TOPOLOGICAL EULERIAN SYNTHESIS OF SLOW MOTION PERIODIC VIDEOS

Christopher Tralie, ctralie@alumni.princeton.edu
Postdoctoral Associate, Department of Mathematics, Duke University
Matthew Berger, matthew.berger@vanderbilt.edu
Assistant Professor, Department of Electrical Engineering And Computer Science, Vanderbilt University

Overview

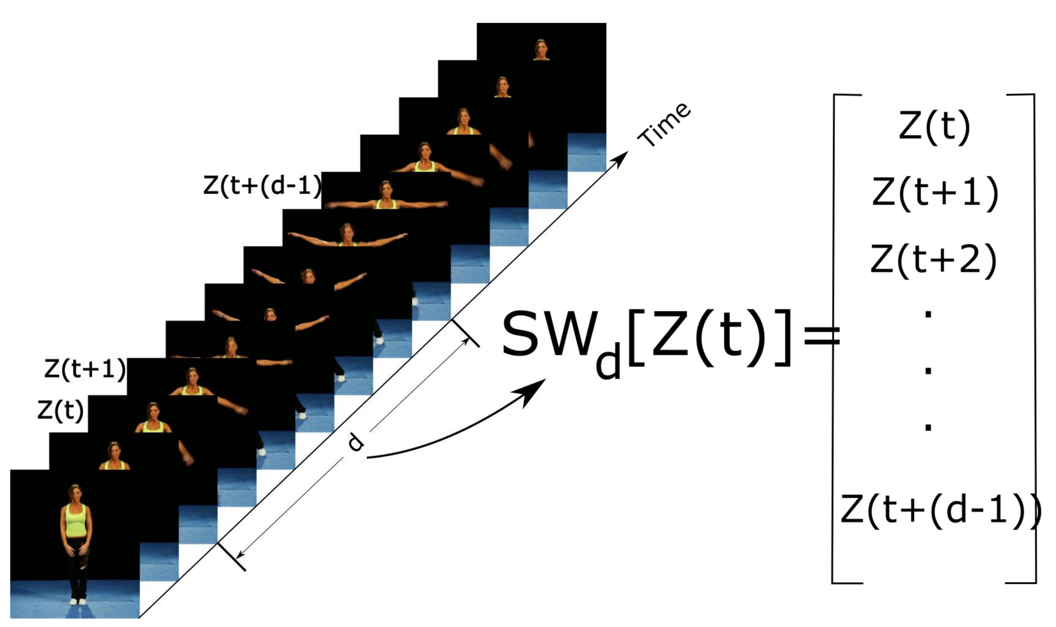
Goal: Take a video with multiple periods of repetitive motion and reorder frames into a single fine detail period

Applications: Heartbeat monitoring[9], repetitive motion stress analysis[10], fine scale motion analysis for optimizing sports performance or detecting onset of mechanical failure, autism stereotypical repetitive motion analysis[11]

Challenges: Noise, drift, occlusions, background motion

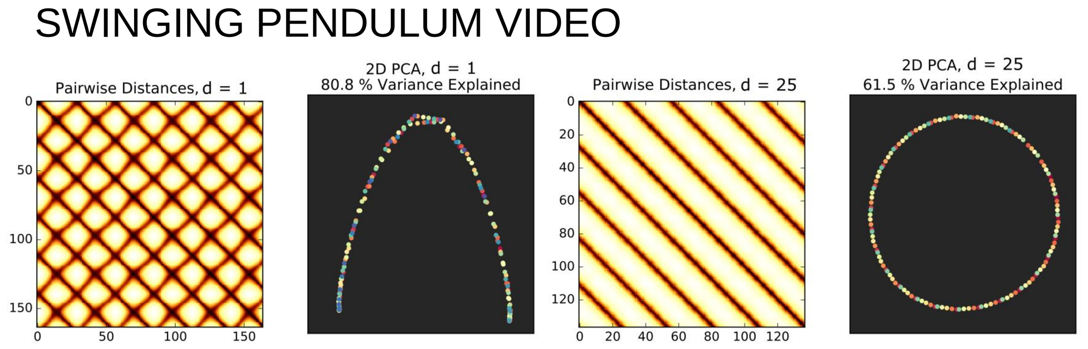
Main Approach: Parameterize period topologically with sliding windows and pixel by pixel median vote where sliding windows line up

Sliding Window Videos

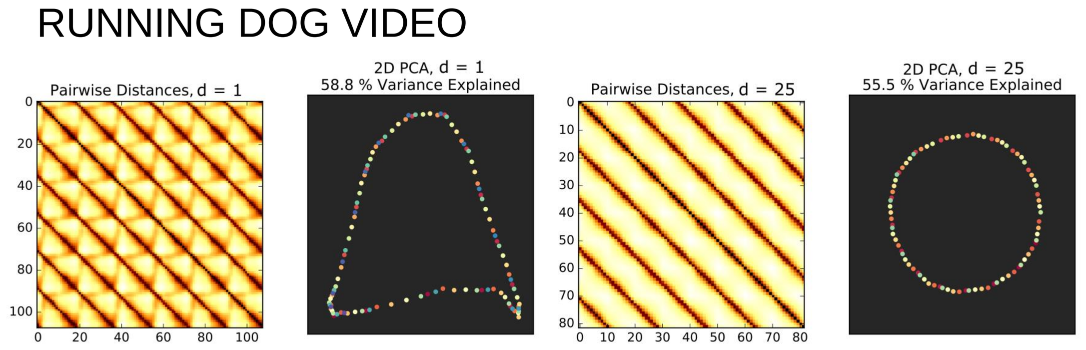


- Stack delay frames of video into one large vector[5, 6]
- Leads to a point cloud which lies on a topological loop for all types of periodic videos[5]
- Provides a sort of "time regularization"

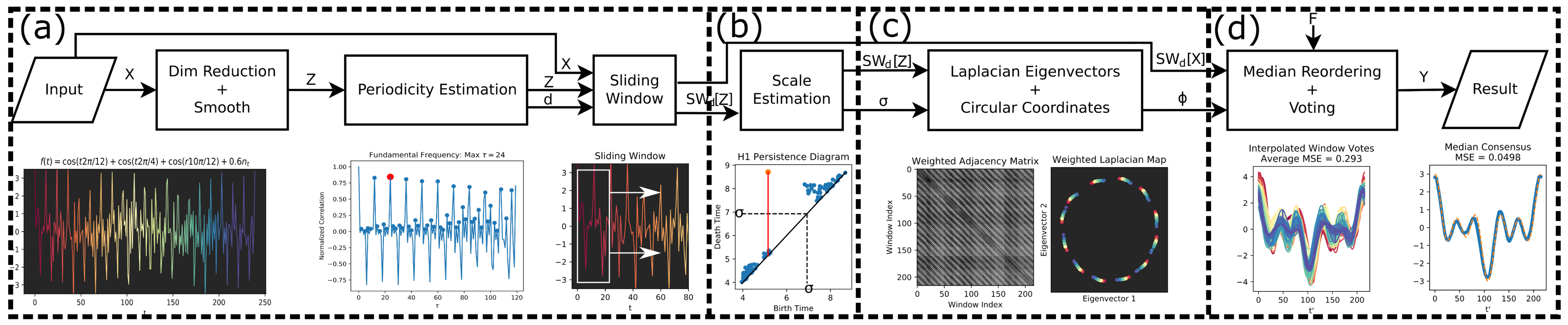
SWINGING PENDULUM VIDEO



RUNNING DOG VIDEO



Pipeline

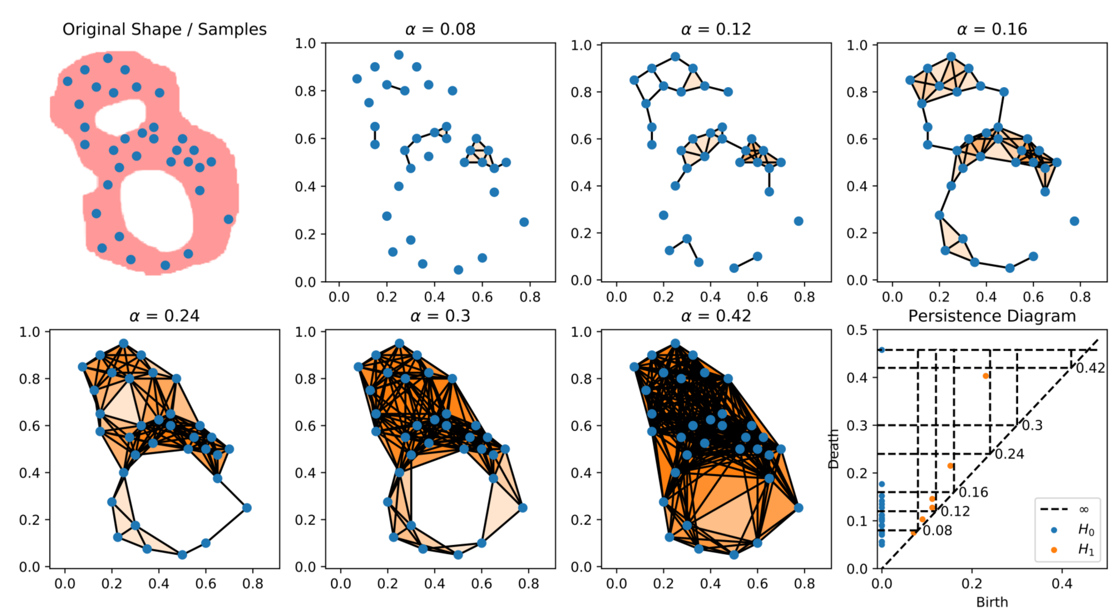


Code / Supplementary Material

https://github.com/ctralie/SloMoLoops

http://www.ctralie.com/Research/SloMoLoops/

Topological Data Analysis / Persistent Homology[2]



- A tool for quantifying multiscale topological features in point cloud data
- Persistence diagram: births scale at which feature forms, death scale at which feature dies. *Persistence* is death - birth.
- Used to find the scale at which the graph Laplacian (see below) should be built on the sliding window embedding
- Let scale σ be $\alpha b_i + (1-\alpha) d_i \quad \alpha \in [0,1]$ where d_i and b_i are the death and birth times corresponding to max persistence dot (similar to [4])

Weighted Graph Laplacian Circular Coordinates

$$\mathbf{A}_{ij} = exp(-||SW_d[\mathbf{Z}(i)] - SW_d[\mathbf{Z}(j)]||_2 / 2\sigma^2)$$

$$\mathbf{D}_{ii} = \sum_{j=1}^{N} \mathbf{A}_{ij}, \mathbf{D}_{i \neq j} = 0$$

$$\mathbf{L} = \mathbf{D} - \mathbf{A}$$

- A tool for nonlinear dimension reduction
- Inspired by [1], we use it to parameterize our sliding window point cloud after building a weighted graph on it
- Use v_1 and v_2 adjacent eigenvectors with smallest number of zero crossings within 10 smallest eigenvalues

$$\phi[n] = \tan^{-1}(v_1[n]/v_2[n])$$

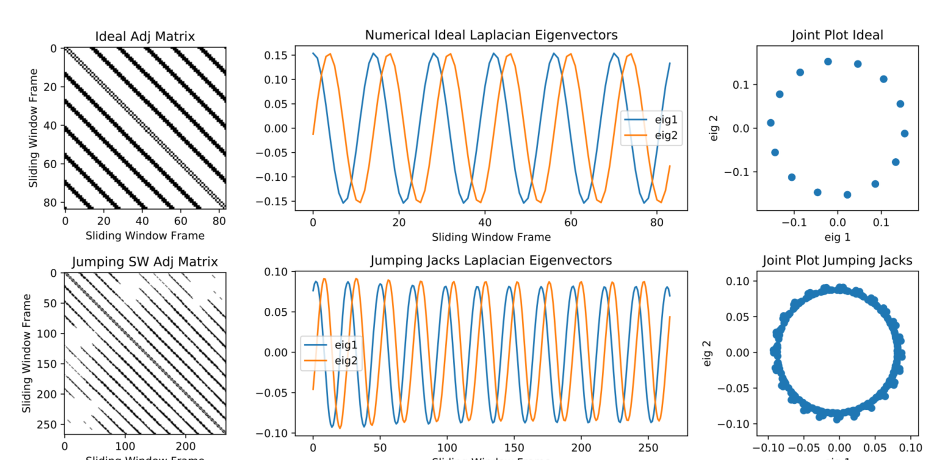
Circulant Graph Model for Periodic Videos

$$L_{ij} = \begin{Bmatrix} 3k-1 & i=j \\ -1 & |i-j| = lT, l \in \mathbb{Z}^+ \\ -1 & |i-j| = lT \pm 1, l \in \mathbb{Z}^+ \\ 0 & \text{otherwise} \end{Bmatrix}$$

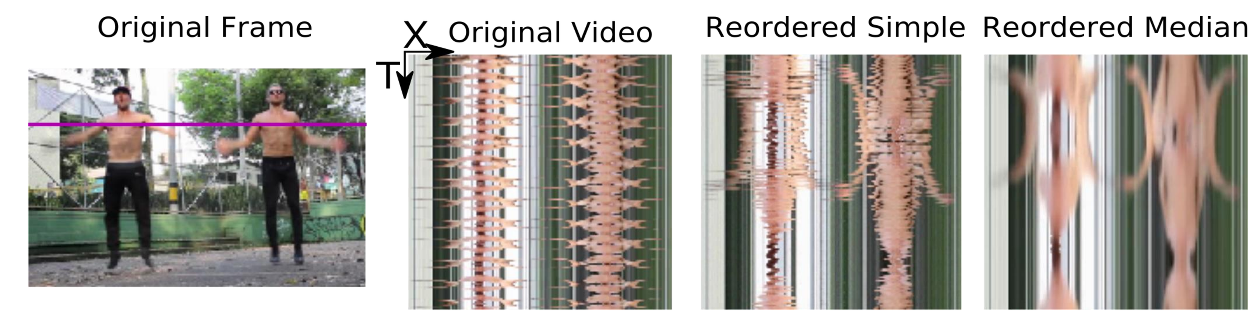
$$v_{2k}[n] = \cos(2\pi n/T), v_{2k+1}[n] = \sin(2\pi n/T)$$

$$\begin{matrix}\lambda_{2m}, \\ \lambda_{2m+1}\end{matrix} = \left\{ \begin{matrix} 3k - k\left(1 + 2\cos\left(\frac{2\pi}{kT}m\right)\right) & m = lk, l \in \mathbb{Z}^+ \\ 3k & \text{otherwise} \end{matrix} \right\}$$

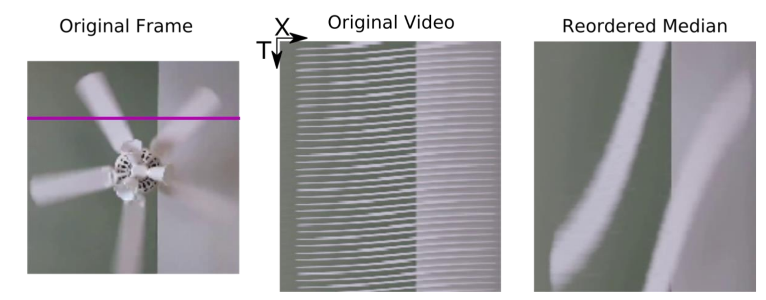
- Assume a video with period **T** going through **k** periods
- Eigenvectors come in cosine/sine pairs with eigenvalue of multiplicity 2
- Eigenvectors with smallest nonzero eigenvalue go through one period. When plotted jointly, they form a circle



Qualitative Results on Full Pipeline

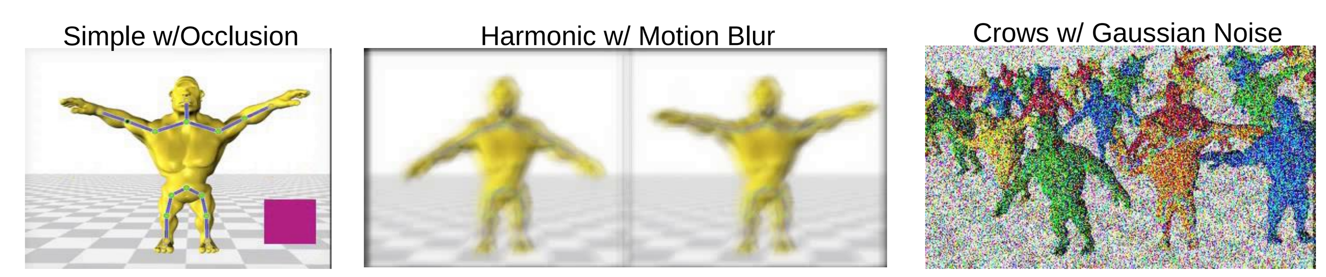


- Simple frame reordering by circular coordinates is visually choppy
- Median voting removes background objects and cuts mitigates motion drift between periods

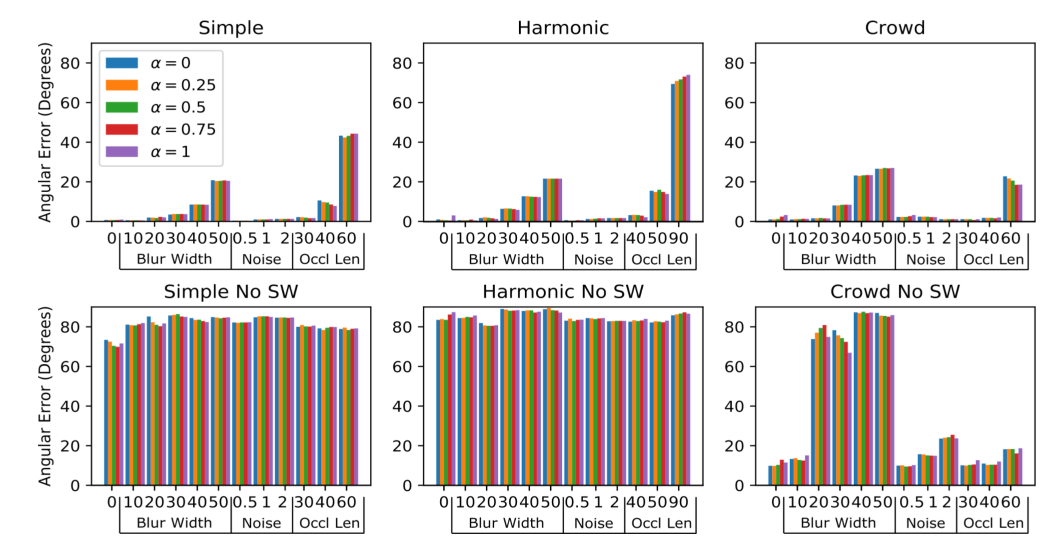


Quantitative Experiment on Circular Coordinates

- 3 Synthetic Periodic Videos: Simple, harmonic, crowd
- 3 Noise Types: AWGN, Motion Blur, Dynamic Occlusion/Background



- With sliding window, errors are low for severe noise and for moderate shake and occlusions
- Without sliding window, errors high for nearly all noise ranges



SLIDING WINDOW

NO SLIDING WINDOW

References

[1] Hadar Averbuch-Elor and Daniel Cohen-Or. Ringit: Ring-ordering casual photos of a temporal event. ACM Trans. Graph., 34(3):33, 2015
[2] Herbert Edelsbrunner and John Harer. Computational Topology: an introduction. American Mathematical Soc., 2010.
[3] Chris Godsil and Gordon F Royle. Algebraic graph theory, volume 207. Springer Science & Business Media, 2013
[4] Jose A Perea and John Harer. Sliding windows and persistence: An application of topological methods to signal analysis. Foundations of Computational Mathematics, 15(3):799–838, 2015
[5] Christopher J. Tralie and Jose A. Perea. (quasi)periodicity quantification in video data, using topology. SIAM Journal on Imaging Sciences, 11(2):1049–1077, 2018.
[6]Christopher John Tralie. Geometric Multimedia Time Series. PhD thesis, Duke University Department of Electrical And Computer Engineering, 2017.
[7] Vinay Venkataraman, Karthikeyan Natesan Ramamurthy, and Pavan Turaga. Persistent homology of attractors for action recognition. In Image Processing (ICIP), 2016 IEEE International Conference on, pages 4150–4154. IEEE, 2016.
[8] ZichengLiao,NeelJoshi,andHuguesHoppe, "Automatedvideolooping with progressive dynamism," ACM Trans. Graph., vol. 32, no. 4, pp. 77:1–77:10, July 2013.
[9] Mayank Kumar, Ashok Veeraraghavan,and Ashutosh Sabharwal, "DistancePPG: Robust non-contact vital signs monitoring using a camera," Biomedical optics express, vol. 6, no. 5, pp. 1565–1588, 2015.
[10] Runyu L Greene, David P Azari, Yu Hen Hu, and Robert G Radwin, "Visualizing stressful aspects of repetitive motion tasks and opportunities for ergonomic improvements using computer vision," Applied ergonomics, vol. 65, pp. 461–472, 2017.
[11] Ulf Großekathöfer, Nikolay V Manyakov, Vojkan Mihajlović, Gahan Pandina, Andrew Skalkin, Seth Ness, Abigail Bangerter, Matthew S Goodwin, "Automated detection of stereotypical motor movements in autism spectrum disorder using recurrence quantification analysis." Frontiers in neuroinformatics 2017
Please see our paper for a more complete list of references

Acknowledgements

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