On the Age of Information in Multi-Source Multi-Hop Wireless Status Update Networks

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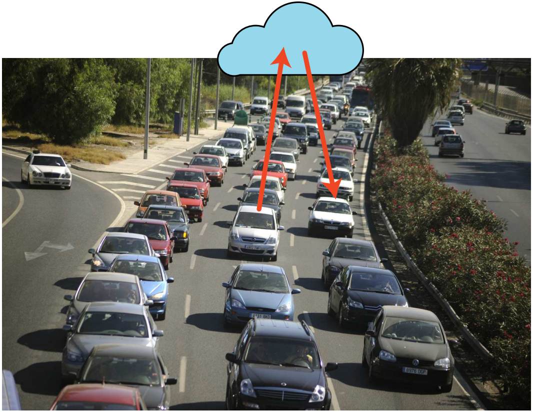
The Importance of Timely Information



- Cars send status updates to other cars
- Cars want the newest state information with low delay
- Metric: Age (staleness) of the most recent update

System Model / Assumptions

- We consider a **multi-source multi-hop** status update network with N nodes modeled by an undirected graph $\mathcal{G} = (\mathcal{V}, \mathcal{E})$.
- Each node keeps a table of all N time-varying processes in the network.
- Out of the N parameters in each node's table, 1 is obtained directly from its local process and the remaining $N-1$ parameters are obtained indirectly from other nodes by dissemination.
- Transmission is slotted, where during each time slot only one packet containing information on one process can be transmitted.

Goal: Derive fundamental bounds on the **peak** and **average** AoI and develop **schedules** that achieve near-optimal performance.

AoI Metrics

Age: Assume the most recent status update of the H_i process received at node j was timestamped at time t'. The age of status update $H_i^{(j)}$ at time $t \geq t'$ is defined as $\Delta_i^{(j)}(t) \triangleq t - t'$ for $j \neq i$.
Since each node is assumed to have zero-delay access to the status of its local process, we have $\Delta_i^{(i)}(t) = 0$ for any $i \in \mathcal{V}$ and t.

Average Age: The average age is defined as

$$\Delta_{avg} \triangleq \lim_{\mathcal{T}\to\infty} \left[\frac{1}{N^2 - N} \sum_{i,j\in\mathcal{V}, i\neq j} \frac{1}{\mathcal{T} - \bar{t}} \int_{\bar{t}}^{\mathcal{T}} \Delta_i^{(j)}(t)\ dt \right]$$

for $\bar{t}$ sufficiently large such that all nodes have complete status update tables.

AoI Metrics

Peak Age: The peak age is defined as

$$\Delta_{peak} \triangleq \sup_{\substack{t \geq \bar{t} \\ i,j \in \mathcal{V}, i \neq j}} \Delta_i^{(j)}(t)$$

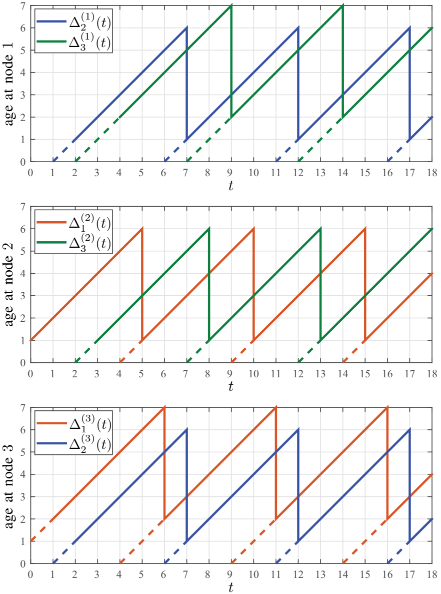
Schedule: We refer to a schedule as an ordered sequence of transmitting nodes and the corresponding update parameter that they disseminate in each time slot.

Example Line Network

Table: Example schedule for the 3-node line network.

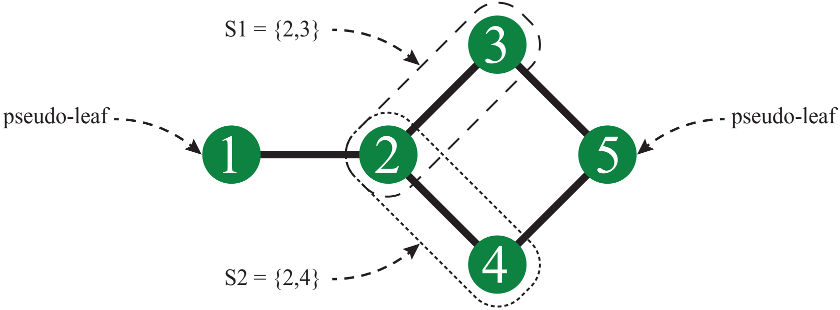
time slot	transmitting node	disseminated update
$n = 0, 5, 10, \ldots$	1	$H_1^{(1)}$
$n = 1, 6, 11, \ldots$	2	$H_1^{(2)}$
$n = 2, 7, 12, \ldots$	2	$H_2^{(2)}$
$n = 3, 8, 13, \ldots$	3	$H_3^{(3)}$
$n = 4, 9, 14, \ldots$	2	$H_3^{(2)}$

Operation of a schedule for the 3-node line network



Key Graph Parameters

- Average distance $\ell_{\mathcal{G}} \triangleq \frac{1}{N^2-N}\sum_{i,j\in\mathcal{V},i\neq j} d(i,j)$
- Minimum connected dominating set (MCDS) $\mathcal{S}$
- Connected domination number γ_c
- Pseudo-leaf vertex u, $u \in \mathcal{L} \triangleq \mathcal{V} - \{\mathcal{S}_1 \cup \mathcal{S}_2 \cup \ldots \cup \mathcal{S}_M\}$



Fundamental Bounds on AoI of T^*-periodic Schedules

Lemma 1.[Lower bound on the schedule length to refresh all tables] To update the status of all parameters throughout the network, at least $T \geq T^* \triangleq \gamma_c N + |\mathcal{L}|$ status update packets need to be disseminated.

Theorem 1. [Lower bound on peak age] The peak age of information for any T^*-periodic schedule is lower-bounded by

$$\Delta_{peak} \geq \Delta^*_{peak} \triangleq \begin{cases} \gamma_c(N+1) & |\mathcal{L}| = 0 \\ \gamma_c(N+1) + |\mathcal{L}| + 1 & |\mathcal{L}| \geq 1 \end{cases}.$$

Theorem 2. [Lower bound on average age] The average age of information for any T^*-periodic schedule is lower-bounded by

$$\Delta_{avg} \geq \Delta^*_{avg} \triangleq \frac{T^*}{2} + \ell_{\mathcal{G}}.$$

Near-Optimal Schedule

Algorithm 1: Schedule design to disseminate status updates throughout the network

Step I: initialize time, $t \leftarrow -1$.

Step II: **for** node $i = 1 : N$ **do**

- **if** $\exists$ MCDS $\bar{\mathcal{S}}$ s.t. $i \in \bar{\mathcal{S}}$ **then**
  - $\mathcal{S} \leftarrow \bar{\mathcal{S}}$.

  else
  - $\mathcal{S} \leftarrow \bar{\mathcal{S}} \cup \{i\}$, for any MCDS $\bar{\mathcal{S}} \subset \mathcal{V}$.

  end
- $\mathcal{S}_{\text{sorted}} = \text{Depth-First Search}(\mathcal{G}[\mathcal{S}], i)$
- **for** $k = 1 : |\mathcal{S}_{sorted}|$ **do**
  - $j = \mathcal{S}_{\text{sorted}}(k)$,
  - node j transmits $H_i^{(j)}(t^+)$,
  - $t \leftarrow t + 1$.

  end

end

Step III: repeat from Step II.

Main Concept of the Near-Optimal Schedule Design

Step 1: Node 1 disseminates its current local information on the H_1 process

Step 2: Disseminate this information on the H_1 process to all nodes in the network through a MCDS

Step 3: Repeat for the local processes of nodes $2, 3, 4, \ldots, N$ from Step 1

Step 4: Repeat from Step 1

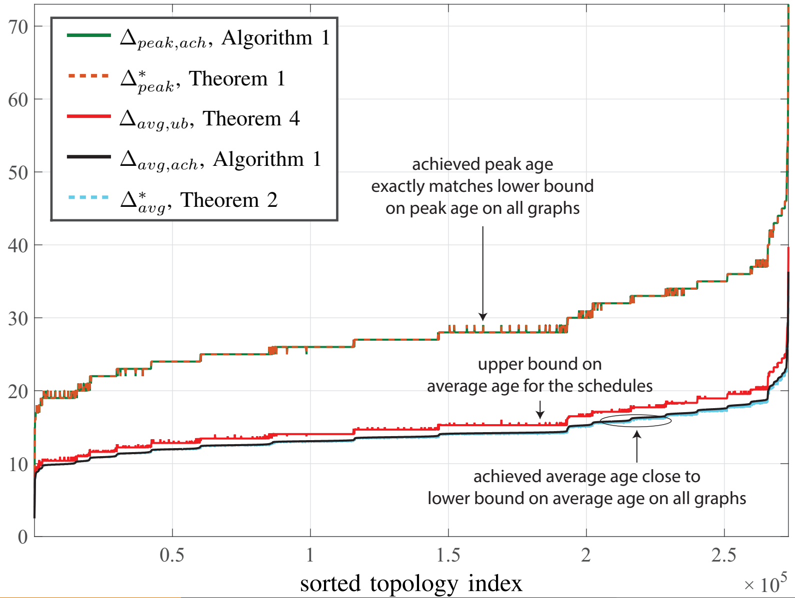
Achievable AoI by the Near-Optimal Schedule

Theorem 3. [Achievable peak age of the near-optimal schedule] The schedule generated by Algorithm 1 achieves $\Delta_{peak} = \Delta^*_{peak}$.

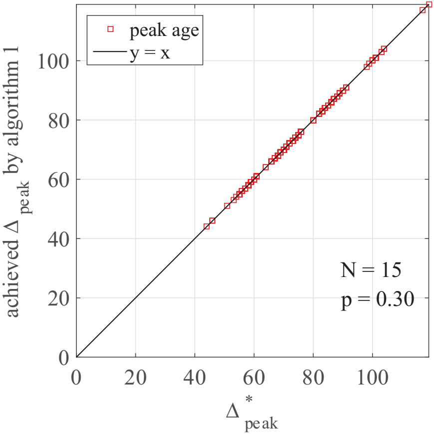
Theorem 4.[Achievable average age of the near-optimal schedule] The average age of the schedule generated by Algorithm 1 is bounded by

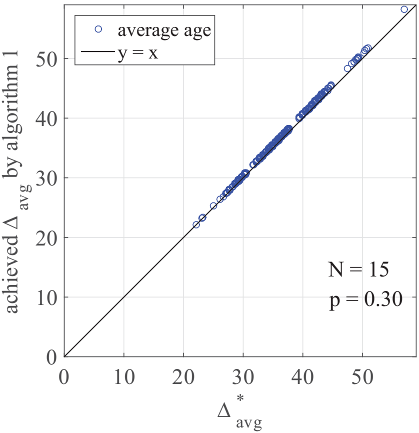
$$\Delta_{avg} \leq \frac{T^*}{2} + \gamma_c + \frac{|\mathcal{L}|}{N} \leq \Delta^*_{avg} + (N-2)$$

Bounds and achievable AoI for every connected graph with $N \leq 9$

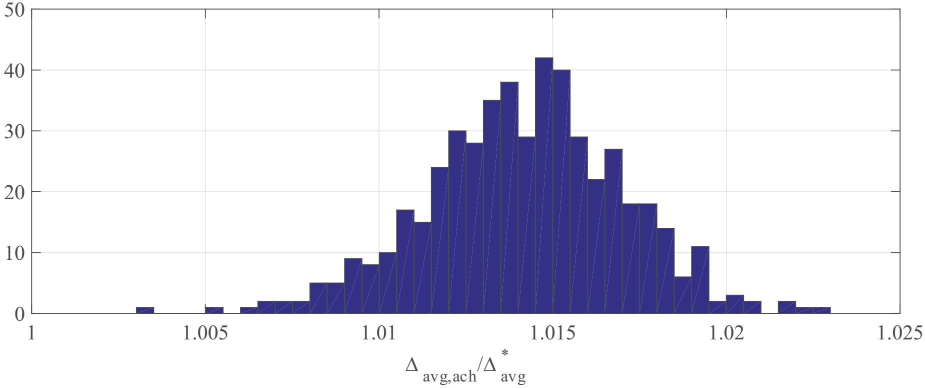


AoI for networks generated based on Erdos-Renyi model





$\frac{\Delta_{avg,ach}}{\Delta^*_{avg}}$ for the Erdos-Renyi-based networks



Future Work

- Generalize the multi-source multi-hop model to consider arbitrary service time distributions
- Derive general bounds that completely characterize the AoI for any network topology and schedule
- Develop near-optimal schedule construction that is shown to analytically achieve the peak age bound and be within constant gap of average bound