PCCA: A Projection method for effective fMRI data analysis

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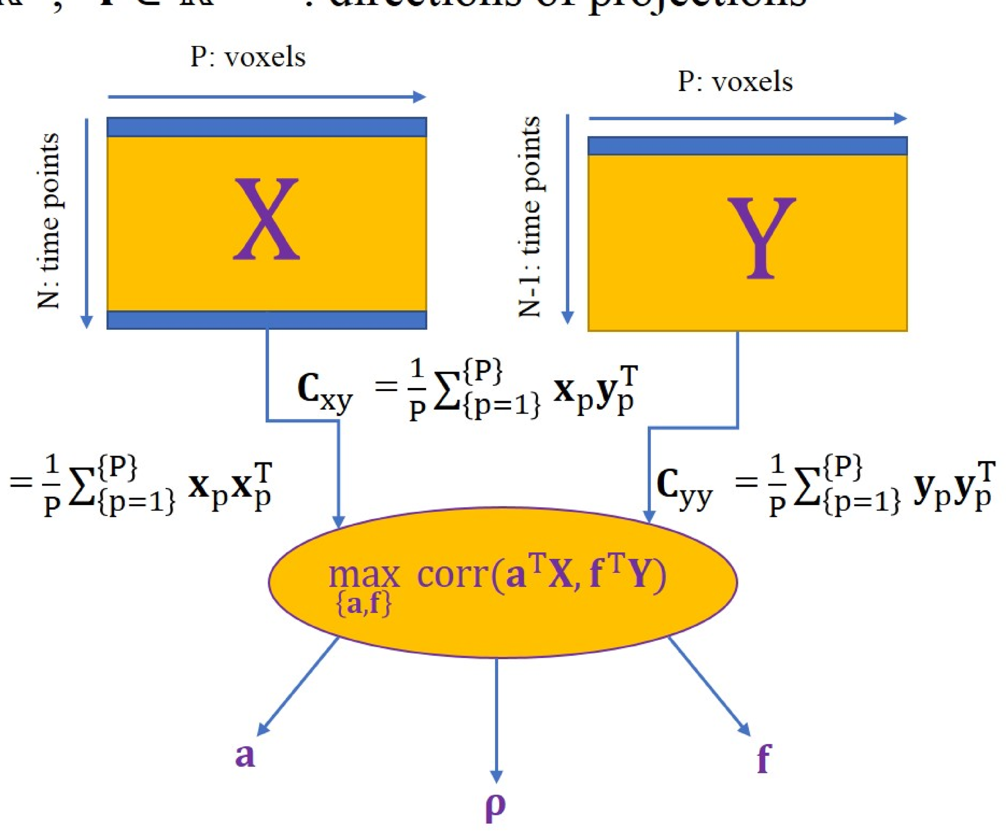
Introduction

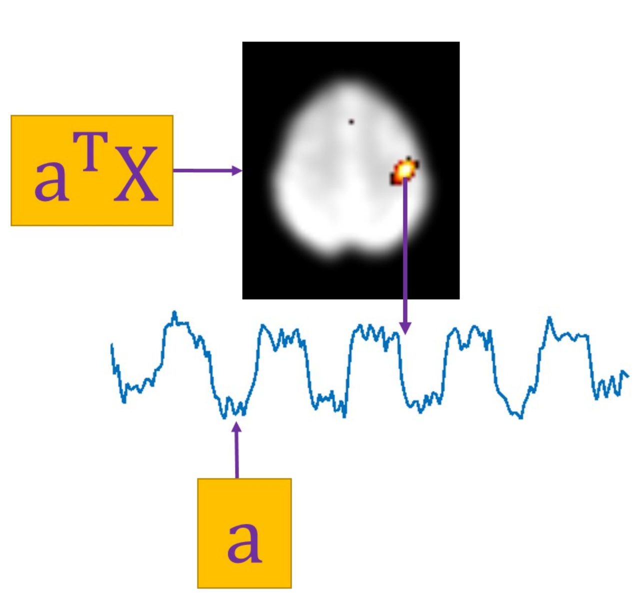
- Canonical correlation analysis (CCA) is a technique for studying the relationship between two sets of variables.
- Few studies on CCA incorporate prior information available for fMRI data.
- Here we propose a projection CCA method by creating a basis for a span that better characterizes the fMRI data-set.
- The proposed method can be seen as a regularized CCA method where regularization is introduced via basis expansion.

Background: CCA

Define:

$\{\mathbf{x}_p \in \mathbb{R}^N\}$
$\{\mathbf{y}_p \in \mathbb{R}^{N-1}\}, p = 1, \cdots, P$
P: voxels, N and N − 1: time points
$\mathbf{a} \in \mathbb{R}^N$, $\mathbf{f} \in \mathbb{R}^{N-1}$: directions of projections





Proposed Method

$$\max_{\mathbf{u},\mathbf{v}} \ \mathbf{u}^\top \mathbf{C}_{xx}^{-1/2}\mathbf{C}_{xy}\mathbf{C}_{yy}^{-1/2}\mathbf{v}$$
$$\text{s.t. } \mathbf{u}^\top\mathbf{u} = \mathbf{v}^\top\mathbf{v} = 1, \quad (1)$$
$$\mathbf{a} = \mathbf{C}_{xx}^{-1/2}\mathbf{u}, \ \mathbf{f} = \mathbf{C}_{yy}^{-1/2}\mathbf{v}$$

- The goal is to incorporate prior information matrix $\mathbf{B} \in \mathbb{R}^{N\times K}$ in the CCA objective function.

$$\mathbf{u} = \mathbf{B}\boldsymbol{\theta} \text{ and } \mathbf{v} = \mathbf{B}\boldsymbol{\psi}. \quad \boldsymbol{\theta}, \boldsymbol{\psi} \in \mathbb{R}^K$$

- Empirical results show that few selected discrete cosine transform (DCT) bases are suitable for $\mathbf{B}$.

Define $\mathbf{T} = \mathbf{C}_{xx}^{-1/2}\mathbf{C}_{xy}\mathbf{C}_{yy}^{-1/2}$

$$\mathcal{L} = \left\|\mathbf{T} - s\mathbf{u}\mathbf{v}^\top\right\|_F^2 = \left\|\mathbf{T} - s\mathbf{B}\boldsymbol{\theta}\boldsymbol{\psi}^\top\mathbf{B}^\top\right\|_F^2 \quad (2)$$

The objective function (2) is solved in two stages:

1. Minimizing $\mathcal{L}$ over $\boldsymbol{\psi}$ and setting $\mathbf{B}^\top\mathbf{B} = \mathbf{R}_B^\top\mathbf{R}_B$ yields $\tilde{\boldsymbol{\theta}}$ as right singular vector of $(\mathbf{R_B^{-1}})^\top\mathbf{B}^\top\mathbf{T}\mathbf{B}\mathbf{R_B^{-1}}$

2. Minimizing $\mathcal{L}$ over $\boldsymbol{\theta}$ and setting $\mathbf{B}^\top\mathbf{B} = \mathbf{R}_B^\top\mathbf{R}_B$ yields $\tilde{\boldsymbol{\psi}}$ as right singular vector of $(\mathbf{R_B^{-1}})^\top\mathbf{B}^\top\mathbf{T}^\top\mathbf{B}\mathbf{R_B^{-1}}$

Statistical significance analysis

Barlett's statistical test is employed to calculate the significance of each recovered voxel time series.

$$L = -\left[P - \frac{1}{2}(N + (N-1) + 3)\right] \sum_{j=1}^{r} \log(1 - s_j^2) \quad (3)$$

- s_j: j^{th} nonzero canonical correlation coefficient
- $L \sim \chi^2$ with $N \times N - 1$ degrees of freedom

Algorithm Overview

Algorithm 1: PCCA Algorithm
Input: Training data $\mathbf{X}$, $\mathbf{Y}$.
Output: $\boldsymbol{\Theta}$ and $\boldsymbol{\Psi}$ containing r pairs of right singular vectors $\boldsymbol{\theta}_r$ and $\boldsymbol{\psi}_r$.
$\mu_x = \frac{1}{P}\sum_{p=1}^{P}\mathbf{x}_p$ and $\mu_y = \frac{1}{P}\sum_{p=1}^{P}\mathbf{y}_p$
for $p \in [1, P]$ **do**
  $\mathbf{x}_p \leftarrow \mathbf{x}_p - \mu_x$
  $\mathbf{y}_p \leftarrow \mathbf{y}_p - \mu_y$
end
Perform singular value decomposition (SVD) of $(\mathbf{R_B^{-1}})^\top\mathbf{B}^\top\mathbf{T}\mathbf{B}\mathbf{R_B^{-1}} = \mathbf{U}\mathbf{S}\boldsymbol{V}^\top$;
$\tilde{\boldsymbol{\Theta}} = \boldsymbol{V}(:, 1:r)$ and estimate $\boldsymbol{\Theta} = \mathbf{R_B^{-1}}\tilde{\boldsymbol{\Theta}}$
$\tilde{\boldsymbol{\Psi}} = \mathbf{U}(:, 1:r)$ and estimate $\boldsymbol{\Psi} = \mathbf{R_B^{-1}}\tilde{\boldsymbol{\Psi}}$

Simulation Results

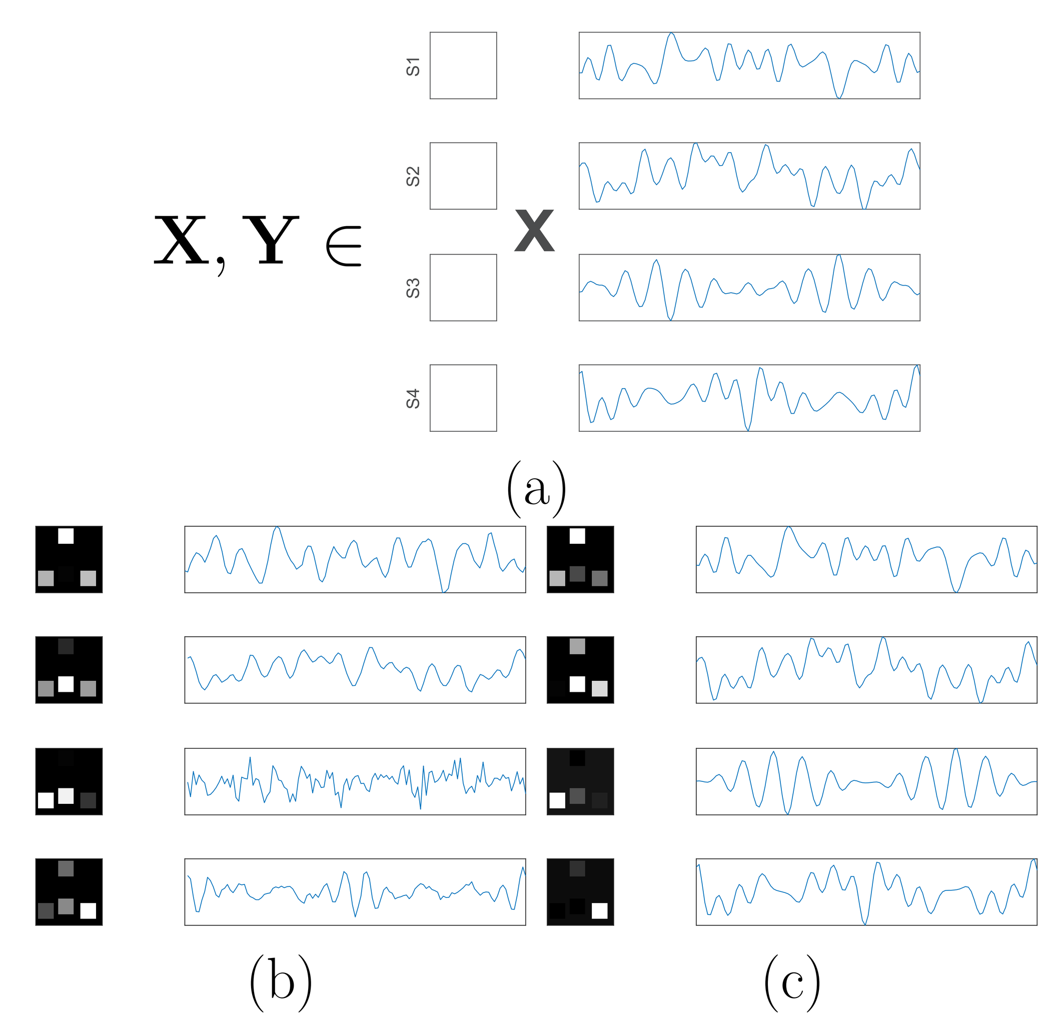


Figure 1: a) Ground truth b) standard CCA, c) the proposed PCCA at $SNR = 10$ dB.

Table 1: Average correlations of recovered source signals.

SNR (dB)	Standard CCA				Projection CCA			
	$\hat{R}_1$	$\hat{R}_2$	$\hat{R}_3$	$\hat{R}_4$	$\hat{R}_1$	$\hat{R}_2$	$\hat{R}_3$	$\hat{R}_4$
0	0.493	0.835	0.353	0.816	0.964	0.939	0.937	0.928
5	0.531	0.949	0.542	0.862	0.938	0.941	0.926	0.956
10	0.855	0.960	0.666	0.699	0.940	0.950	0.925	0.955
15	0.895	0.701	0.717	0.682	0.921	0.960	0.907	0.905
Mean	0.693	0.861	0.569	0.765	**0.941**	**0.947**	**0.924**	**0.936**

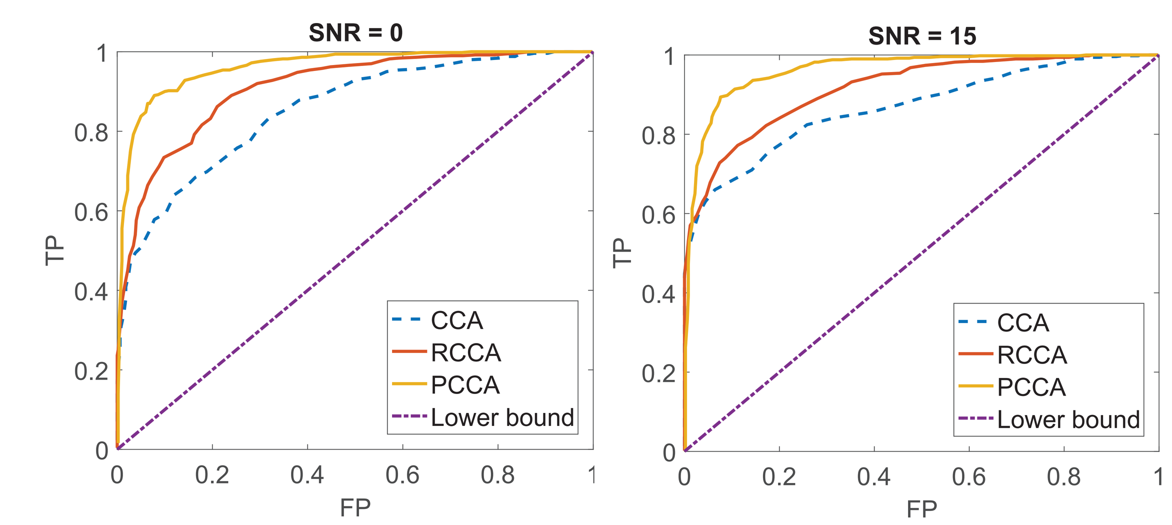


Figure 2: ROC curves obtained using CCA, regularized CCA (RCCA) and PCCA (proposed) shown at SNR = 0(left) and SNR= 15(right), where TP : true positive, FP: false positives.

Resting-state fMRI analysis

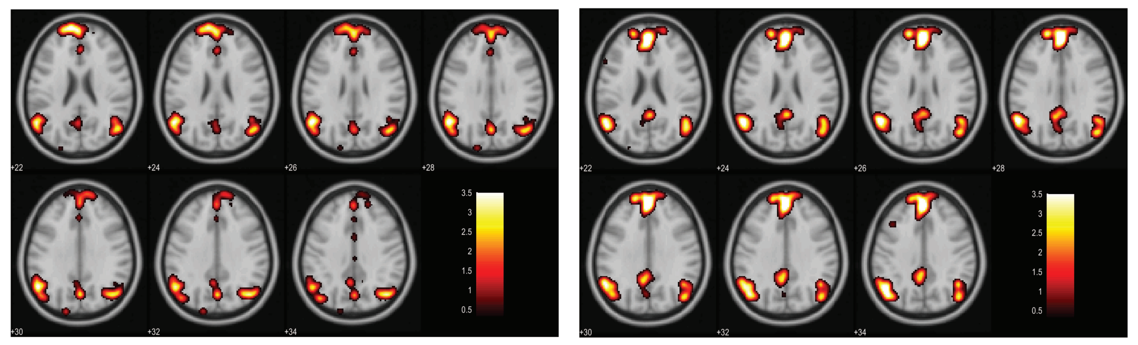


Figure 3: Activation maps of default mode networks (DMN) obtained using CCA (left) and PCCA (proposed) (right).

Table 2: MNI coordinates of selected seed voxels of DMN and their correlation coefficients with estimated time series.

	MNI				
	x	y	z	CCA	PCCA
Ventral medial prefrontal cortex	6	70	14	0.922	0.997
Precuneus cortex	-8	-60	14	0.876	0.960
Dorsal posterior cingulate	0	-24	38	0.937	0.945
Ventral posterior cingulate	2	-46	28	0.924	0.992
Superior frontal gyrus	8	50	38	0.886	0.912
	Mean			0.909	**0.961**

Conclusion

- Spatio-temporal fMRI datasets are structurally smooth.
- Classical CCA methods ignore this structure.
- A regularized rank-1 matrix approximation problem is proposed for CCA via basis expansion.
- To estimate canonical variates this problem was solved through alternating least squares.