IMPROVING LIDAR DEPTH RESOLUTION WITH DITHER

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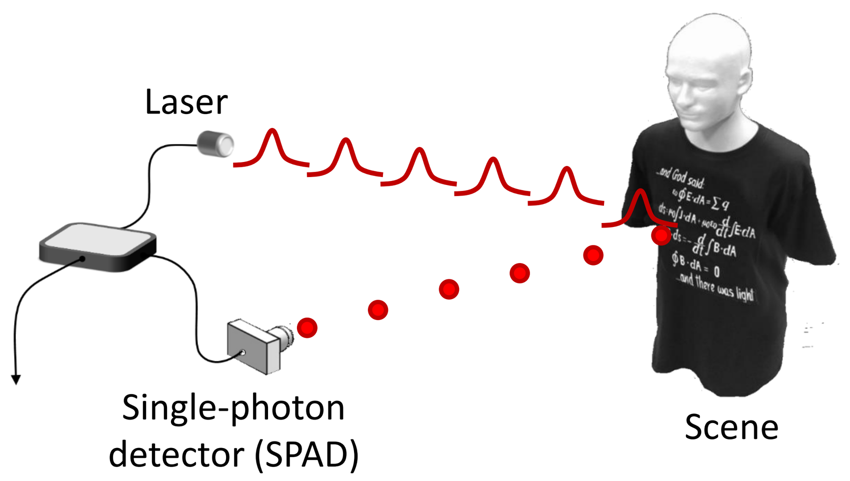
Introduction

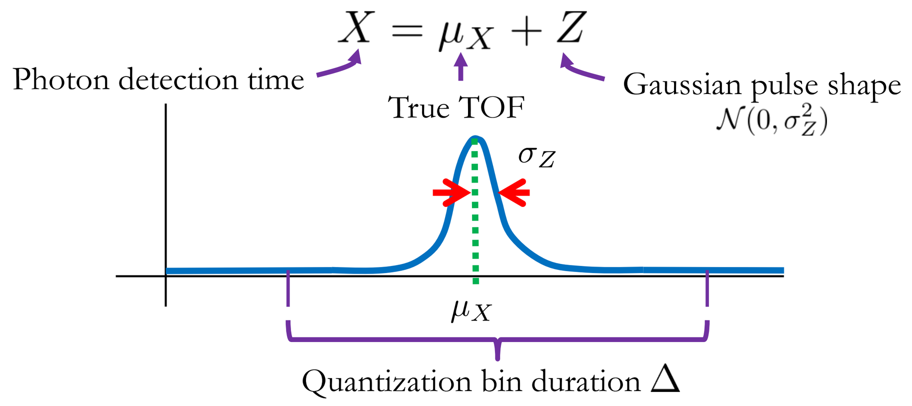
- Fast and accurate **depth imaging** is critical for autonomous navigation
- Raster scanning is too slow for real-time applications
- Array detectors **parallelize acquisition**, but timing resolution is worse than for single-pixel detectors



Contributions

1. Introduce concept of subtractive dither to improve time resolution for time-correlated single photon counting
2. Propose a generalized Gaussian approximation for measurements obtained from subtractively-dithered quantization of a Gaussian signal
3. Design and implement a dithered photon-counting lidar system

Measurement Model



- Laser returns modeled as Gaussian pulse
- Photon arrival times are independent samples of pulse distribution
- Detector electronics quantize photon arrival times into coarse bins
- Dither useful for improving resolution when $\sigma_Z/\Delta < 0.3$ [1]

Estimation with Subtractive Dither

Dither

- **Quantization:** $q(\cdot)$ rounds to nearest multiple of Δ
- **Dither:** varies signal by small amount d_i before quantization

Subtractively-dithered measurements [2]:
If $D \sim \text{uniform}[-\Delta/2, \Delta/2]$ and independent of X, then measurements $y_i = q(x_i + d_i) - d_i$ are equal in distribution to $y_i = x_i + w_i$, where $W \sim \text{uniform}[-\Delta/2, \Delta/2]$ and independent of X.

Generalized Gaussian Approximation

- Dithered lidar measurement model:

$$y_i = \mu_X + z_i + w_i$$

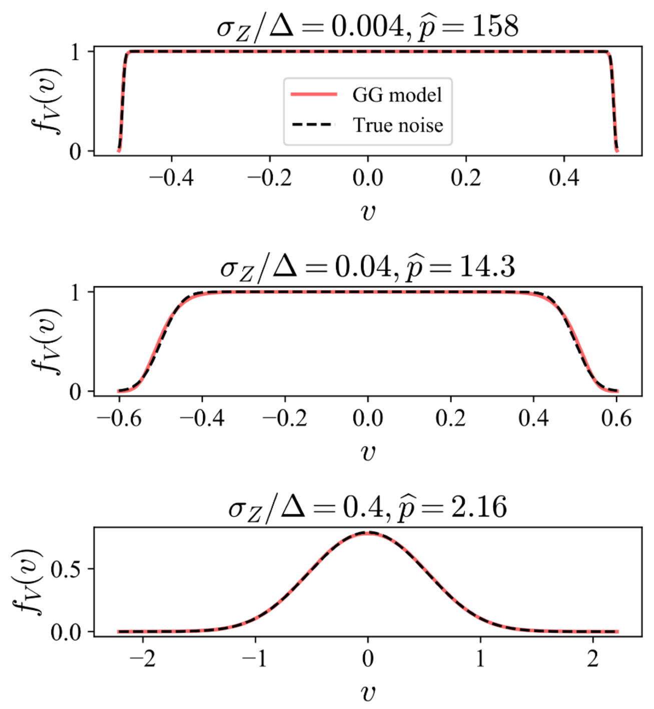
- Approximate total noise $V = Z + W$ as **generalized Gaussian** (GG):

$$f(v; \mu, \sigma, p) = \frac{1}{2\Gamma(1+1/p)A(p)} \times \exp\left\{-\left(\frac{|v-\mu|}{A(p)}\right)^p\right\}$$

where $A(p) = \sqrt{\sigma^2\Gamma(1/p)/\Gamma(3/p)}$

- Shape parameter $p \geq 2$ fit via kurtosis-matching [3]:

$$\frac{\Gamma(1/\widehat{p})\Gamma(5/\widehat{p})}{\Gamma(3/\widehat{p})^2} = 3\frac{\sigma_z^4}{\sigma_v^4} + \frac{9}{5}\frac{\sigma_w^4}{\sigma_v^4} + 6\frac{\sigma_z^2\sigma_w^2}{\sigma_v^4}$$



Estimators

Sample mean (no dither):

$$\widehat{\mu}_{\text{QM}} = \frac{1}{K}\sum_{i=1}^{K} q(x_i)$$

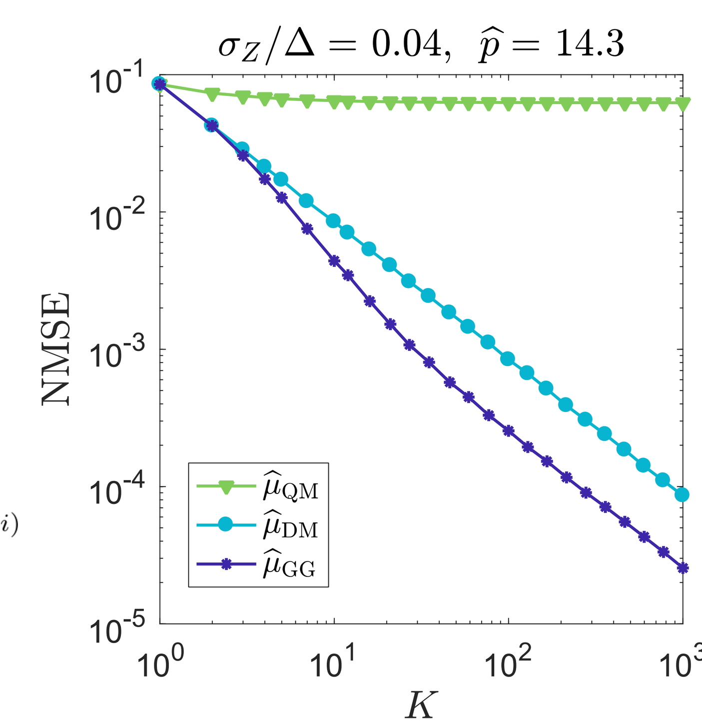
Sample mean (dithered):

$$\widehat{\mu}_{\text{DM}} = \frac{1}{K}\sum_{i=1}^{K} y_i$$

GG estimator [4]:

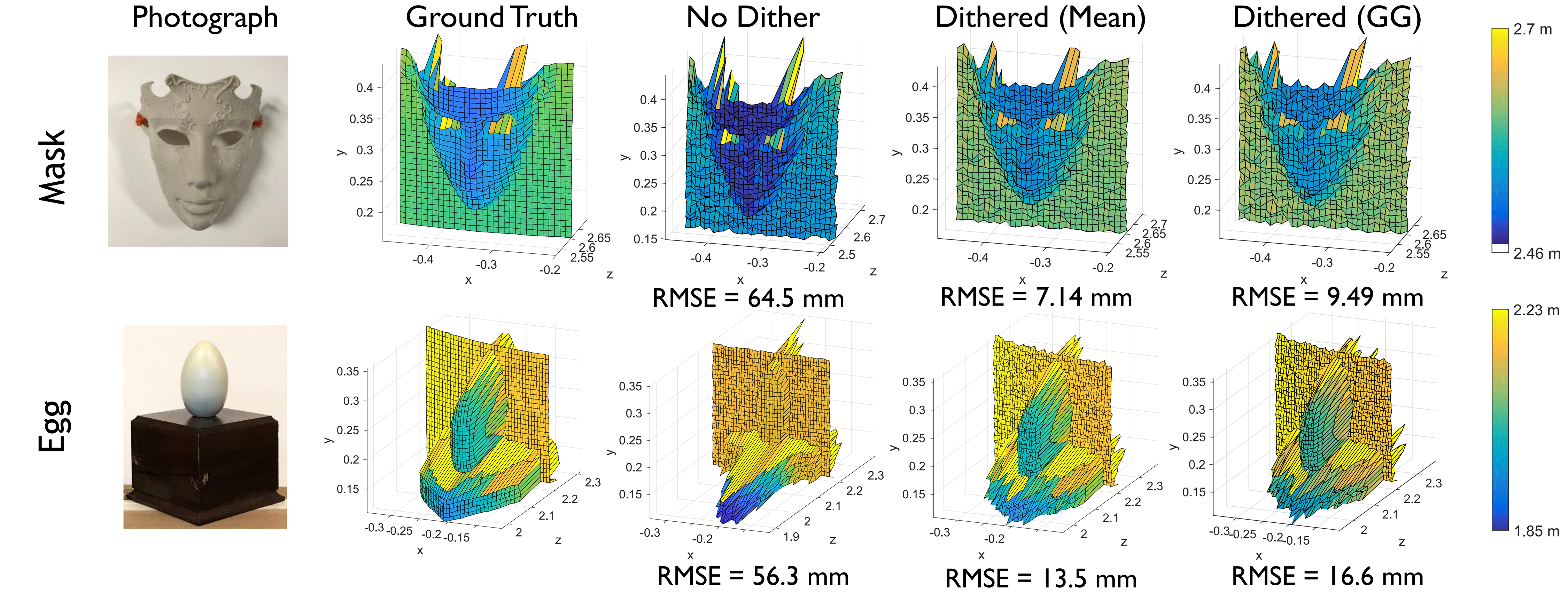
- Order statistics $y_{(1)} \leq y_{(2)} \leq \cdots \leq y_{(K)}$
- Symmetric pairwise ranges $R_i = y_{(K-i+1)} - y_{(i)}$

$$\widehat{\mu}_{\text{GG}} = \sum_{i=1}^{K} y_{(i)} \frac{R_i^{p-2}}{2\sum_{j=1}^{K} R_j^{p-2}}$$



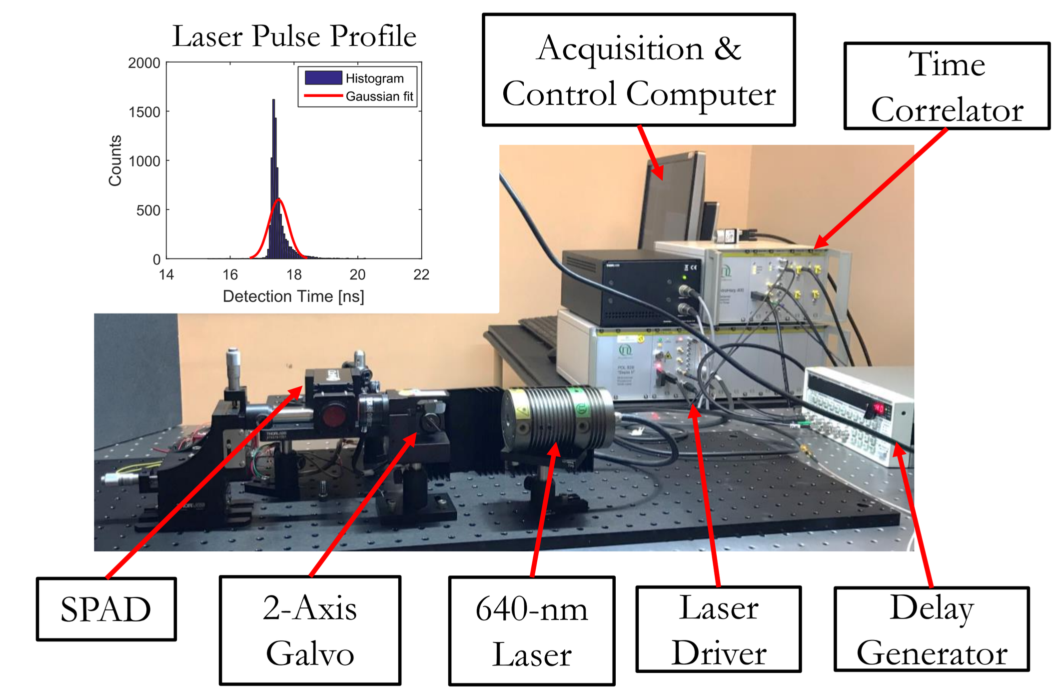
Simulation results show significant improvement using GG estimator

Experimental Results



Up to 9x reduction in Root Mean Square Error (RMSE)

Experimental setup



Discrete dither implementation:
equal number of trials per dither step

Parameters	Value
Ground truth resolution (Δ)	4 ps (≈0.6 mm)
Emulated SPAD array resolution (Δ)	2048 ps (≈0.31 m)
Laser pulse width (σ_Z)	300 ps (≈45 mm)
Dither step size	10 ps (≈1.5 mm)

	Mask	Egg
Scan size (pixels)	32 x 32	40 x 30
Per pixel acquisition time	8.5 ms	10.2 ms
Mean photon count per pixel	267	362

Conclusions

1. Subtractive dither enables resolution of features smaller than the quantization bin size.
2. Dithered lidar implemented with delays added to electronic control signals.
3. Generalized Gaussian noise model improves results for simulations, although the same improvement does not extend to experimental data due to model mismatch.

References

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