Approximate Message Passing in Coded Aperture Snapshot Spectral Imaging

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Hyperspectral Images

RGB image

3D cube

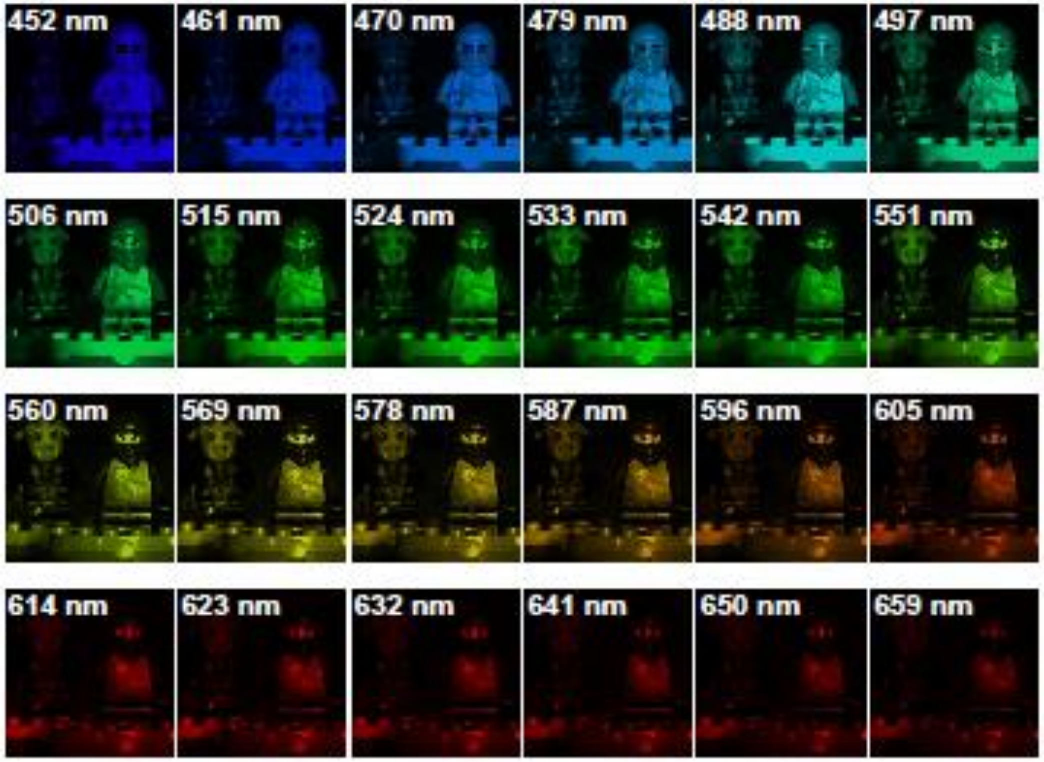
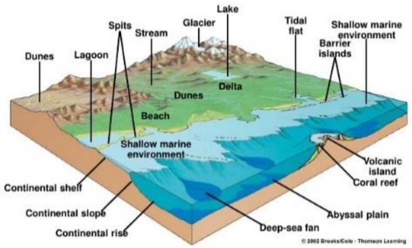


Image slices at different wavelengths

Hyperspectral Images

- Obtain spectrum information of a scene
- Applications include
  - Medical imaging

  - Geology



  - Astronomy
  - Remote sensing

Conventional Hyperspectral Imaging

- Acquire and store entire image in all spectrum bands
- Disadvantages
  - Long imaging time
  - Large storage

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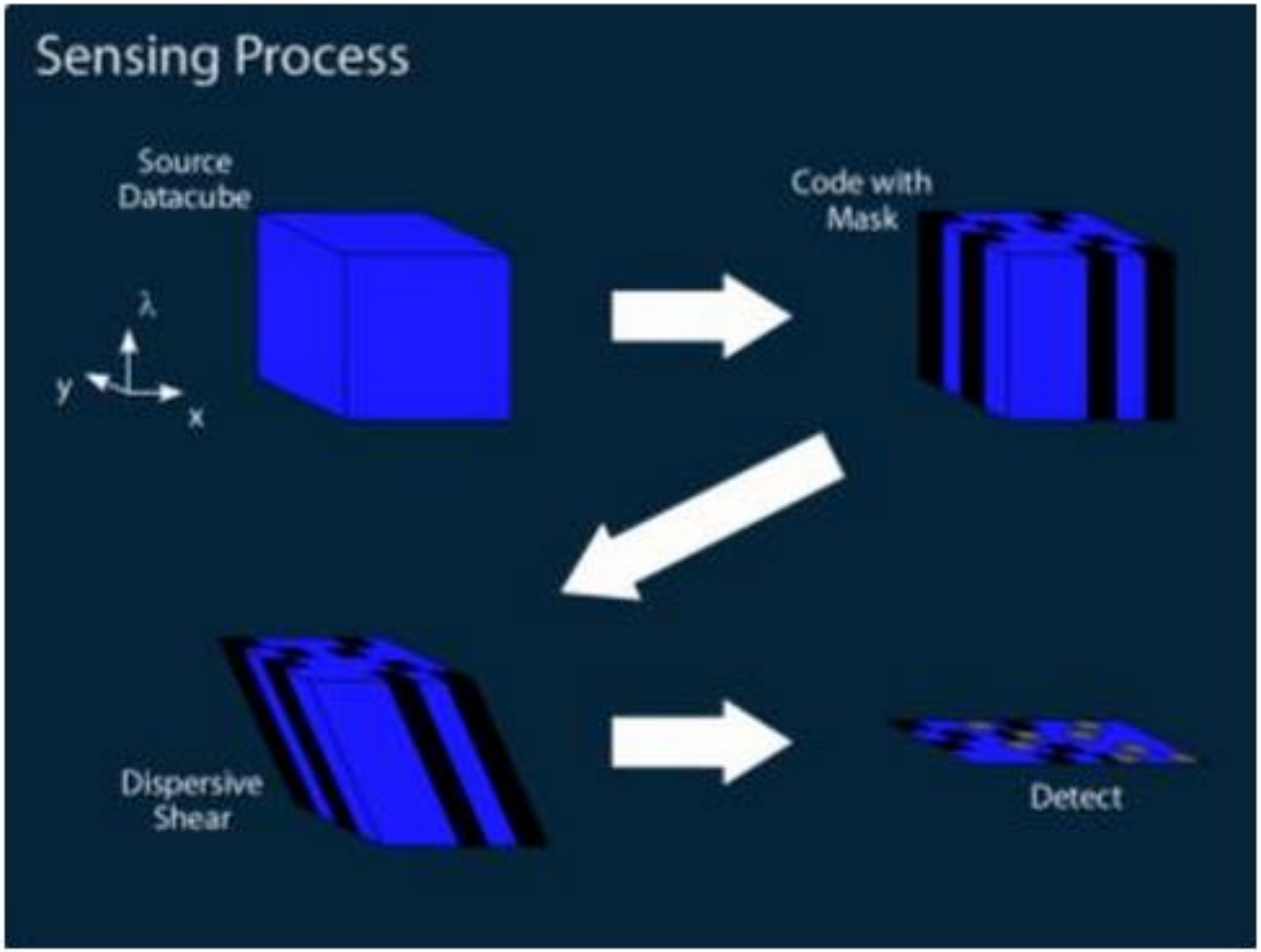
Better imaging system?

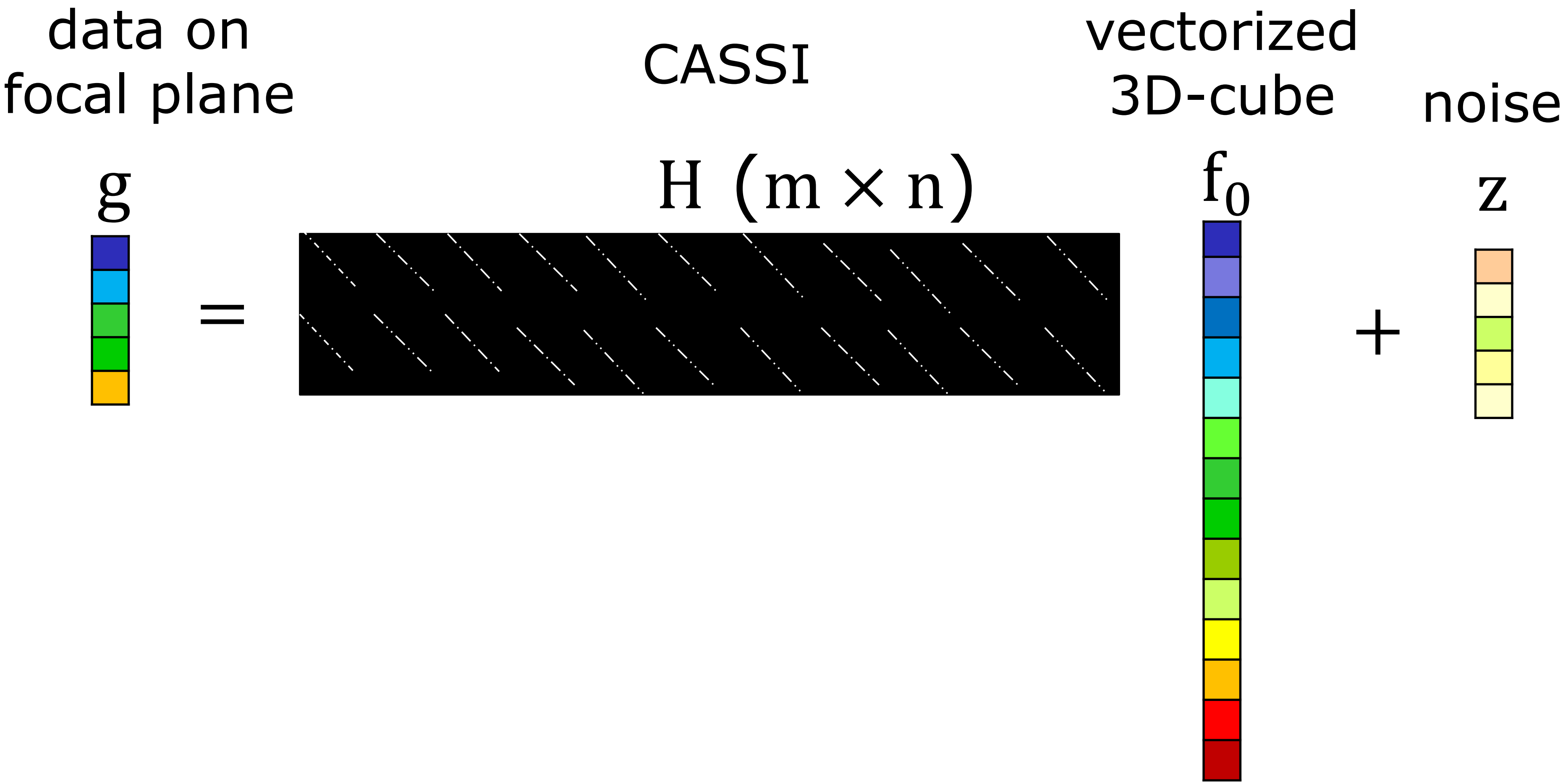
Compressive Hyperspectral Imaging

Coded Aperture Snapshot Spectral Imaging (CASSI) [Wagadarikar et al. 2008]

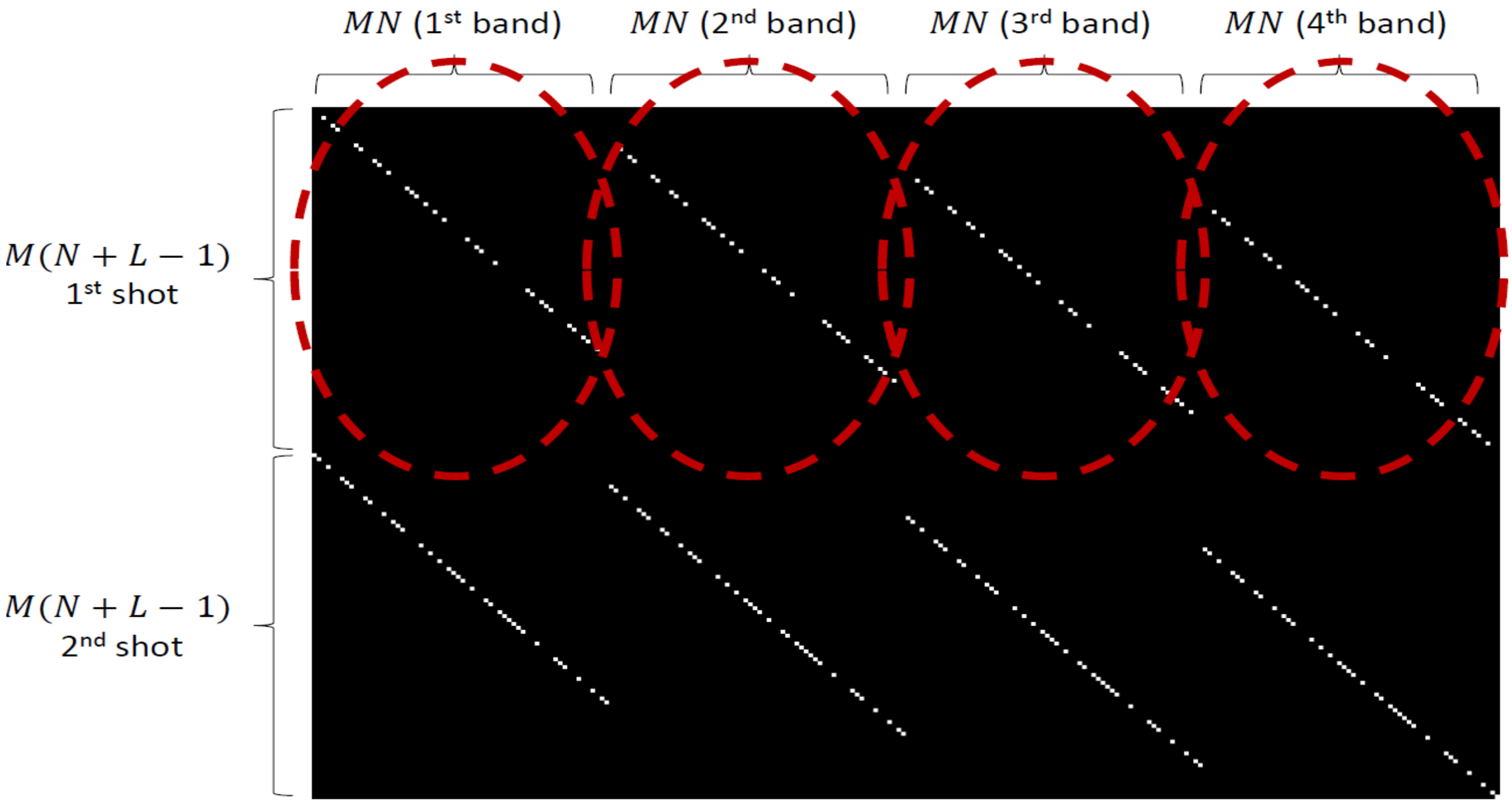


Compressive Sensing Formulation

$$\mathrm{g} = \mathrm{H}\mathrm{f}_0 + \mathrm{z}$$

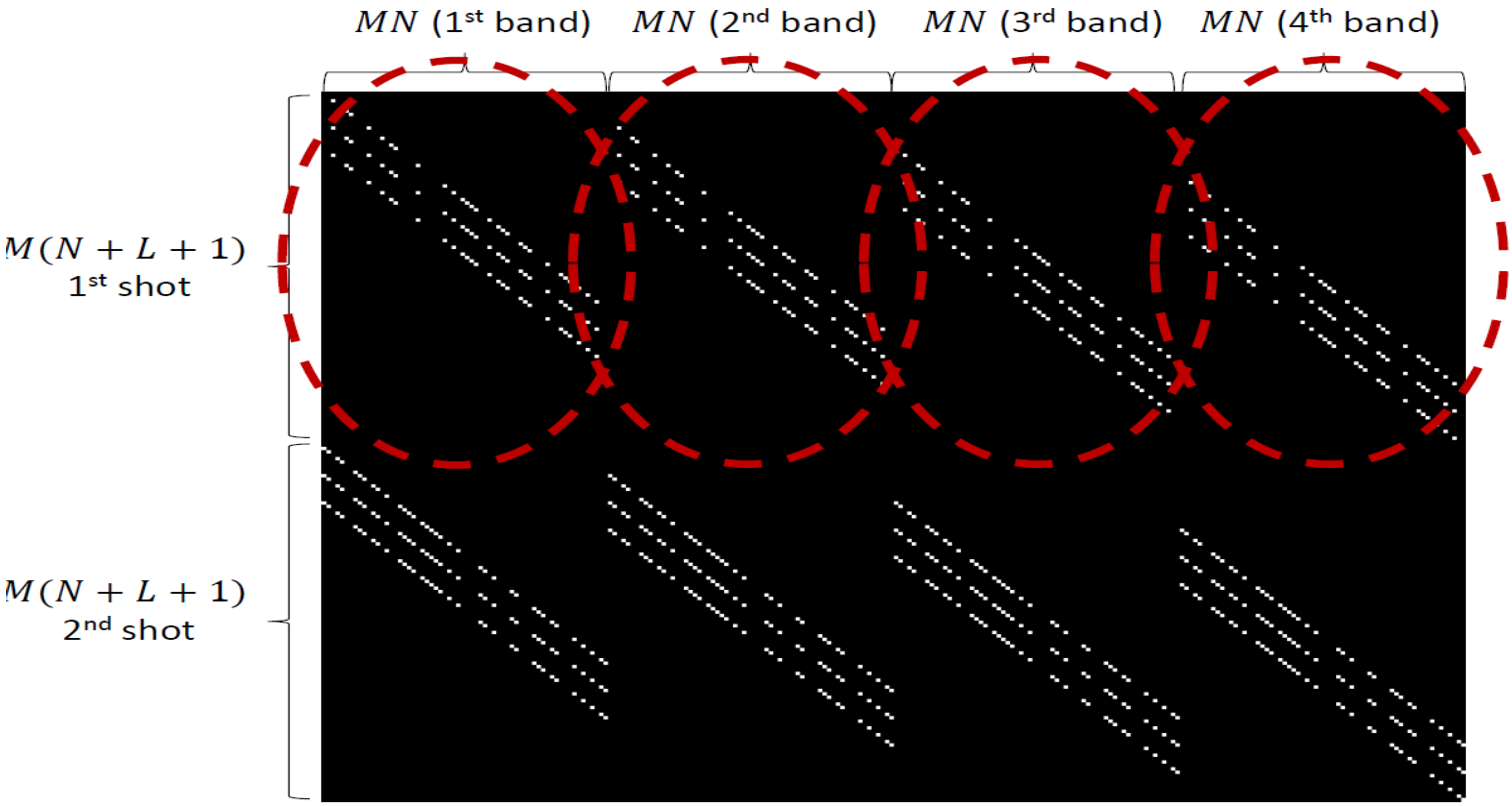


Multi-shot CASSI [Arguello et al. 2011]



measurement rate ≈ $2/\mathrm{L}$, L: #spectrum bands

Higher Order CASSI [Arguello et al. 2013]



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Challenges

- Highly compressed measurements
- Structured sensing matrix
- Large signal dimension (need fast algorithm)

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- Structured sensing matrix
- Large signal dimension (need fast algorithm)

no parameter tuning   runtime

Approximate Message Passing

[Donoho et al. 2009]

Approximate Message Passing (AMP)

compressive sensing → denoising

$$g = Hf_0 + z \in \mathbb{R}^m \qquad q^t = f_0 + v^t \in \mathbb{R}^n$$

Pseudo-data
(noisy data)

Approximate Message Passing (AMP)

compressive sensing → denoising

$g = Hf_0 + z \in \mathbb{R}^m$ → $q^t = f_0 + v^t \in \mathbb{R}^n$

If sensing matrix H is i.i.d. Gaussian, asymptotically

- Noise v^t uncorrelated with input f_0
- Noise v^t distributed as i.i.d. Gaussian $\mathcal{N}(0, \sigma_t^2)$
- Noise variance σ_t^2 can be accurately estimated

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May break down for structured matrix!

AMP Pseudocode

Initialize $f^{t=0} \leftarrow \mathbf{0}$

At iteration t, do

Residual: $r^t \leftarrow g - Hf^t + \frac{r^{t-1}}{m/n}\langle \eta'_{t-1}(f^{t-1} + H^T r^{t-1}) \rangle$

Noisy signal: $q^t \leftarrow f^t + H^T r^t \ (= f_0 + v^t)$

Noise (v^t) level: $\sigma_t^2 \leftarrow ||r^t||_2^2/m$

Denoising: $f^{t+1} \leftarrow \eta_t(q^t; \sigma_t^2)$

AMP Pseudocode

Initialize $f^{t=0} \leftarrow \mathbf{0}$

At iteration t, do

Onsager correction

Residual: $r^t \leftarrow g - Hf^t + \boxed{\frac{r^{t-1}}{m/n}\langle \eta'_{t-1}(f^{t-1} + H^T r^{t-1})\rangle}$

Noisy signal: $q^t \leftarrow f^t + H^T r^t \ (= f_0 + v^t)$

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AMP for Hyperspectral Image Recovery

Initialize $f^{t=0} \leftarrow \mathbf{0}$

At iteration t, do

Residual: $r^t \leftarrow g - Hf^t + \frac{r^{t-1}}{m/n}\langle \eta'_{t-1}(f^{t-1} + H^T r^{t-1})\rangle$

Noisy signal: $q^t \leftarrow f^t + H^T r^t$

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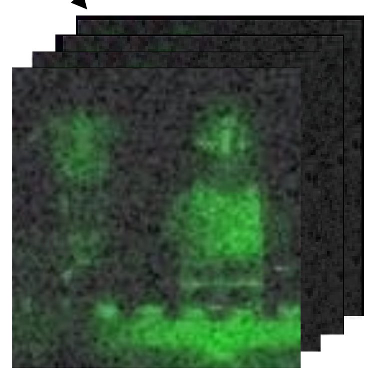
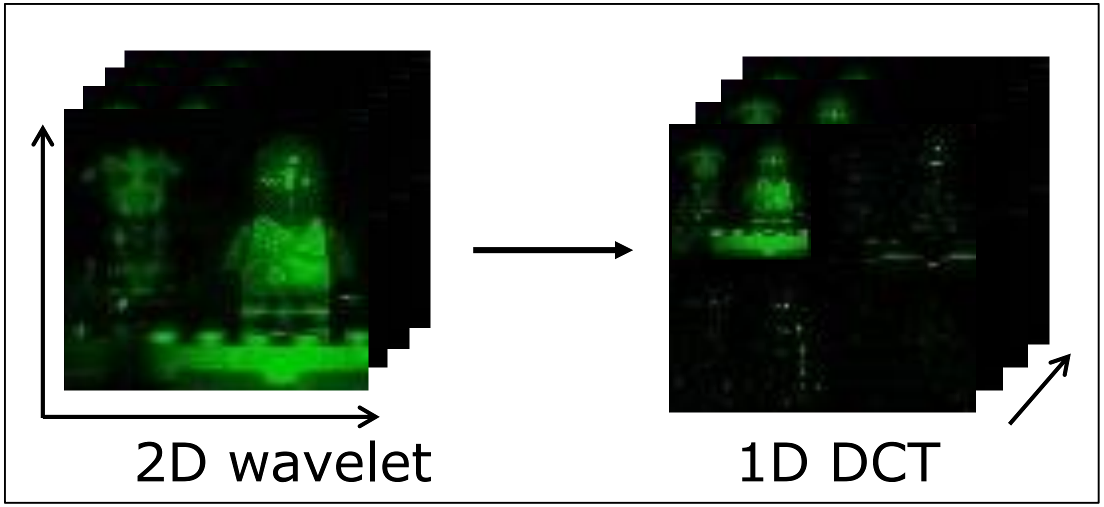
3D image denoiser

Denoising: $f^{t+1} \leftarrow \eta_t(q^t; \sigma_t^2)$



Denoising (3D-Wiener)

$$q^t = f_0 + v^t$$

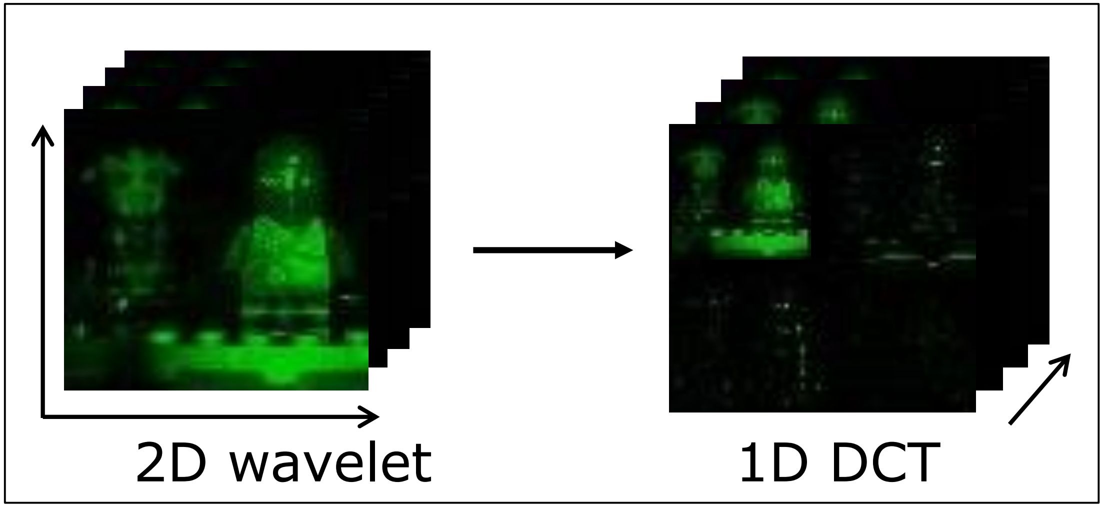


$$\theta_q^t = \theta_{f_0} + \theta_v^t$$

- 2D wavelet + 1D discrete cosine transform (DCT)

Denoising (3D-Wiener)

$q^t = f_0 + v^t$

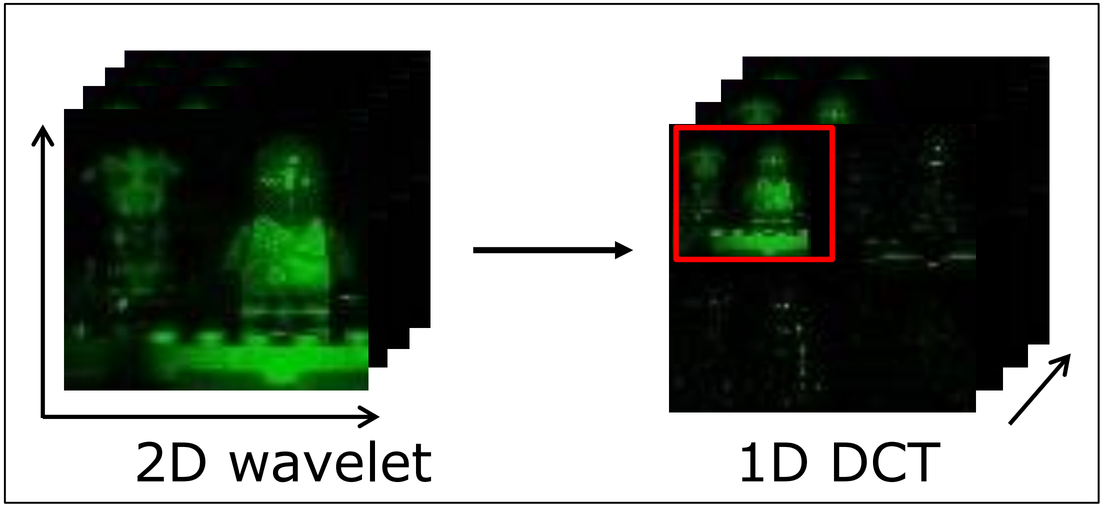


$\theta_q^t = \theta_{f_0} + \theta_v^t$

- 2D wavelet + 1D discrete cosine transform (DCT)
- Assume Var($\theta_{v,i}^t$) = σ_t^2, $i = 1, .., n$

Denoising (3D-Wiener)

$q^t = f_0 + v^t$

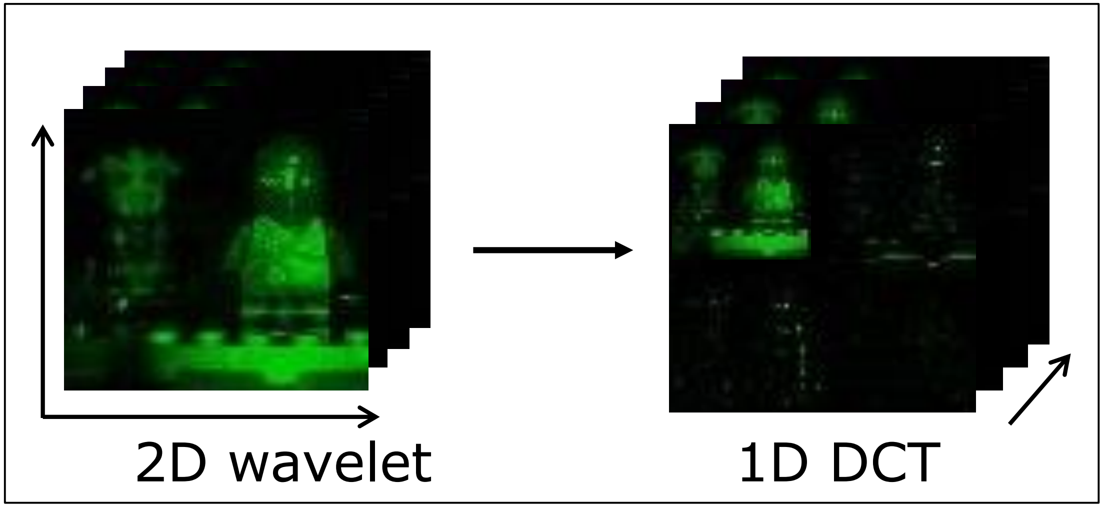


$\theta_q^t = \theta_{f_0} + \theta_v^t$

- 2D wavelet + 1D discrete cosine transform (DCT)
- Assume $\mathrm{Var}(\theta_{v,i}^t) = \sigma_t^2,\ i = 1,..,n$
- Empirical variance σ_i^2 and mean μ_i of $\theta_{f_0,i}$ estimated using θ_q^t in wavelet subband

Denoising (3D-Wiener)

$q^t = f_0 + v^t$ →

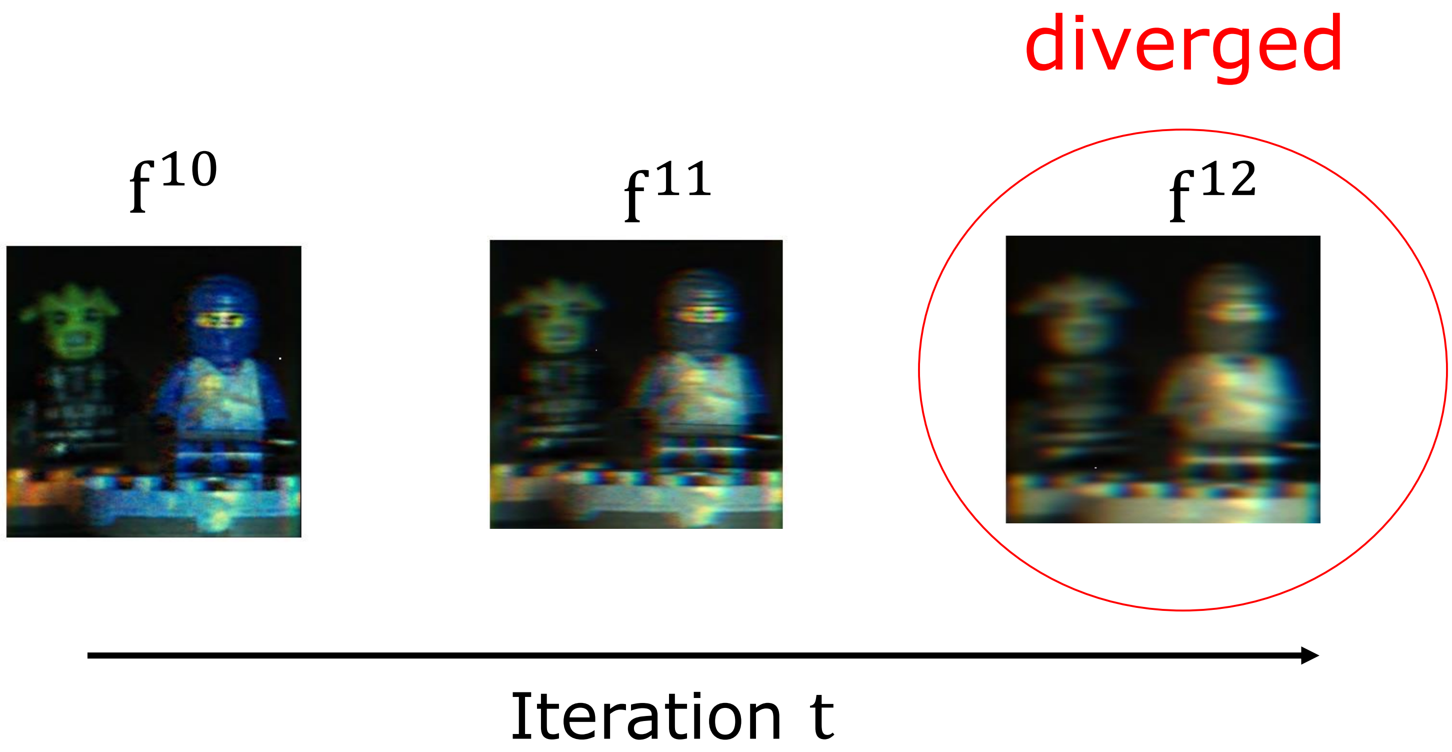


→ $\theta_q^t = \theta_{f_0} + \theta_v^t$

- 2D wavelet + 1D discrete cosine transform (DCT)
- Assume $\mathrm{Var}(\theta_{v,i}^t) = \sigma_t^2$, $i = 1, .., n$
- Empirical variance σ_i^2 and mean μ_i of $\theta_{f_0,i}$ estimated using θ_q^t in wavelet subband
- Wiener filter: $\theta_{f,i}^{t+1} = \frac{\sigma_i^2}{\sigma_i^2 + \sigma_t^2} \cdot \left(\theta_{q,i}^t - \mu_i\right) + \mu_i$

Divergence Problem

- Structured matrix H
- Inaccurate model assumption in denoising problem



AMP-3D-Wiener

Initialize $f^0 \leftarrow \mathbf{0}$

At iteration t, do

Residual: $r^t \leftarrow g - Hf^t + \frac{r^{t-1}}{m/n} \langle \eta'_{t-1}(f^{t-1} + H^T r^{t-1}) \rangle$

Noisy image: $q^t \leftarrow f^t + H^T r^t$

Noise level: $\sigma_t^2 \leftarrow ||r^t||_2^2/m$

Denoising: $f^{t+1} \leftarrow \eta_t(q^t; \sigma_t^2)$

3D-Wiener

Damping: $f^{t+1} \leftarrow \alpha \cdot f^{t+1} + (1-\alpha) \cdot f^t$

$(0 < \alpha \leq 1)$

Numerical Results

Numerical Results

Lego toy example

Original

Iteration 1

Numerical Results

Lego toy example

Original

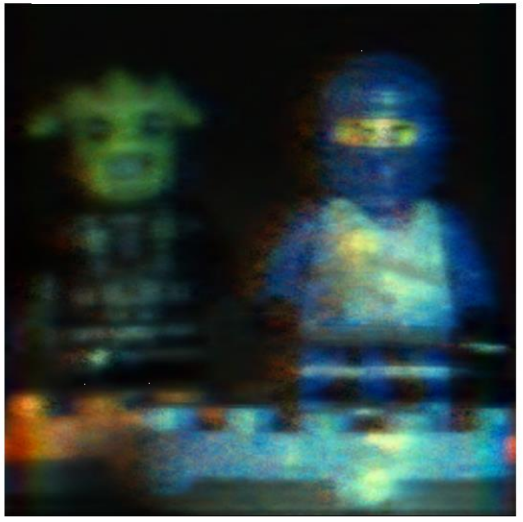
Iteration 4

Numerical Results

Lego toy example

Original

Iteration 7



Numerical Results

Lego toy example

Original

Iteration 10

Numerical Results

Lego toy example

Original

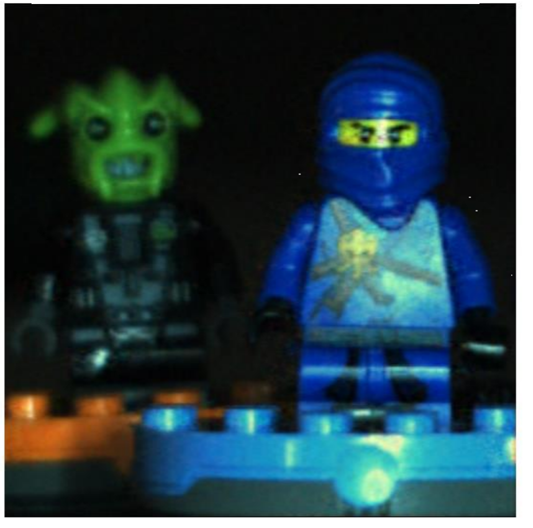
Iteration 20

Numerical Results

Lego toy example

Original

Iteration 50



Numerical Results

Lego toy example

Original

Iteration 100

Numerical Results

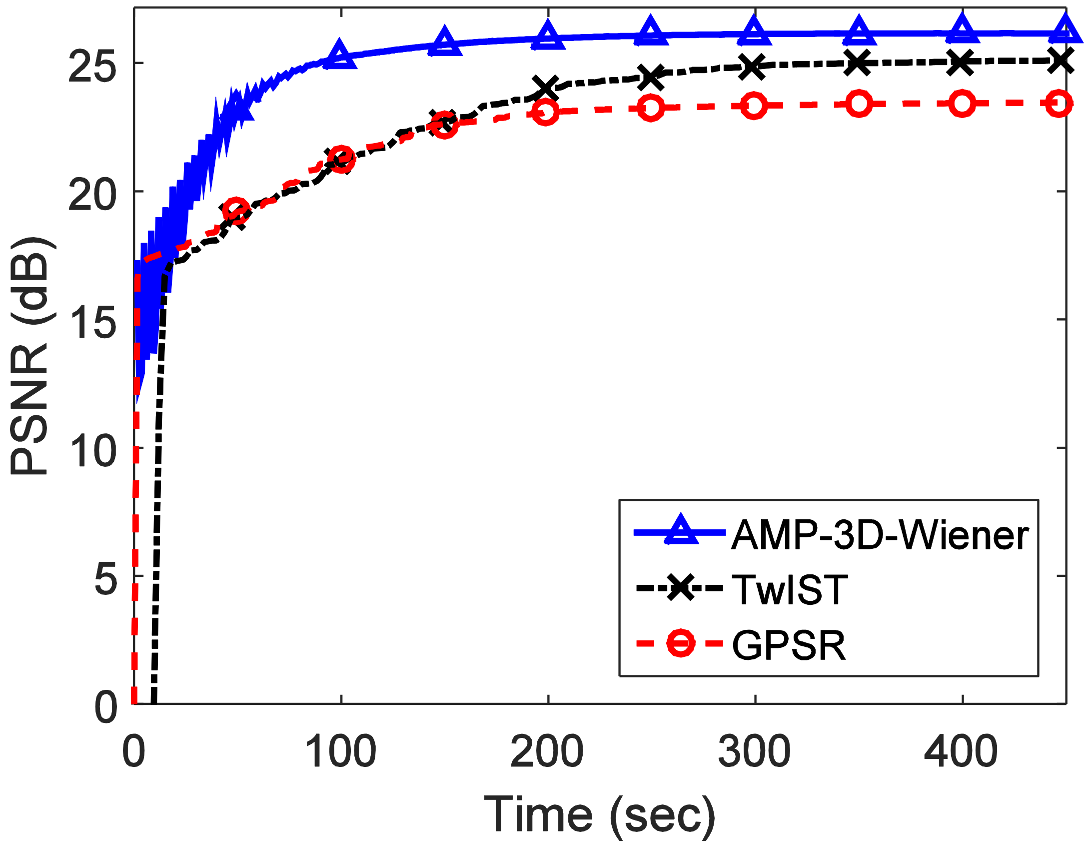
Lego toy example

Original

Iteration 400

Numerical Results

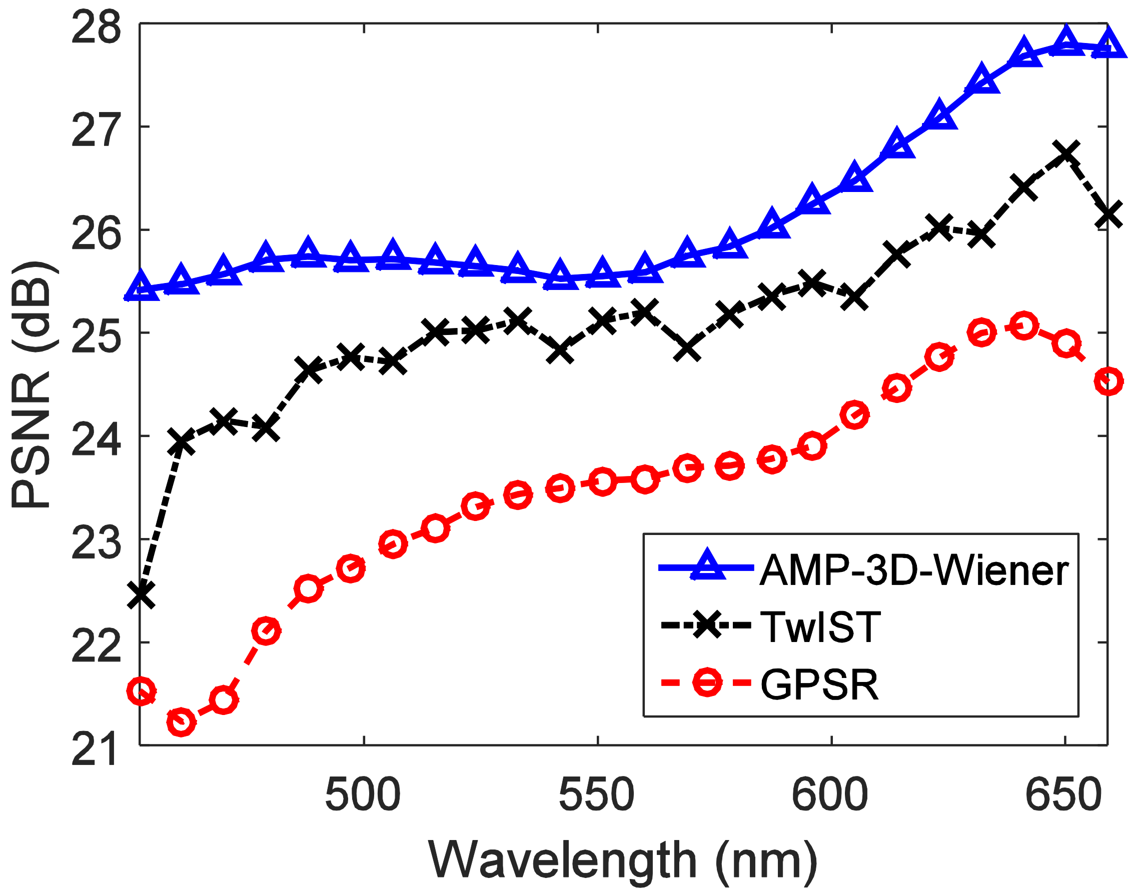
- Lego toy example
- 2 shots; complementary coded aperture; 20dB noise
- No parameter tuning for AMP-3D-Wiener



TwIST [Bioucas-Dias & Figueiredo 2007, Wagadarikar 2008]
GPSR [Figueiredo et al. 2007]

Numerical Results

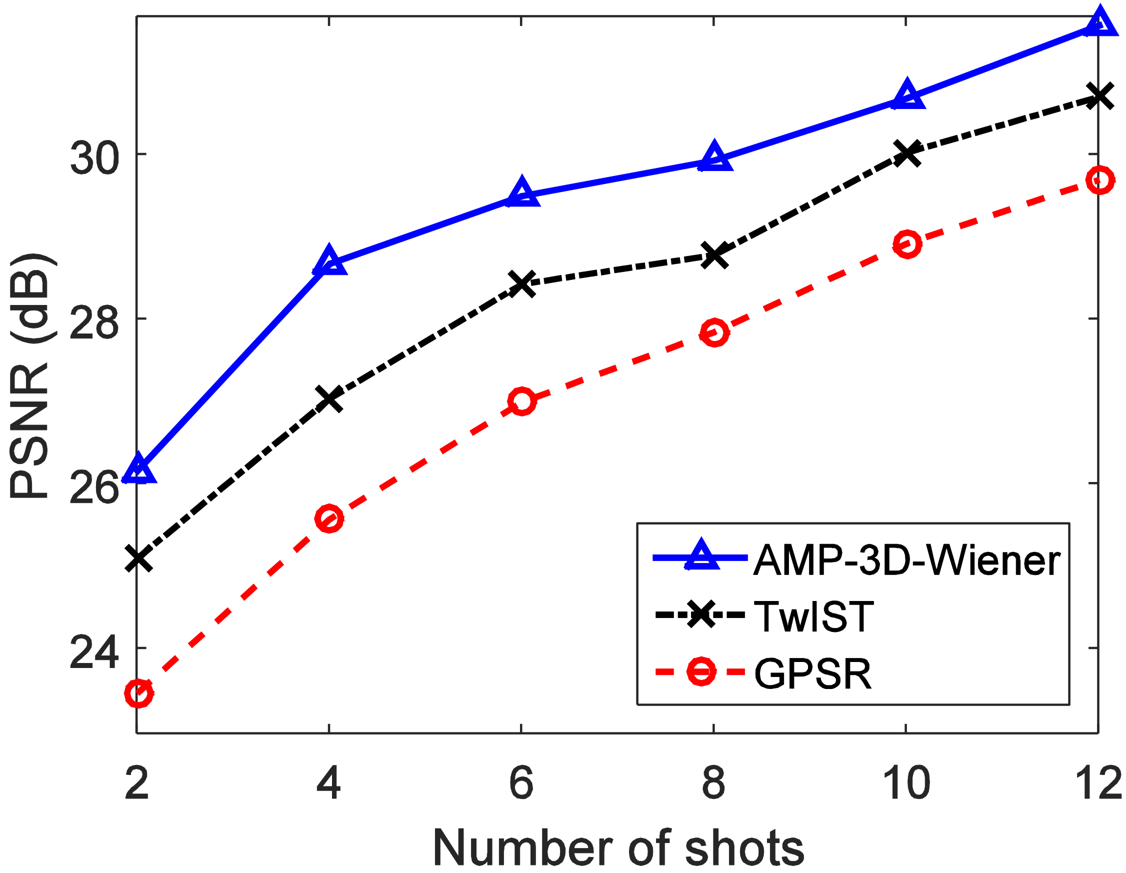
- Lego toy example
- 2 shots; complementary coded aperture; 20dB noise



TwIST [Bioucas-Dias & Figueiredo 2007, Wagadarikar 2008]
GPSR [Figueiredo et al. 2007]

Numerical Results

- Lego toy example
- 2-12 shots; complementary coded aperture; 20dB noise



TwIST [Bioucas-Dias & Figueiredo 2007, Wagadarikar 2008]
GPSR [Figueiredo et al. 2007]

Numerical Results

Natural scenes [personalpages.manchester.ac.uk/staff/d.h.foster/]

AMP reconstructs better in all tested scenes.

SNR	15 dB			20 dB		
Algorithm	AMP	GPSR	TwIST	AMP	GPSR	TwIST
Scene 1	**30.48**	28.43	30.17	**30.37**	28.53	30.31
Scene 2	**27.34**	24.71	27.03	**27.81**	24.87	27.35
Scene 3	**33.13**	29.38	31.69	**33.12**	29.44	31.75
Scene 4	**32.07**	26.99	31.69	**32.14**	27.25	32.08
Scene 5	**27.44**	24.25	26.48	**27.83**	24.60	26.85
Scene 6	**29.15**	24.99	25.74	**30.00**	25.53	26.15
Scene 7	**36.35**	33.09	33.59	**37.11**	33.55	34.05
Scene 8	**32.12**	28.14	28.22	**32.93**	28.82	28.69

TwIST [Bioucas-Dias & Figueiredo 2007, Wagadarikar 2008]
GPSR [Figueiredo et al. 2007]

Summary

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- **Problem**: Hyperspectral image reconstruction in CASSI
- **Algorithm**: Approximate message passing with adaptive Wiener filter in 2D wavelet + 1D DCT domain
- **Challenges**:
  - Highly compressed measurements
  - Structured sensing matrix
- **Results**:
  - Improved PSNR and runtime
  - No parameter tuning

Future Work

- Why simple denoiser helps convergence?
- Improve AMP convergence

→ better denoiser (e.g. BM4D [Maggioni et al. 2012])

Thank you!