电子科技大学
University of Electronic Science and Technology of China

Semidefinite Programming for TDOA Localization with Locally Synchronized Anchor Nodes

{ Yanbin Zou, Qun Wan, and Huaping Liu }
Yanbin Zou and Qun Wan are with University of Electronic Science and Technology of China
Yanbin Zou and Huaping Liu are with Oregon State University

Oregon State University

Problems

- Source localization using TDOA measurements in the system of nodes part synchronization.
- Robust localization for the presence of nodes' position errors.

Contributions

- Present a source localization model based on TDOA measurements with locally synchronized anchor nodes.
- Propose two SDP-based localization algorithms for accurate and nonaccurate anchor nodes' position, respectively.

Measurements Model & Proposed Methods

A partly synchronous TDOA localization system has N clusters, and each cluster has M_n sensors. The total number of sensors is $\sum_{n=1}^{N} M_n = f$. $f \geq \max(m+N, 2N)$ is a required condition, where m is equal to 2 or 3.

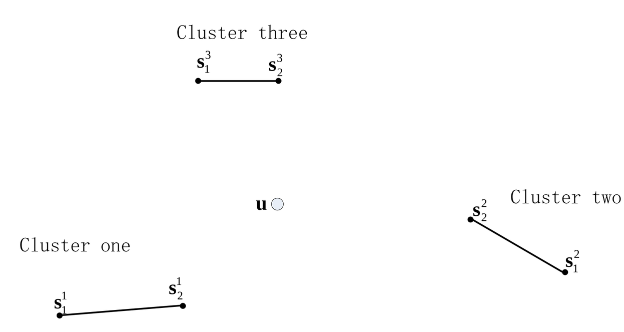


Figure 1: Illustration of nodes partly synchronous TDOA source localization system, where the nodes connected by lines means they are synchronized

In nth cluster, let $\mathbf{s}_1^n$ be the reference node. The TDOA measurements are

$$r_{i1}^n = d_i^n - d_1^n + e_{i1}^n, \quad d_i^n = \|\mathbf{u} - \mathbf{s}_i^n\|, n = 1, 2, \ldots, N, i = 2, 3, \ldots, M_n. \tag{1}$$

Then the maximum likelihood estimator (MLE):

$$\min_{\mathbf{u}} \sum_{n=1}^{N} \sum_{i=2}^{M_n} \sum_{j=2}^{M_n} \left(r_{i1}^n - \|\mathbf{u} - \mathbf{s}_i^n\| + \|\mathbf{u} - \mathbf{s}_1^n\|\right) [Q_n^{-1}]_{(i-1),(j-1)} \cdot \left(r_{j1}^n - \left\|\mathbf{u} - \mathbf{s}_j^n\right\| + \|\mathbf{u} - \mathbf{s}_1^n\|\right) \tag{2}$$

Next, (2) can be written as

$$\min_{\mathbf{u}, \mathbf{d}^n} \sum_{n=1}^{N} (\mathbf{r}_d^n - \mathbf{A}_n \mathbf{d}^n)^T \mathbf{Q}_n^{-1} (\mathbf{r}_d^n - \mathbf{A}_n \mathbf{d}^n) \tag{3a}$$

$$s.t.\ d_i^n = \|\mathbf{u} - \mathbf{s}_i^n\|, \ n = 1, 2, \ldots, N, i = 1, 2, \ldots, M_n. \tag{3b}$$

where $\mathbf{r}_d^n = [r_{21}^n, r_{31}^n, \ldots, r_{M_n 1}^n]^T$, $\mathbf{d}^n = [d_1^n, d_2^n, \ldots, d_{M_n}^n]^T$, $\mathbf{A}_n = [-\mathbf{1}_{M_n-1}, \mathbf{I}_{M_n-1}]$.
The objective function in (3a) can be rewritten as

$$\sum_{n=1}^{N} \left(tr(\mathbf{D}^n \mathbf{A}_n^T \mathbf{Q}_n^{-1} \mathbf{A}_n) - 2\mathbf{r}_d^{nT} \mathbf{Q}_n^{-1} \mathbf{A}_n \mathbf{d}^n + \mathbf{r}_d^{nT} \mathbf{Q}_n^{-1} \mathbf{r}_d^n\right) \tag{4}$$

where $\mathbf{D}^n = \mathbf{d}^n \mathbf{d}^{nT}$.

The constraints in (3b) can be expressed as

$$D_{i,i}^n = \|\mathbf{u} - \mathbf{s}_i^n\|^2 = y_s - 2\mathbf{u}^T \mathbf{s}_i^n + \mathbf{s}_i^{nT} \mathbf{s}_i^n, i = 1, 2, \ldots, M_n. \tag{5}$$

where $y_s = \mathbf{u}^T \mathbf{u}$.
Using the Cauchy-Schwartz inequality, we can obtain

$$D_{i,j}^n \geq |y_s - \mathbf{u}^T (\mathbf{s}_i^n + \mathbf{s}_j^n) + \mathbf{s}_i^{nT} \mathbf{s}_j^n|, \quad 1 \leq i < j \leq M_n. \tag{6}$$

Note that $\mathbf{A}_n^T \mathbf{Q}_n^{-1} \mathbf{A}_n$ in (4) is singular. To improve the accuracy, as in [1], we also introduce a penalty term $\sum_{n=1}^{N} tr(\mathbf{D}^n)$ into the objective function and add the second-order-cone (SOC) constraints

$$\|\mathbf{u} - \mathbf{s}_i^n\| \leq d_i^n, \ n = 1, 2, \ldots, N, i = 1, 2, \ldots, M_n. \tag{7}$$

Now, (3) can be relaxed into the following convex problem

$$\min_{\mathbf{d}^n, \mathbf{D}^n, \mathbf{u}, y_s} \sum_{n=1}^{N} \left(tr(\mathbf{D}^n \mathbf{A}_n^T \mathbf{Q}_n^{-1} \mathbf{A}_n) - 2\mathbf{r}_d^{nT} \mathbf{Q}_n^{-1} \mathbf{A}_n \mathbf{d}^n + \eta tr(\mathbf{D}^n)\right) \tag{8a}$$

$$s.t.\ D_{i,i}^n = y_s - 2\mathbf{u}^T \mathbf{s}_i^n + \mathbf{s}_i^{nT} \mathbf{s}_i^n, n = 1, 2, \ldots, N, i = 1, 2, \ldots, M_n. \tag{8b}$$

$$\|\mathbf{u} - \mathbf{s}_i^n\| \leq d_i^n, \ n = 1, 2, \ldots, N, i = 1, 2, \ldots, M_n. \tag{8c}$$

$$D_{i,j}^n \geq |y_s - \mathbf{u}^T (\mathbf{s}_i^n + \mathbf{s}_j^n) + \mathbf{s}_i^{nT} \mathbf{s}_j^n|, \ n = 1, 2, \ldots, N, 1 \leq i < j \leq M_n. \tag{8d}$$

$$\begin{bmatrix} 1 & \mathbf{d}^{nT} \\ \mathbf{d}^n & \mathbf{D}^n \end{bmatrix} \succeq \mathbf{0}, n = 1, 2, \ldots, N, \tag{8e}$$

$$\begin{bmatrix} \mathbf{I}_m & \mathbf{u} \\ \mathbf{u}^T & y_s \end{bmatrix} \succeq \mathbf{0}. \tag{8f}$$

where η is the regularization parameter which is difficult to determine. To alleviate this problem, first, we need to choose K different η, $\{\eta_k\}_{k=1}^K$, and then solve (8) with the K different choice of η, $\{\eta_k\}_{k=1}^K$, finally, from the K estimates $\{\hat{\mathbf{u}}_k\}_{k=1}^K$ to select $\hat{\mathbf{u}}$ that gives the minimum cost function J_k

$$J_k = \sum_{n=1}^{N} (\mathbf{r}_d^n - \mathbf{A}_n \hat{\mathbf{d}}_k^n)^T \mathbf{Q}_n^{-1} (\mathbf{r}_d^n - \mathbf{A}_n \hat{\mathbf{d}}_k^n), \ k = 1, 2, \ldots, K. \tag{9}$$

Robust localization algorithm for nodes' position errors

In the previous discussion, the sensor positions are accurate. However, in practical, there exist the sensor position errors [2]. The obtained but erroneous sensor position can be expressed as

$$\mathbf{b}_i^n = \mathbf{s}_i^n + \boldsymbol{\beta}_i^n \tag{10}$$

where $\boldsymbol{\beta}_i^n$ is the sensor position error, which is modeled as Gaussian white noise with covariance matrix $\delta_i^{n2} \mathbf{I}_m$.
Under the condition of independent noises $\boldsymbol{\beta}_i^n$ and e_{i1}^n, the MLE problem can be written as

$$\min_{\mathbf{u}, \mathbf{s}_i^n} \sum_{n=1}^{N} \sum_{i=2}^{M_n} \sum_{j=2}^{M_n} \left(r_{i1}^n - \|\mathbf{u} - \mathbf{s}_i^n\| + \|\mathbf{u} - \mathbf{s}_1^n\|\right) [Q_n^{-1}]_{(i-1),(j-1)} \cdot \left(r_{j1}^n - \left\|\mathbf{u} - \mathbf{s}_j^n\right\| + \|\mathbf{u} - \mathbf{s}_1^n\|\right) +$$

$$\sum_{n=1}^{N} \sum_{i=1}^{M_n} \frac{\|\mathbf{b}_i^n - \mathbf{s}_i^n\|^2}{\delta_i^{n2}} \tag{11}$$

The above formulation can be reshaped as

$$\min_{\mathbf{X}, \mathbf{d}^n} \sum_{n=1}^{N} (\mathbf{r}_d^n - \mathbf{A}_n \mathbf{d}^n)^T \mathbf{Q}_n^{-1} (\mathbf{r}_d^n - \mathbf{A}_n \mathbf{d}^n) + \left\| (\mathbf{X}(:, 2 : f+1) - \mathbf{B}) \mathbf{W}^{\frac{1}{2}} \right\|_F^2 \tag{12a}$$

$$s.t.\ d_i^n = \left\| \mathbf{X}(:, 1) - \mathbf{X}(:, 1 + i + \sum_{q=0}^{n-1} M_q) \right\|, \ n = 1, 2, \ldots, N, i = 1, 2, \ldots, M_n. \tag{12b}$$

where $M_0 = 0$, $\mathbf{X} = [\mathbf{u}, \mathbf{s}_1^1, \ldots, \mathbf{s}_{M_1}^1, \ldots, \mathbf{s}_1^N, \ldots, \mathbf{s}_{M_N}^N]$, $\mathbf{B} = [\mathbf{b}_1^1, \ldots, \mathbf{b}_{M_1}^1, \ldots, \mathbf{b}_1^N, \ldots, \mathbf{b}_{M_N}^N]$, and $\mathbf{W} = diag\left([\delta_1^{1\,-2}, \ldots, \delta_{M_1}^{1\,-2}, \ldots, \delta_1^{N\,-2}, \ldots, \delta_{M_N}^{N\,-2}]\right)$.

Let $\mathbf{Y} = \mathbf{X}^T \mathbf{X}$. Similar to the deviation of (8), we give the robust SDP localization algorithm

$$\min_{\mathbf{d}^n, \mathbf{D}^n, \mathbf{X}, \mathbf{Y}} \sum_{n=1}^{N} \left(tr(\mathbf{D}^n \mathbf{A}_n^T \mathbf{Q}_n^{-1} \mathbf{A}_n) - 2\mathbf{r}_d^{nT} \mathbf{Q}_n^{-1} \mathbf{A}_n \mathbf{d}^n + \eta tr(\mathbf{D}^n)\right) +$$

$$tr\left(\mathbf{W}\mathbf{Y}(2 : f+1, 2 : f+1)\right) - 2tr\left(\mathbf{W}\mathbf{X}(:, 2 : f+1)^T \mathbf{B}\right) \tag{13a}$$

$$s.t.\ D_{i,i}^n = Y(1,1) - 2Y(1, 1 + i + \sum_{q=0}^{n-1} M_q) + Y(1 + i + \sum_{q=0}^{n-1} M_q, 1 + i + \sum_{q=0}^{n-1} M_q), \tag{13b}$$

$$\left\| \mathbf{X}(:, 1) - \mathbf{X}(:, 1 + i + \sum_{q=0}^{n-1} M_q) \right\| \leq d_i^n, \ n = 1, 2, \ldots, N, i = 1, 2, \ldots, M_n. \tag{13c}$$

$$D_{i,j}^n \geq |Y(1,1) - Y(1, 1 + i + \sum_{q=0}^{n-1} M_q) - Y(1, 1 + j + \sum_{q=0}^{n-1} M_q) +$$

$$Y(1 + i + \sum_{q=0}^{n-1} M_q, 1 + j + \sum_{q=0}^{n-1} M_q)|, \ n = 1, 2, \ldots, N, 1 \leq i < j \leq M_n. \tag{13d}$$

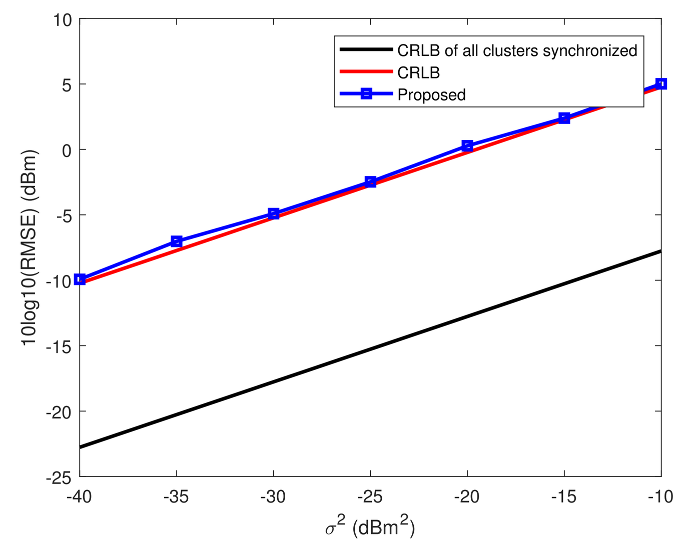
$$\begin{bmatrix} 1 & \mathbf{d}^{nT} \\ \mathbf{d}^n & \mathbf{D}^n \end{bmatrix} \succeq \mathbf{0}, n = 1, 2, \ldots, N, \tag{13e}$$

$$\begin{bmatrix} \mathbf{I}_m & \mathbf{X} \\ \mathbf{X}^T & \mathbf{Y} \end{bmatrix} \succeq \mathbf{0}. \tag{13f}$$

Simulations

There are four clusters TDOA measurements, and each cluster has two nodes. The positions of the sensor nodes are $[0,0]$,$[10,0]$,$[90,0]$,$[100,0]$, $[0,90]$, $[0,100]$, $[90,90]$, $[90,100]$. We set $K = 5$, $\eta_1 = 10^{-4}$, $\eta_2 = 10^{-3}$, $\eta_3 = 10^{-2}$, $\eta_4 = 10^{-1}$, $\eta_5 = 10^{0}$ for the computation of (8) and (13). The proposed SDP algorithm is implemented by CVX toolbox, using SeDuMi as a solver, and the precision is set to *best*.

Figure 2: RMSE vs σ^2, $\mathbf{u} = [74, 60]^T m$.



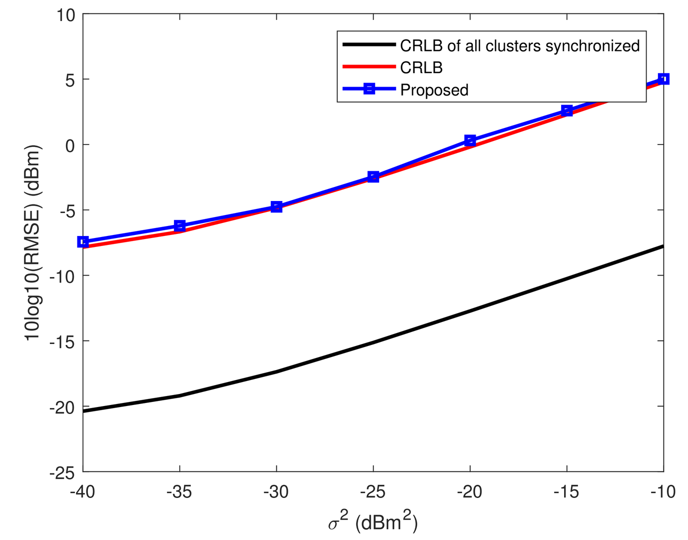


Figure 3: RMSE vs σ^2, $\mathbf{u} = [74, 60]^T m$, $\delta = 0.01m$.

References

[1] Yanbin Zou, and Qun Wan, "Asynchronous Time-of-Arrival-Based Source Localization with Sensor Position Uncertainties," *IEEE Communications Letters*, vol. 20, no. 9, pp. 1860-1863, Sep. 2016.

[2] Yanbin Zou, Huaping Liu, Wei Xie, and Qun Wan, "Semidefinite Programming Methods for Alleviating Sensor Position Error in TDOA Localization," *IEEE Access*, vol. 5, pp. 23111–23120, Sep. 2017.