Compound Multiple Access Channel with Full-Duplex Amplify-and-Forward Transmitter Cooperations

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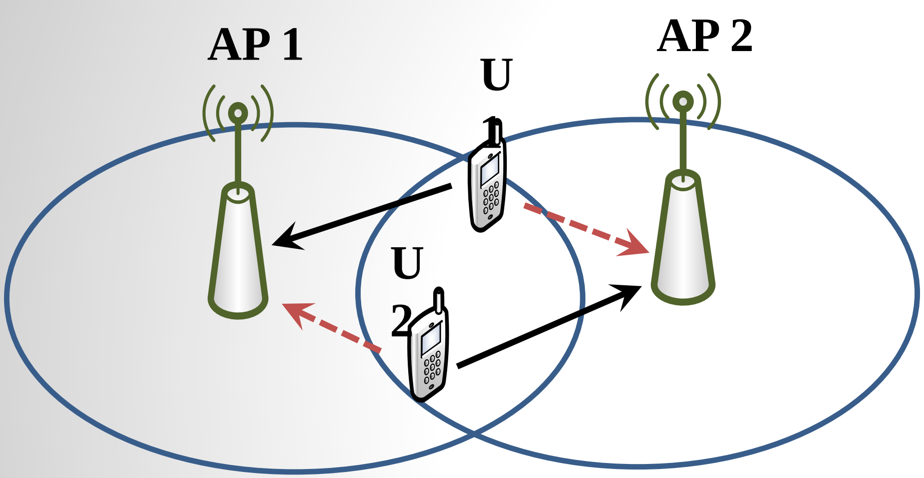
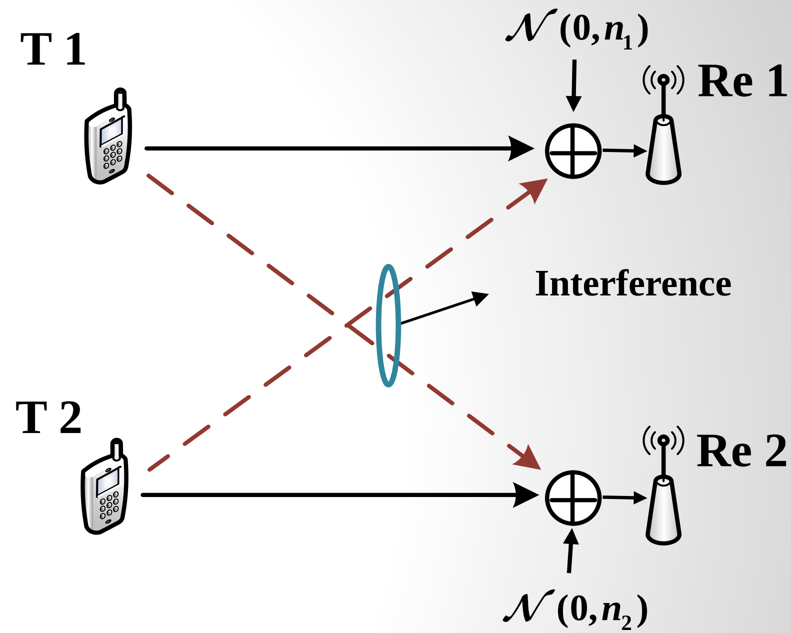
Contents

Interference in cellular networks



Cellular networks



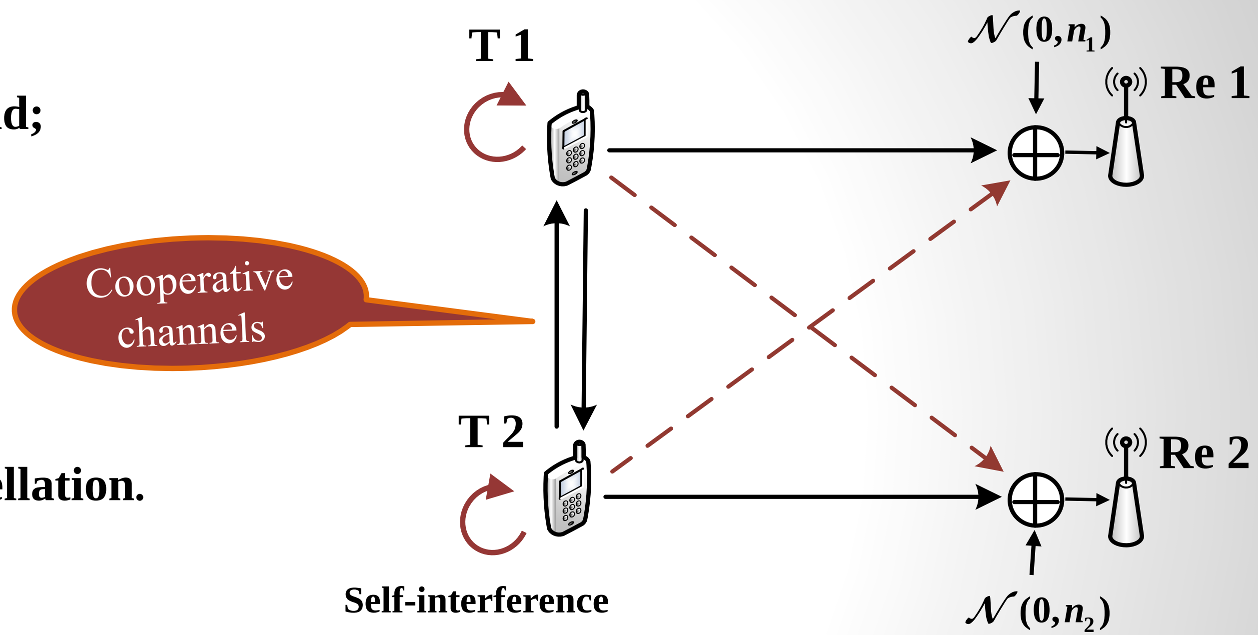
Two-user Gaussian interference channel

When the interference channels are very strong, it is optimal to jointly decode both the two users' messages at each receiver. In this case, this interference channel is so-called compound multiple access channel.

Ref: H. Sato, "The capacity of the Gaussian interference channel under strong interference," IEEE Trans. Inf. Theory,, vol. 27,no. 6, pp. 786–788, 1981.

Transmitter cooperations

- **Communication setup**
  - **Share the same frequency band;**
  - **Full-duplex model.**
- **Pramod *et al* works**
  - **Decode-and-forward (DF);**
  - **Perfect self-interference cancellation.**
- **Our work**
  - **Amplify-and-forward (AF);**
  - **Residual self-interference accumulation**



Interference channel with transmitter cooperations

Ref. V. M. Prabhakaran and P. Viswanath, "Interference channels with source cooperation," IEEE Trans. Inf. Theory, vol. 57, no. 1, pp. 156–186, Jan. 2011

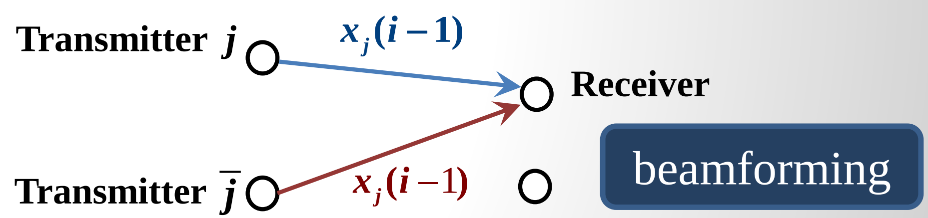
Transmission and reception at transmitter

Time Slots#	i
Transmitted signals	$w_{j1}x_j(i)+w_{j2}x_j(i-1)+w_{j3}(x_{\bar{j}}(i-1)+\hat{y}_j(i-1))$
Received signals	$t_j(i)+t_{\bar{j}}(i)+n_j(i)$
Signals after SI cancellation	$x_{\bar{j}}(i)+\hat{y}_j(i)$

Forward (one-block delay)

Transmissions and receptions for transmitter *j* at the *i*-th time slot

- **Transmitted signals at transmitter *j***
  - The new signal $x_j(i)$;
  - The old signal $x_j(i-1)$ again;
  - The received signals $x_{\bar{j}}(i-1)+\hat{y}_j(i-1)$ after processing.



- **Received signals at transmitter *j***
  - The signal $t_{\bar{j}}(i)$ from transmitter $\bar{j}$;
  - The signal $t_j(i)$ from itself as self-interference;
  - The additive Gaussian noise $n_j(i)$.

Self-interference (SI) cancellation and process

■ **SI cancellation for the signal $t_j(i)$**

SI at the *i*-th time slot:

$$t_j(i) = w_{j1}x_j(i) + w_{j2}x_j(i-1) + w_{j3}(y_j(i-1))$$

after SI cancellation, the residual part treated as

$$\hat{t}_j(i) \sim \mathcal{N}(0, \hat{P}_j)$$

constant $\hat{P}_j$ is the residual power of SI.

Time Slots#	i
Received signals	$t_j(i) + t_{\bar{j}}(i) + n_j(i)$

Ref. H. Cui, M. Ma, L. Song, and B. Jiao, "Relay selection for two-way full duplex relay networks with amplify-and-forward protocol," IEEE Trans. Wireless Commun., vol. 13, no. 7, pp. 3768–3777, Jul. 2014.

■ **Cancellation process for the signal $t_{\bar{j}}(i)$**

$$t_{\bar{j}}(i) = w_{\bar{j}1}x_{\bar{j}}(i) + w_{\bar{j}2}x_{\bar{j}}(i-1) + w_{\bar{j}3}\left(w_{j1}h_{\bar{j}j}(x_j(i-1) + \hat{y}_j(i-1))\right)$$

- **Cancel $x_j(i-1)$**
  known to transmitter *j*
- **Cancel $x_{\bar{j}}(i-1)$**
  estimate from $(1/h_{\bar{j}j}w_{\bar{j}1})y_j(i-1)$

$$y_j(i-1) = w_{\bar{j}1}h_{\bar{j}j}x_{\bar{j}}(i-1) + \hat{y}_j(i-1)$$

■ **The accumulated residual SI**

Final: $$y_j(i) = w_{\bar{j}1}h_{\bar{j}j}x_{\bar{j}}(i) + \hat{y}_j(i)$$

$$\hat{y}_j(i) = \alpha_{\bar{j}}\hat{y}_j(i-1) + h_{\bar{j}j}w_{\bar{j}3}\hat{y}_{\bar{j}}(i-1) + \hat{t}_j(i) + n_j(i),$$

with $\alpha_{\bar{j}} = w_{\bar{j}2}/w_{\bar{j}1}$. Markov process

The Residual SI accumulation

- **The statistics of the residual SI**
  - **Matrix form:**

$$\hat{\mathrm{Y}}(i) = A\hat{\mathrm{Y}}(i-1) + \mathrm{F}(i)$$

where $\hat{\mathrm{Y}}(i) = \left[\hat{y}_1(i), \hat{y}_2(i)\right]^T$ **and** $\mathrm{F}(i) = \left[\hat{t}_1(i), \hat{t}_2(i)\right]^T$.

Recursively it follows:

$$\hat{\mathrm{Y}}(i) = \sum_{n=0}^{i-1} A^n \mathrm{F}(i-n)$$

Matrix power series:

$$a_1 x_1 + a_2 x_2^{\,2} + \cdots a_n x_n^{\,n} = \sum_{i=1}^{n} a_n x_n^{\,n}$$

  - **Stationary state:**

Condition: when the time slot i tends to infinity, $\max|\lambda(A)| < 1$

The covariance matrix of the residual SI will converge to a finite constant.

$$\mathrm{E}\left[\hat{\mathrm{Y}}(i)\cdot\hat{\mathrm{Y}}(k)^H\right] = P^H \begin{bmatrix} \lambda_1 & 0 \\ 0 & \lambda_2 \end{bmatrix}^{(i-k)^+} \cdot \begin{bmatrix} c_1 & c_3 \\ c_3^{*} & c_2 \end{bmatrix} \cdot \begin{bmatrix} \lambda_1^{*} & 0 \\ 0 & \lambda_2^{*} \end{bmatrix}^{(k-i)^+} P$$

Reception at each receiver

- **Received signals at receiver *k***

$$r_k(i) = \sum_{j=1}^{2} h_{jk} t_j(i) + n_k(i)$$

It follows,

$$r_k(i) = \sum_{j=1}^{2} h_{jk} w_{j1} x_j(i) + (h_{jk} w_{j2} + h_{\bar{j}k} h_{\bar{j}j} w_{\bar{j}3} w_{j1}) x_j(i-1) + v_k(i)$$

Produced by beamforming

with $v_k(i) = \sum_{j=1}^{2} h_{jk} w_{j3} \hat{y}_j(i-1) + n_k(i)$.

Equivalent channel model: Two-tap multiple access channel.

Treat $x_j(i-1)$ **as interference to decode the *i*-th time slot messages is bad !!**

Decoding scheme

- **The optimal decoding scheme**

Unite N time slots to decode: $Y_k = \sum_{j=1}^{2} \mathrm{H}_{jk} X_j + Z_k$, **with** $Y = \left[r_k(N) \ \cdots \ r_k(i) \ \cdots \ r_k(1) \right]^T$

- **Tradeoff: unite two time slots**

Channel input and output model:

$$\begin{bmatrix} r_k(i) \\ r_k(i-1) \end{bmatrix} = \begin{bmatrix} h_{jk} w_{j2} + h_{\bar{j}k} h_{\bar{j}j} w_{\bar{j}3} w_{j1} \\ h_{jk} w_{j1} \end{bmatrix} x_j(i-1) + \begin{bmatrix} h_{\bar{j}k} w_{\bar{j}2} + h_{jk} h_{j\bar{j}} w_{j3} w_{\bar{j}1} \\ h_{\bar{j}k} w_{\bar{j}1} \end{bmatrix} x_{\bar{j}}(i-1) + \begin{bmatrix} z_{k1} \\ z_{k2} \end{bmatrix}$$

- **Joint forward decoding:** $x_j(i-2)$ **known;** $x_j(i)$ **treated as interference.**
- **Joint backward decoding:** $x_j(i)$ **known;** $x_j(i-2)$ **treated as interference.**

Forward:

interference

$$z_{k1} = n_k(i) + \sum_{j=1}^{2} h_{jk} w_{j1} x_j(i) + h_{\bar{j}k} h_{\bar{j}j} w_{j3} w_{\bar{j}1} \hat{y}_j(i)$$

$$z_{k2} = n_k(i-1) + \sum_{j=1}^{2} h_{\bar{j}k} h_{\bar{j}j} w_{j3} w_{\bar{j}1} \hat{y}_j(i-1)$$

Achievable rate region

■ Achievable rate region

- **Stationary condition:** $i \to \infty$ **and** $\max\{|\lambda_1|, |\lambda_2|\} < 1$

$$\mathcal{C}(\mathcal{P}) = \bigcup_{\substack{\{w_{ij}\}\in\mathcal{P} \\ \max\{|\lambda_1|,|\lambda_2|\}<1}} \left\{ (R_1, R_2) \middle| \begin{array}{c} R_1 \le \log\left(1 + \mathbf{H}_{1k}^H \mathbf{Q}_k^{-1} \mathbf{H}_{1k}\right) \\ R_2 \le \log\left(1 + \mathbf{H}_{2k}^H \mathbf{Q}_k^{-1} \mathbf{H}_{2k}\right) \\ R_1 + R_2 \le \log\left(1 + \mathbf{H}_{1k}^H \mathbf{Q}_k^{-1} \mathbf{H}_{1k} + \mathbf{H}_{2k}^H \mathbf{Q}_k^{-1} \mathbf{H}_{2k}\right) \end{array} \right\}$$

Power region: $\mathcal{P} = \left\{ w_{ij} : |w_{j1}|^2 + |w_{j2}|^2 + |w_{j3}|^2 (\mathbf{g}_{\bar{j}\bar{j}} |w_{\bar{j}1}|^2 + \hat{\alpha}_j) \le \mathbf{P}_j \right\}$

Q_k **denotes the covariance matrix of** Z_k **:** $Q_k = \mathrm{E}\left(Z_k \cdot Z_k^H\right)$

Ref. A. Goldsmith, S. A. Jafar, N. Jindal, and S. Vishwanath, "Capacity limits of MIMO channels," IEEE J. Sel. areas Commun., vol. 21, no. 5, pp. 684–702, June 2003.

Optimization Problem

- **Maximize sum rate**
  - **Rate-profile approach: choose a** $0 \le \alpha \le 1,\ \bar{\alpha} = 1 - \alpha$

    Define $R_{sum} = R_1 + R_2,\ R_1 = \alpha R_{sum},\ R_2 = \bar{\alpha} R_{sum}$.
  - **Sum rate maximum problem for a fixed** α

$$
\begin{aligned}
\max_{\{w_{ij}\}} \quad & R_1 (R_2) \\
\text{s.t.} \quad & R_1 \le \log\left(1 + \mathbf{H}_{1k}^H Q^{-1} \mathbf{H}_{1k}\right) \\
& R_2 \le \log\left(1 + \mathbf{H}_{2k}^H Q^{-1} \mathbf{H}_{2k}\right) \\
& R_1 + R_2 \le \log\left(1 + \mathbf{H}_{1k}^H Q^{-1} \mathbf{H}_{1k} + \mathbf{H}_{2k}^H Q^{-1} \mathbf{H}_{2k}\right) \\
& \max\{|\lambda_1|, |\lambda_2|\} < 1, \\
& \{w_{ij}\} \in \mathcal{P},\ k = 3,4.
\end{aligned}
$$

$\Rightarrow$

$$
\begin{aligned}
\max_{R_{sum}, \{w_{ij}\}} \quad & R_{sum} \\
\text{s.t.} \quad & \alpha R_{sum} \le \log\left(1 + \mathbf{H}_{1k}^H Q^{-1} \mathbf{H}_{1k}\right) \\
& \bar{\alpha} R_{sum} \le \log\left(1 + \mathbf{H}_{2k}^H Q^{-1} \mathbf{H}_{2k}\right) \\
& R_{sum} \le \log\left(1 + \mathbf{H}_{1k}^H Q^{-1} \mathbf{H}_{1k} + \mathbf{H}_{2k}^H Q^{-1} \mathbf{H}_{2k}\right) \\
& \max\{|\lambda_1|, |\lambda_2|\} < 1, \\
& \{w_{ij}\} \in \mathcal{P},\ k = 3,4.
\end{aligned}
$$

All the constraints are not convex !!

Algorithm

- **Iterative method**
  - **Fix Q_k, and optimize the transmission parameters $\{w_{ij}\}$.**
  - **Update Q_k with the obtained transmission parameters.**

Iteratively solve the first step: optimize w_{j1}, w_{j2}, w_{j3} with the fixed $w_{\bar{j}1}, w_{\bar{j}2}, w_{\bar{j}3}$

Define

$$W_{j1} = \begin{bmatrix} w_{j2} & w_{j1} \end{bmatrix}^T \qquad W_{j2} = \begin{bmatrix} w_{j3} & 1 \end{bmatrix}^T$$

$$H_{jk} = G_{\bar{j}k1} W_{j1} = G_{jk2} W_{\bar{j}2}$$

Hermitian matrix: $T_{jkj} = G_{jkj}^H Q^{-1} G_{jkj}$

$$\max_{R_{sum}, \{W_{ij}\}} \quad R_{sum}$$

$$\text{s.t.} \quad \alpha R_{sum} \le \log\left(1 + \mathrm{W}_{jj}^H T_{\bar{j}kj} \mathrm{W}_{jj}\right)$$

$$\bar{\alpha} R_{sum} \le \log\left(1 + \mathrm{W}_{\bar{j}\bar{j}}^H T_{\bar{j}k\bar{j}} \mathrm{W}_{\bar{j}\bar{j}}\right)$$

$$R_{sum} \le \log\left(1 + \mathrm{W}_{\bar{j}\bar{j}}^H T_{\bar{j}k\bar{j}} \mathrm{W}_{\bar{j}\bar{j}} + \mathrm{W}_{jj}^H T_{\bar{j}kj} \mathrm{W}_{jj}\right)$$

$$\max\{|\lambda_1|, |\lambda_2|\} < 1,$$

$$\mathrm{W}_{j1}^H \mathrm{W}_{j1} + (\mathrm{W}_{j1}^H \mathrm{W}_{j1} - 1)\mathrm{m}_j \le \mathrm{P}_j, \; k = 3,4.$$

Quadratic constraints

Eigenvalue constraint

Eigenvalue constraint approximation

- **Difficult: matrix A is not Hermitian.**

$$A=\begin{bmatrix} \frac{w_{j2}}{w_{j1}} & h_{j\bar{j}}w_{j3} \\ h_{\bar{j}j}w_{\bar{j}3} & \frac{w_{\bar{j}2}}{w_{\bar{j}1}} \end{bmatrix}$$

- **Approximation:**

$$W_{j1}^H E_0 W_{j1} \le 0,\ \ g_{j\bar{j}} W_{j2}^H E_1 W_{j2} \le 1$$

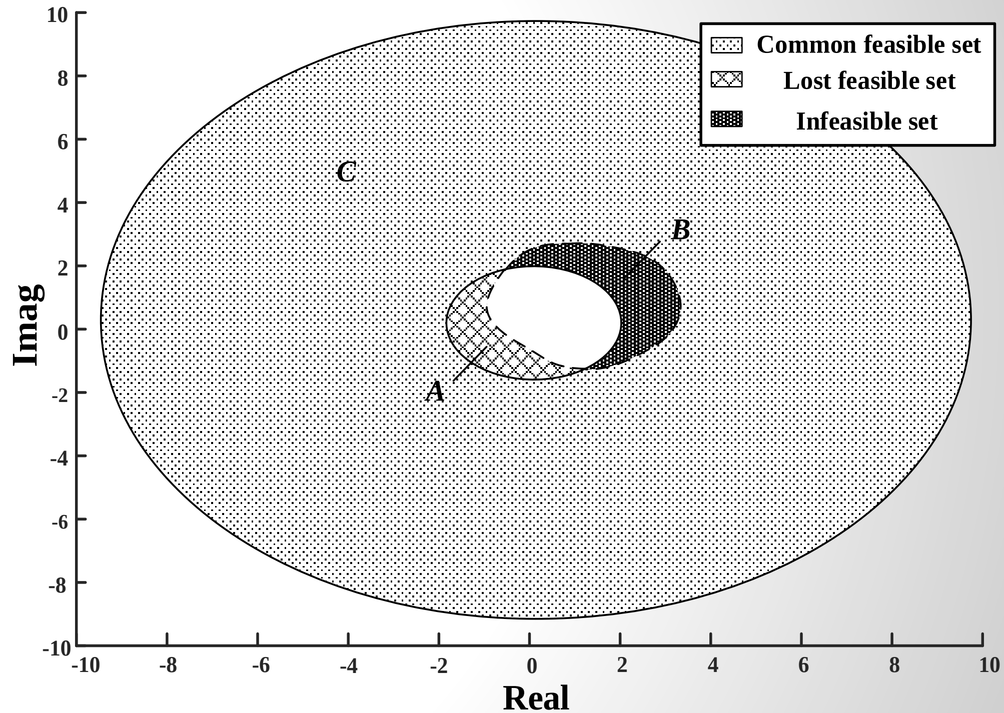
with $E_0 = diag(1,-1),\ \ E_1 = diag(1,0)$

limit the norm of each element.

$$\left|h_{j\bar{j}}w_{j3}\right| \ge \left|\lambda_j + \frac{w_{j2}}{w_{j1}}\right| \ge \left|\lambda_j\right| - \left|\frac{w_{j2}}{w_{j1}}\right| \to \left|\lambda_j\right| \le \left|\frac{w_{j2}}{w_{j1}}\right| + \left|h_{j\bar{j}}w_{j3}\right|$$

By Gershgorin circle theorem.

Example



Feasible set of w_{11}

max $|\lambda_j|$: $A \cup C$ Approximation: $B \cup C$

Semidefinite Relaxation

Equivalent change

$$\max_{R_{sum},\{W_{ij}\}} \quad R_{sum}$$

$$\text{s.t.} \quad 2^{\alpha R_{sum}} \leq \left(1+Tr(T_{\bar{j}kj}\tilde{\mathrm{W}}_{jj})\right)$$

$$2^{\bar{\alpha} R_{sum}} \leq \left(1+Tr(T_{\bar{j}k\bar{j}}\tilde{\mathrm{W}}_{\bar{j}\bar{j}})\right)$$

$$2^{R_{sum}} \leq \left(1+Tr(T_{\bar{j}k\bar{j}}\tilde{\mathrm{W}}_{\bar{j}\bar{j}})+Tr(T_{\bar{j}kj}\tilde{\mathrm{W}}_{jj})\right)$$

$$\mathrm{Tr}\,(\mathrm{E}_0\tilde{\mathrm{W}}_{j1}) \leq 0,\ \mathrm{Tr}(\mathrm{E}_1\tilde{\mathrm{W}}_{j1}) \leq 1,$$

$$\mathrm{rank}(\tilde{\mathrm{W}}_{j1})=\mathrm{rank}(\tilde{\mathrm{W}}_{j2})=1$$

$$Tr(\tilde{\mathrm{W}}_{j1})+(Tr(\tilde{\mathrm{W}}_{j1})-1)\mathrm{m}_j \leq \mathrm{P}_j,\ k=3,4.$$

Define new 2 by 2 matrix:

$$\tilde{W}_{ij} = W_{ij}W_{ij}^H, \text{ with } \tilde{W}_{ij} \in H^{2\times 2}$$

Gaussian Randomization

- **Obtain $\tilde{W}_{ij}^*$ by solving the left problem**
- **Generate samples $W_{ij} \sim \mathcal{CN}(0,\ \tilde{W}_{ij}^*)$**
- **Check the generated samples.**
- **Choose the best couple samples.**

$$R_{sum} = \min\left\{ \frac{1}{\alpha}\log\left(1+\mathrm{W}_{jj}^H T_{\bar{j}kj}\mathrm{W}_{jj}\right),\ \frac{1}{\bar{\alpha}}\log\left(1+\mathrm{W}_{\bar{j}\bar{j}}^H T_{\bar{j}k\bar{j}}\mathrm{W}_{\bar{j}\bar{j}}\right),\ \log\left(1+\mathrm{W}_{\bar{j}\bar{j}}^H T_{\bar{j}k\bar{j}}\mathrm{W}_{\bar{j}\bar{j}} + \mathrm{W}_{jj}^H T_{\bar{j}kj}\mathrm{W}_{jj}\right) \right\}$$

Z. Q. Luo, W. K. Ma, A. So, Y. Ye, and S. Zhang, "Semidefinite relaxation of quadratic optimization problems," IEEESignal Process. Mag., vol. 27, no. 3, pp. 20–34, May 2010.

Gaussian Algorithm

TABLE I Algorithm I: Gaussian randomization method for optimizing $\mathbf{w}_{j1}$ and $\mathbf{w}_{j2}$.

Input: the total sample number I, $\mathbf{Q}_k$, $\mathbf{w}_{\bar{j}1}$, and $\mathbf{w}_{\bar{j}2}$

Output: $\hat{\mathbf{w}}_{j1i^*}$, $\hat{\mathbf{w}}_{j2i^*}$

```
Compute the SDR solutions W*_{j1}, W*_{j2} by solving the convex problem (53) with the fixed Q_k, w_{j̄1}, and w_{j̄2}.
for i = 1 to I do
    Generate samples ŵ_{j1i} ~ N(0, W*_{j1}) and ŵ_{j2i} ~ N(0, W*_{j2});
    Set the second element of ŵ_{j2i} as 1;
    if ŵ^H_{j1} ŵ_{j1} + (ŵ_{j2}^H ŵ_{j2} − 1) m_j > P_j then
        Compute ŵ_{j1i}, ŵ_{j2i} in (55) and (56).
    end if
    if ŵ_{j1i}, ŵ_{j2i} do not satisfy max(|λ_1|, |λ_2|) < 1 then
        R̂_sum(ŵ_{j1i}, ŵ_{j2i}) = 0.
    else
        Compute R̂_sum(ŵ_{j1i}, ŵ_{j2i}) in (54).
    end if
    i = i + 1.
end for
Search i* = arg max_{i=1,...,I} R̂_sum(ŵ_{j1i}, ŵ_{j2i}).
```

Final Algorithm

TABLE II Algorithm II: Two-step iterative algorithm for optimizing transmission parameters.

Input: $l = 0$, tolerances $\xi_0 > 0$, $\xi_1 > 0$ and feasible initial transmission parameters $\mathbf{w}_{11}^0$, $\mathbf{w}_{12}^0$, $\mathbf{w}_{21}^0$, $\mathbf{w}_{22}^0$

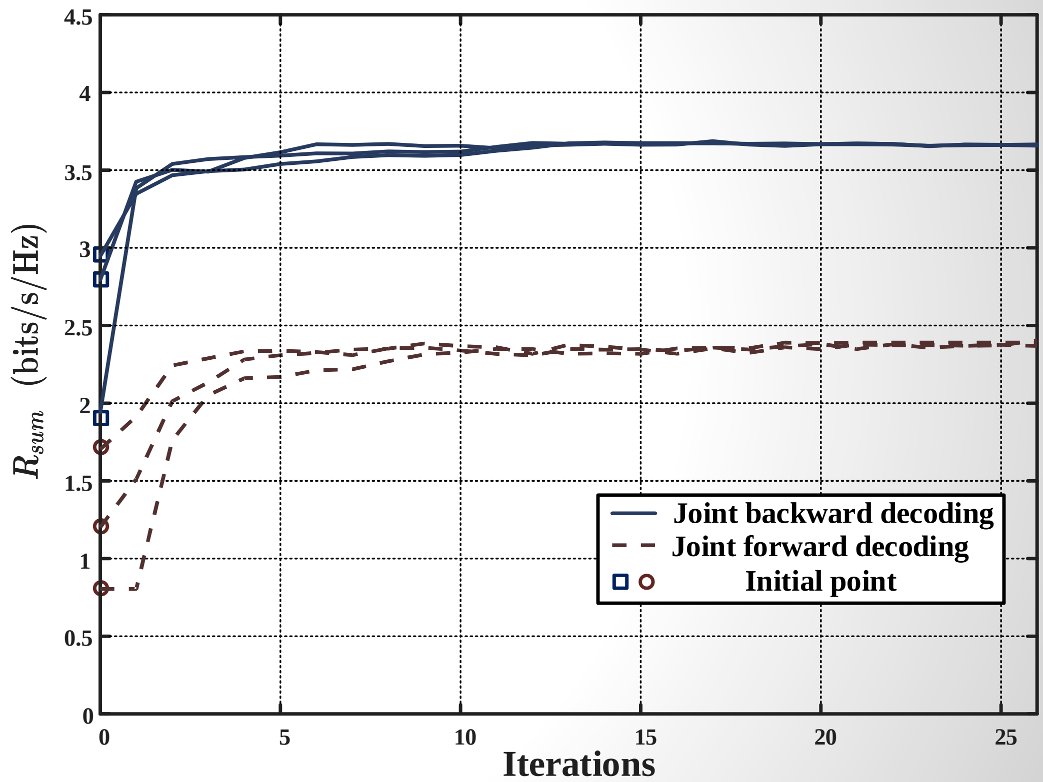
Output: $\mathbf{w}_{11}^*$, $\mathbf{w}_{12}^*$, $\mathbf{w}_{21}^*$, $\mathbf{w}_{22}^*$

repeat
    Compute $\mathbf{Q}_k^l$ in (41) and (43) with $\mathbf{w}_{11}^l$, $\mathbf{w}_{12}^l$, $\mathbf{w}_{21}^l$, and $\mathbf{w}_{22}^l$;
    $(\mathbf{w}_{11}^{l+1}, \mathbf{w}_{12}^{l+1}, \mathbf{w}_{21}^{l+1}, \mathbf{w}_{22}^{l+1}) = (\mathbf{w}_{11}^l, \mathbf{w}_{12}^l, \mathbf{w}_{21}^l, \mathbf{w}_{22}^l)$.
    repeat
        Obtain $(\mathbf{w}_{11}^{l+1}, \mathbf{w}_{12}^{l+1}) = (\hat{\mathbf{w}}_{11i^*}, \hat{\mathbf{w}}_{12i^*})$ via Algorithm I with the fixed $\mathbf{w}_{21}^{l+1}$, $\mathbf{w}_{22}^{l+1}$, and $\mathbf{Q}_k^l$;
        Obtain $(\mathbf{w}_{21}^{l+1}, \mathbf{w}_{22}^{l+1}) = (\hat{\mathbf{w}}_{21i^*}, \hat{\mathbf{w}}_{22i^*})$ via Algorithm I with the fixed $\mathbf{w}_{11}^{l+1}$, $\mathbf{w}_{12}^{l+1}$, and $\mathbf{Q}_k^l$.
    until $|\hat{R}_{sum}(\mathbf{w}_{11}^{l+1}, \mathbf{w}_{12}^{l+1}) - \hat{R}_{sum}(\mathbf{w}_{21}^{l+1}, \mathbf{w}_{22}^{l+1})| < \xi_0$.
    $l = l + 1$.
until $|R_{sum}(\mathbf{w}_{11}^l, \mathbf{w}_{12}^l, \mathbf{w}_{21}^l, \mathbf{w}_{22}^l) - R_{sum}(\mathbf{w}_{11}^{l-1}, \mathbf{w}_{12}^{l-1}, \mathbf{w}_{21}^{l-1}, \mathbf{w}_{22}^{l-1})| < \xi_1$
$(\mathbf{w}_{11}^*, \mathbf{w}_{12}^*, \mathbf{w}_{21}^*, \mathbf{w}_{22}^*) = (\mathbf{w}_{11}^l, \mathbf{w}_{12}^l, \mathbf{w}_{21}^l, \mathbf{w}_{22}^l)$.

Convergence

P_1	20dB
P_2	20dB
$h_{12} = h_{21}$	3
$\hat{P}_j$ (Residual SI)	-10dB

channel	value
$\lvert h_{13} \rvert$	0.1
$\lvert h_{24} \rvert$	0.1
$\lvert h_{14} \rvert$	0.4
$\lvert h_{23} \rvert$	0.4



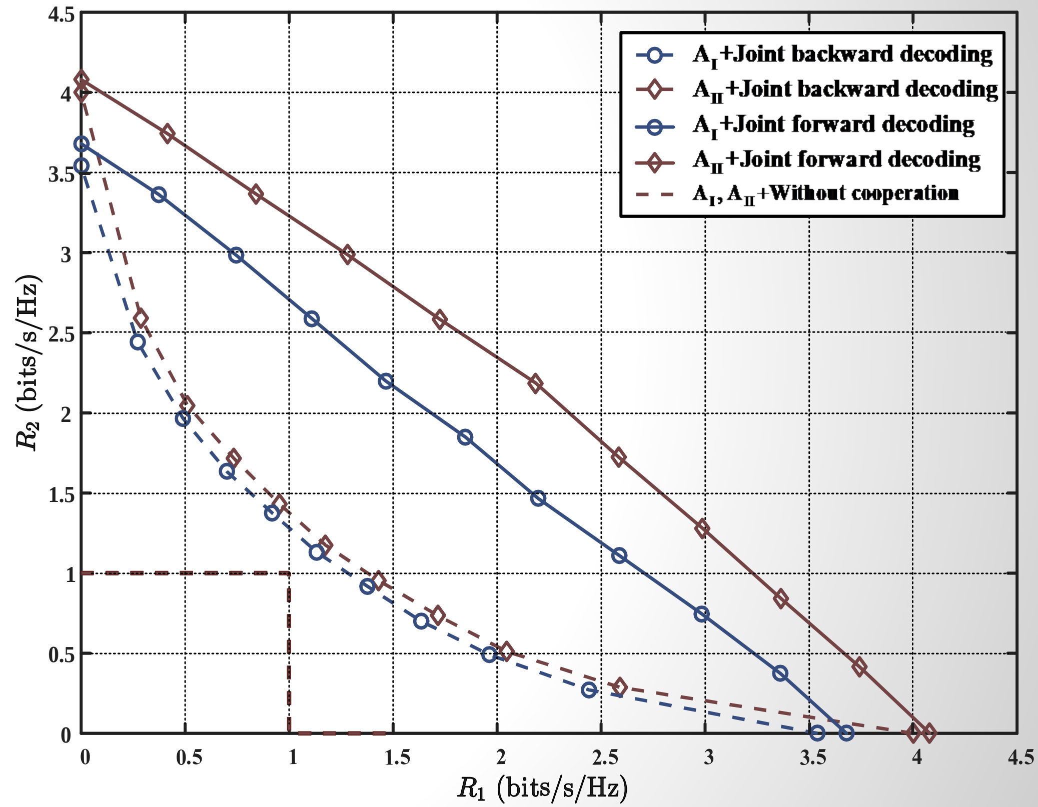
The convergence of algorithm

Achievable Rate region

P_1	20dB
P_2	20dB
$h_{12} = h_{21}$	3
$\hat{P}_j$(Residual SI)	-10dB

channel	value
$\lvert h_{13} \rvert$	0.1
$\lvert h_{24} \rvert$	0.1

$\mathbf{A}_{\mathrm{I}}$	$\lvert h_{14} \rvert = \lvert h_{23} \rvert$	0.4
$\mathbf{A}_{\mathrm{II}}$	$\lvert h_{14} \rvert = \lvert h_{23} \rvert$	0.5



Achievable rate regions

Conclusion

- **AF-based scheme has been proposed;**
- **Accumulated residual self-interference has been studied;**
- **Joint forward decoding and backward decoding were proposed to characterize the achievable rate regions;**
- **SDR-based algorithm was studied.**