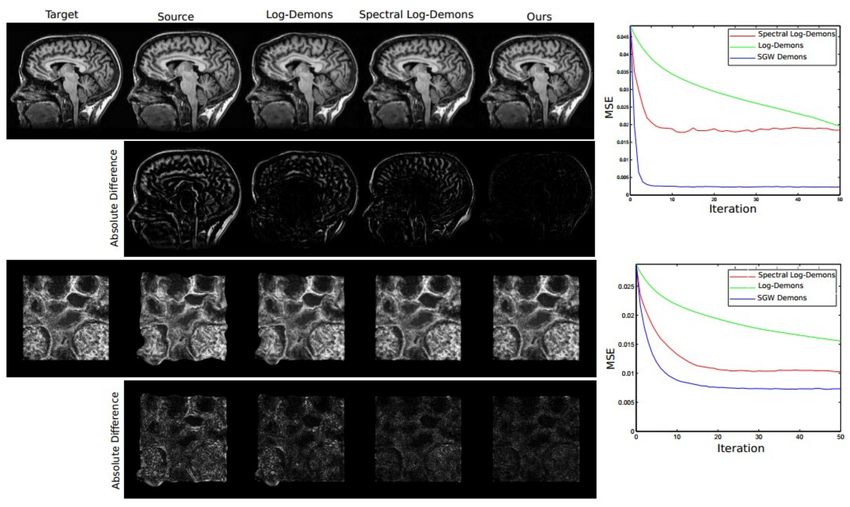


0.6	-0.1		-0.5					
-0.1	0.7	-0.2		-0.4				
	-0.2	0.4			-0.2			
-0.5			0.9	-0.1		-0.3		
	-0.4		-0.1	0.8	-0.1		-0.2	
		-0.2		-0.1	0.7			-0.4
			-0.3			0.8	-05	
				-0.2		-05	0.9	-0.2
					-0.4		-0.2	0.6

Adjacency matrix: $A_{ij} = \begin{cases} \exp\left(-\frac{(I_i - I_j)^2}{k^2}\right) & \text{if } (i,j) \in \mathcal{E} \\ 0, & \text{otherwise} \end{cases}$

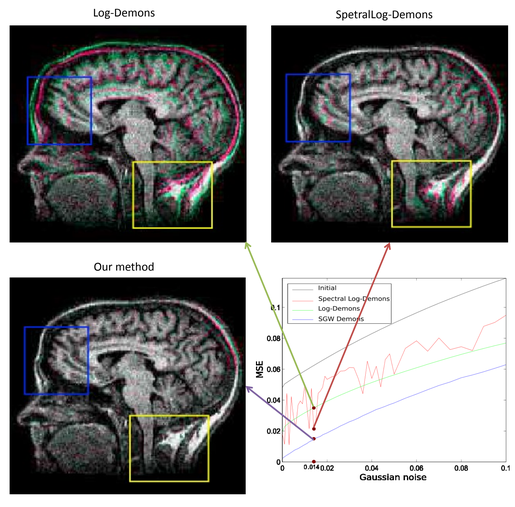
Results

Experiments on brain T1 images and endomicroscopic images



Comparisin between our proposal and the existing methods when no noise is added

Robustness to noise



MSE with respect to Gaussian noise (σ_{noise})

[Ham2011] Hammond et al (2011). Wavelets on graphs via spectral graph theory. Applied and Computational Harmonic Analysis, Elsevier.
[Ver2008] Vercauteren et al (2008). Symmetric log-domain diffeomorphic registration: A demons-based approach. International conference in medical image computing and computer-assisted intervention, Springer.

Conclusion

❖SGWs can be applied to spectral matching, and then adapted to the Log-Demons framework.

❖Our method outperforms the existing methods on similar framework

❖Our method is more robust to noise