RECOVERING TEXTURE OF DENOISED IMAGE VIA ITS STATISTICAL ANALYSIS

MQ.P2.7

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Introduction

- Existing image denoising methods **smooth a texture component** with noise.
- Our proposed method estimates the texture component which has been lost by WNNM [S. Gu+, 2017].
  state of the art of image denoising
- We extend the observation model:

$$\mathbf{y} = \mathbf{x} + \mathbf{n} \longrightarrow \mathbf{y} = \mathbf{s} + \mathbf{t} + \mathbf{n}$$

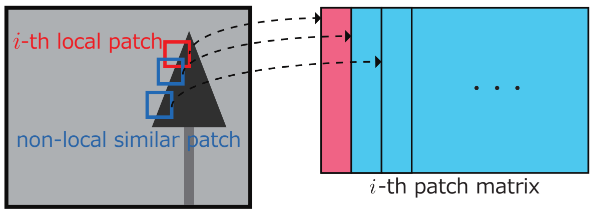
observation ($\mathbf{y}$), original image ($\mathbf{x}$), additive white gaussian noise ($\mathbf{n}$) — Zero mean and the standard deviation is σ; denoised image (by WNNM) ($\mathbf{s}$), texture component ($\mathbf{t}$)

- In this method, we apply a **Wiener filter** to estimate the texture component.
  It needs statistics about texture and noise which are difficult to estimate...
- We estimate the statistics by **Stein's lemma** and some assumptions.

Our proposed method

proposed mehtod

$\mathbf{y}$ → denosing by WNNM → $\mathbf{s}$, $\mathbf{S}_i$ → decompose $\mathbf{y}$ into patches → $\mathbf{y}_i$ → estimate the statistics by Stein's lemma → $\mathbf{R}_{\mathbf{t}_i\mathbf{n}_i}$, $\mathbf{R}_{\mathbf{t}_i\mathbf{t}_i}$ → apply Wiener filter $\mathbf{W}_i^*$ to each patch martrix $\mathbf{T}_i + \mathbf{N}_i$ → $\hat{\mathbf{T}}_i$ → reconstruct each matrix to estimated the texture → $\hat{\mathbf{t}}$ → estimated image $\hat{\mathbf{x}} = \mathbf{s} + \alpha\hat{\mathbf{t}}$



i-th patch			
$\mathbf{y}_i$	$\mathbf{s}_i$	$\mathbf{t}_i$	$\mathbf{n}_i$

i-th patch matrix			
$\mathbf{Y}_i$	$\mathbf{S}_i$	$\mathbf{T}_i$	$\mathbf{N}_i$

We arrange local patch and their each non-local similar patch into patch matrix.

Wiener filter for texture recovery

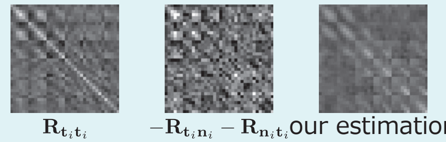
$$\mathbf{W}_i^* = \underset{\mathbf{W}}{\operatorname{argmin}}\, \mathbb{E}[\|\mathbf{W}(\mathbf{t}_i + \mathbf{n}_i) - \mathbf{t}_i\|_2^2]$$

$$\mathbf{W}_i^* = (\mathbf{R}_{\mathbf{t}_i\mathbf{t}_i} + \mathbf{R}_{\mathbf{t}_i\mathbf{n}_i})(\mathbf{R}_{\mathbf{t}_i\mathbf{t}_i} + \sigma^2\mathbf{I})^{-1}$$

$$\mathbf{R}_{\mathbf{ab}} = \mathbb{E}[(\mathbf{a} - \mathbb{E}[\mathbf{a}])(\mathbf{b} - \mathbb{E}[\mathbf{b}])^\top]$$

The statistics $\mathbf{R}_{\mathbf{t}_i\mathbf{t}_i}$ and $\mathbf{R}_{\mathbf{t}_i\mathbf{n}_i}$ is unavailable since $\mathbf{t}_i$ and $\mathbf{n}_i$ are not observable.

$$\mathbf{R}_{\mathbf{t}_i\mathbf{t}_i} \approx -\mathbf{R}_{\mathbf{t}_i\mathbf{n}_i} - \mathbf{R}_{\mathbf{n}_i\mathbf{t}_i}$$



$$\mathbf{R}_{\mathbf{t}_i\mathbf{n}_i} = \mathbf{R}_{\mathbf{n}_i\mathbf{t}_i}^\top = \sigma^2\mathbb{E}\left[\frac{\partial\mathbf{t}_i}{\partial\mathbf{n}_i}\right]$$

from Stein's lemma

We can obtain the statistics if we can approximate the process of WNNM.

$$\mathbf{S}_i = \mathbf{F}_i(\mathbf{Y}_i - \bar{\mathbf{Y}}_i) + \bar{\mathbf{Y}}_i$$

Each column is the row-wise corresponding average of $\mathbf{Y}_i$.

singular value decomposition

$$\mathbf{Y}_i - \bar{\mathbf{Y}}_i = \mathbf{U}_i\Sigma_{\mathbf{Y}_i-\bar{\mathbf{Y}}_i}\mathbf{V}_i^\top, \text{ and } \mathbf{S}_i - \bar{\mathbf{S}}_i = \mathbf{U}_i\Sigma_{\mathbf{S}_i-\bar{\mathbf{S}}_i}\mathbf{V}_i^\top$$

$$\mathbf{S}_i = \mathbf{U}_i \underline{g(\Sigma_{\mathbf{Y}_i-\bar{\mathbf{Y}}_i})}\mathbf{V}_i^\top + \bar{\mathbf{Y}}_i \qquad \mathbf{F}_i = \mathbf{U}_i\Sigma_{\mathbf{S}_i-\bar{\mathbf{S}}_i}\Sigma_{\mathbf{Y}_i-\bar{\mathbf{Y}}_i}^{-1}\mathbf{U}_i^\top$$

thresholding to the singular values

The partial derivative of $\bar{\mathbf{Y}}_i$ with respect to $\mathbf{n}$ is considered to be very small.

$$\mathbf{R}_{\mathbf{t}_i\mathbf{n}_i} = \sigma^2\mathbb{E}\left[\frac{\partial\mathbf{t}_i}{\partial\mathbf{n}_i}\right] = -\sigma^2\mathbb{E}\left[\frac{\partial\mathbf{s}_i}{\partial\mathbf{n}_i}\right] = -\sigma^2\mathbf{F}_i$$

Finally, we can estimate $\mathbf{T}_i$ as

$$\hat{\mathbf{T}}_i = \mathbf{W}_i^*(\mathbf{Y}_i - \mathbf{S}_i) = (\sigma^2\mathbf{F}_i)(2\sigma^2\mathbf{F}_i + \sigma^2\mathbf{I})^{-1}(\mathbf{Y}_i - \mathbf{S}_i)$$

Thus, we can obtain the final estimated image $\hat{\mathbf{x}}$ as

$$\hat{\mathbf{x}} = \mathbf{s} + \underline{\alpha}\hat{\mathbf{t}}$$

the scaling factor of the texture to add

Experimental result

for more detail, please refer our manuscript.

(a) Original PSNR [dB] / SSIM	(b) Noisy image	(c) WNNM	(d) Proposed $\alpha = 3.26$	(e) Proposed $\alpha = 1$
	22.13 [dB] / 0.763	24.90 [dB] / 0.787	24.82 [dB] / **0.833**	**25.15**[dB] / 0.813

Performance comparison on the cropped image 'kodim01' ($\sigma = 20$).

Our proposed method outperforms WNNM.