Sparsity and Rank Exploitation for Time-Varying Narrowband Leaked OFDM Channel Estimation

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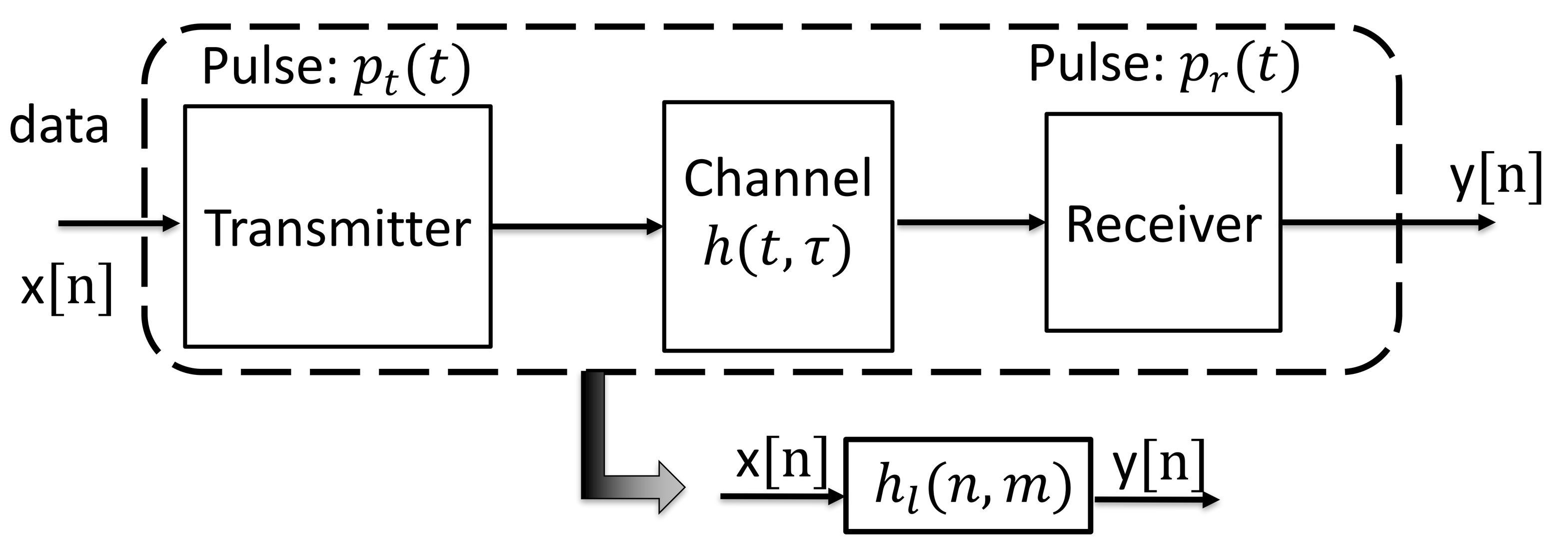
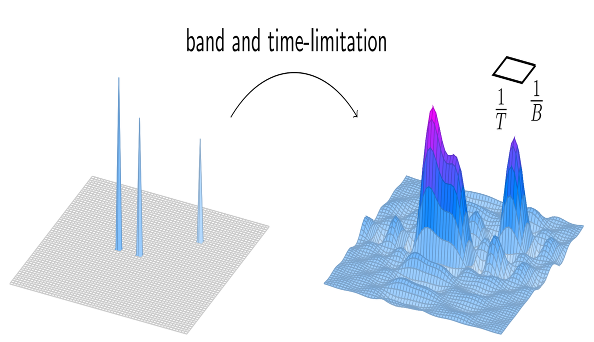




channel in each path

$$h_l[n,m] = \sum_{k=1}^{\#\,paths} \eta_k e^{j2\pi\nu_k((n-m)T_s - t_k)} p(mT_s - t_k)$$

channel gain; Doppler shift; delay; **Leakage**= convolution of transmit and receiver pulses

Problem

$h(t,\tau)$ is sparse in delay-Doppler domain

but the **leaked** channel $h_l(n,m)$ **is not sparse anymore**

Goal

Exploiting a **new structure** to estimate the leaked channels

Previous work: channel estimation

- **Classical methods** (least-squares), numerous works [Letaief, Schniter, Banelli, Leus, Giannakis, Molisch, ...]
- **Sparsity based methods** [Bajwa, Haupt, Sayeed & Rauhut'10, Aktas&Mitra'07, Carbonelli & Beygi, Mitra, Strom'15, . . .]
  Leakage violates sparsity → **Huge gap between theory and practice!**
- **Leakage for single carrier** (Beygi and Mitra, TSP 2017)

Our Contributions

- Extension to multicarrier
- **Effective separability of Doppler and leakage contributions maintained**
- **Optimization via atomic norm**
  - Scaling law for number of measurements
- **Numerical results – up to 8 dB gain over purely sparse and non-sparse methods**

Signal Model

$\boldsymbol{y}[n] = <\boldsymbol{X}, \boldsymbol{H}_l>$ → $\boldsymbol{y} = \Pi(\boldsymbol{H}_l)$

training matrix; **leaked** channel matrix= $H_{l,\tau}\ H_{l,\nu}$

rank-one matrix

$$\sum_{k=1}^{\#\,channel\ path} (\text{Leakage vector})_k \times (\text{Doppler vector})_k$$

Main result

The leaked OFDM channel matrix is low rank and separable in leakage and Doppler domains

Optimization

Set of atoms: parametric Doppler atoms (partial Vandermonde structure) + generic leakage atoms

Atomic norm; noise variance

$$\underset{\boldsymbol{H}_l}{\text{minimize}}\ \|\boldsymbol{H}_l\|_{\mathcal{A}} \quad \text{subject to}\ \|\boldsymbol{y} - \Pi(\boldsymbol{H}_l)\|_2 \le \sigma_z^2$$

has a semidefinite program: Bhaskar, Tang & Recht, TSP 12/13

Main result

The recovery problem is convex

Scaling Law

For random BPSK training signal, if the sampling rate satisfies,

$$n_T \ge c p_0 m_0 L \log^3\left(\frac{n_T p_0 m_0 L}{\delta}\right)$$

then, $\boldsymbol{H}_l$ can be recovered with probability $1\text{-}\delta$ using Atomic norm minimization

Observation: Numerical results suggest that $n_T \sim O(m_0 p_0 L)$

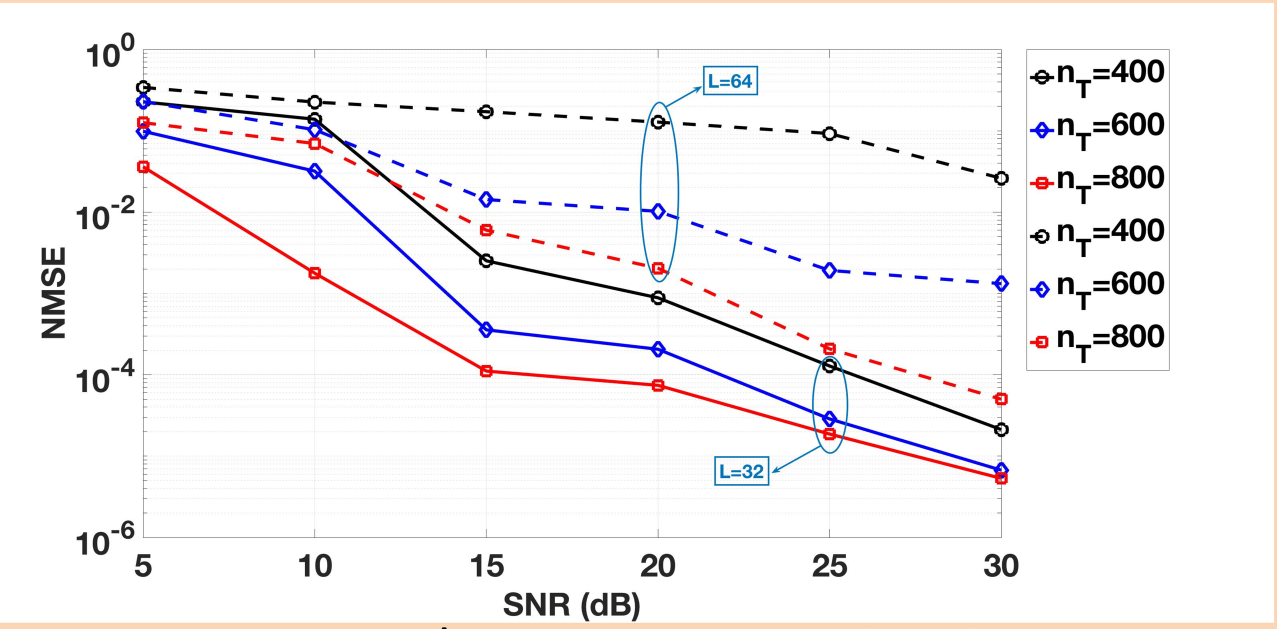
p_0: # channel paths

m_0: maximum channel delay

L: # OFDM subcarriers

n_T: # measurements

Simulations



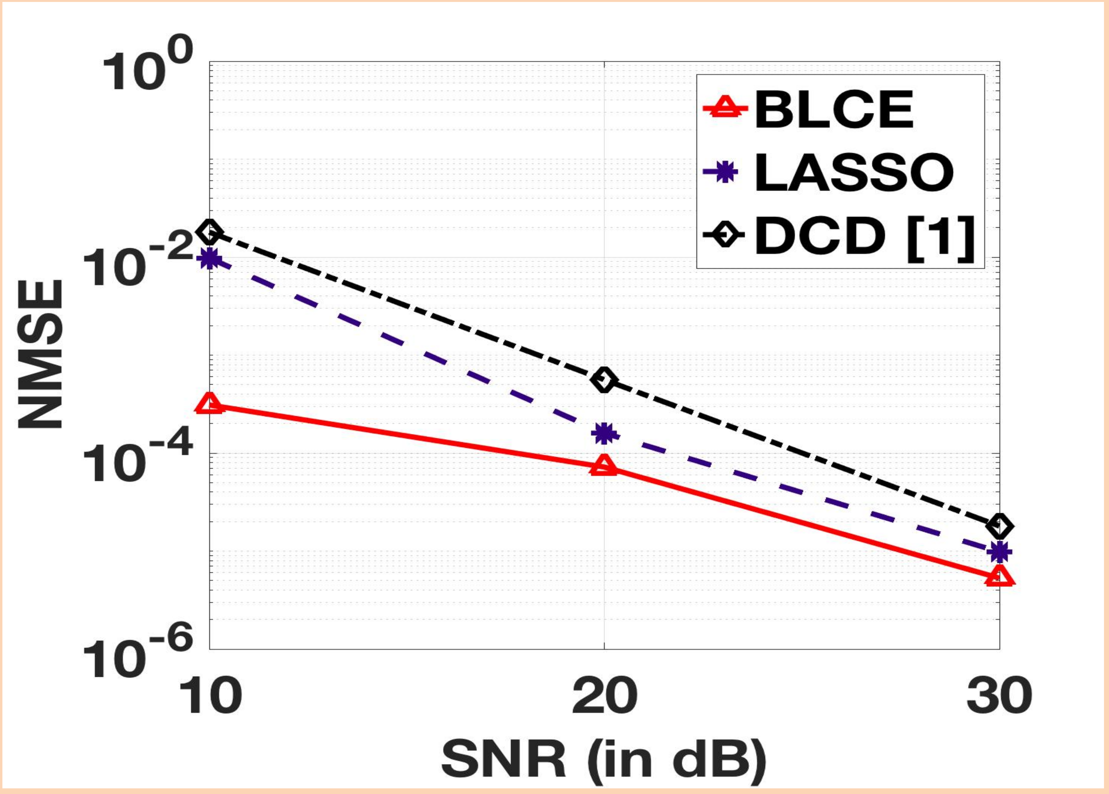
Doppler parameters estimation

$$\text{NMSE} = \frac{\|\boldsymbol{\theta}_0 - \widehat{\boldsymbol{\theta}}\|_F^2}{\|\boldsymbol{\theta}_0\|_F^2}$$

Parameters: $m_0 = 3$ and $p_0 = 2$

- Fixed L, as $n_T \uparrow$, NMSE (ν) ↓
- Fixed n_T, as L ↑, NMSE (ν) ↑, since more parameters need to be estimated (scaling law)

Real Data



channel matrix estimation

MACE'10 Data set for Aval &. Stojanovic JOE 2015

BLCE: our method

DCD: Differentially Coherent Detection

- MACE is designed to challenge the vulnerability of OFDM acoustic signals to motion-induced Doppler effects
- frequency range between 10.5 and 15.5 kHz
- QPSK modulated

Main result

superiority over sparse and non-sparse methods

[1] Aval &. Stojanovic JOE 2015

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