TOTAL VARIATION REGULARIZED REWEIGHTED LOW-RANK TENSOR COMPLETION FOR COLOR IMAGE INPAINTING

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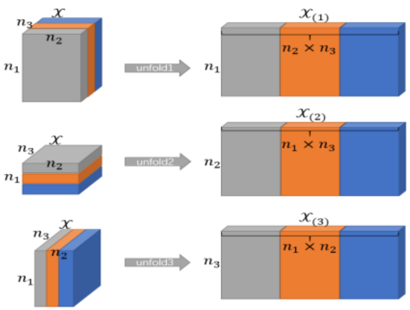
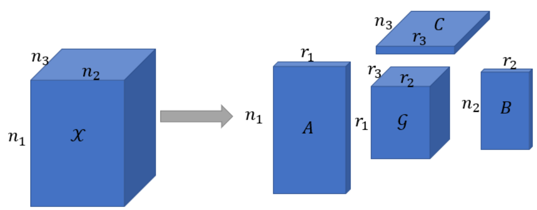
Outline

Shortcomings of traditional tensor completion methods:
Treat each dimension of tensors equally ➔ ignore the difference among dimensions
Advantages of proposed reweighted low-rank tensor completion methods:
Consider the difference of each dimension➔ capture the intrinsic physical meaning

tensor unfolding and Tucker decomposition

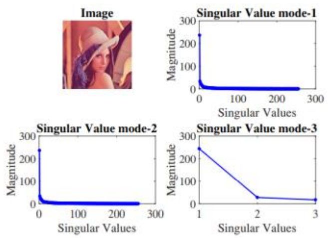




$\mathcal{X} \approx \mathcal{G} \times_1 A \times_2 B \times_3 C$

$$x_{ijk} \approx \sum_{p=1}^{P}\sum_{q=1}^{Q}\sum_{r=1}^{R} g_{pqr}\, a_{ip} b_{jq} c_{kr} \quad \text{for} \quad i=1,\ldots,I,\; j=1,\ldots,J,\; k=1,\ldots,K.$$

difference among each dimension of a tensor



According to Fig. 1, it is obvious that the distributions of singular values are varied among the spatial and channel modes. Specifically, the matrices unfolded by two spatial modes are low-rank, but the matrix unfolded by channel is nearly full rank, i.e., the rank is 3. Therefore, we only impose the low-rank constraints on spatial modes, which can truthfully capture properties among different modes.

TV Regularized Reweighted Low-rank Tensor Completion based on tensor unfolding

$$\min_{\mathcal{Z}} \quad \lambda \sum_{i=1}^{2} \|F_{(i)} Z_{(i)}\|_1 + \sum_{i=1}^{2} \|Z_{(i)}\|_*$$
$$\text{s.t.} \quad [\mathcal{Z}]_\Omega = [\mathcal{Y}]_\Omega \,,$$

$F_i \in \mathbb{R}^{(n_i-1)\times n_i}, [F_i]_{j,j} = 1, [F_i]_{j,j+1} = -1$ is the total variation regularized matrix.

TV Regularized Reweighted Low-rank Tensor Completion based on Tucker decomposition

$$\min_{\mathcal{Z}} \quad \lambda_1 \sum_{i=1}^{2} \|F_i Z_{(i)}\|_1 + \sum_{i=1}^{2} \|U_i\|_* + \lambda_2 \|\mathcal{G}\|_F^2$$
$$s.t. \quad \mathcal{Z} = \mathcal{G} \times_1 U_1 \times_2 U_2 \times_3 U_3,$$
$$[\mathcal{Z}]_\Omega = [\mathcal{Y}]_\Omega \,,$$

$U_i, i = 1,2,3$ are the factor matrices and can be thought of as the principal components in each mode.

alternating direction method of multipliers (ADMM)

$$\mathcal{L} = \sum_{i=1}^{2}\left(\lambda\|Q_i\|_1 + \frac{\rho_1}{2}\|Q_i - F_i R_i + \frac{\Lambda_i}{\rho_1}\|_F^2\right) + \sum_{i=1}^{2}\left(\frac{\rho_2}{2}\|R_i - Z_{(i)} - \frac{\Phi_i}{\rho_2}\|_F^2\right) + \sum_{i=1}^{2}\left(\|M_i\|_* + \frac{\rho_3}{2}\left\|M_i - Z_{(i)} + \frac{\Psi_i}{\rho_3}\right\|\right)$$

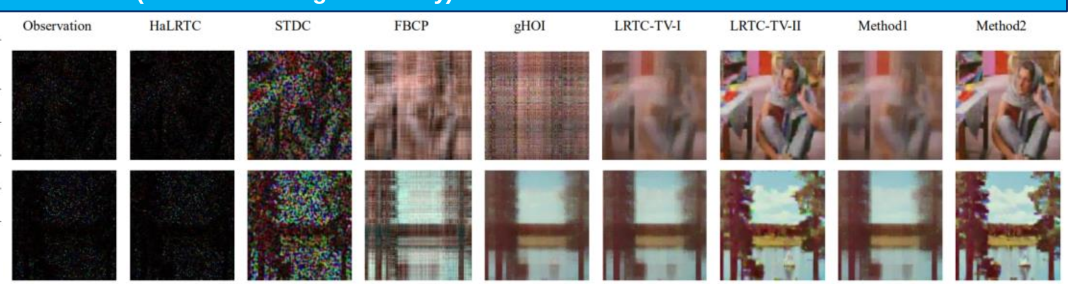
$$\mathcal{L} = \sum_{i=1}^{2}\left(\lambda_1\|Q_i\|_1 + \frac{\rho_1}{2}\left\|Q_i - F_i R_i + \frac{\Lambda_i}{\rho_1}\right\|_F^2\right) + \sum_{i=1}^{2}\left(\frac{\rho_2}{2}\left\|R_i - Z_{(i)} - \frac{\Phi_i}{\rho_2}\right\|_F^2\right) + \frac{1}{2}\sum_{i=1}^{2}\|U_i\|_* + \sum_{i=1}^{2}\frac{\rho_3}{2}\left\|V_i - U_i + \frac{\Psi_i}{\rho_3}\right\|_F^2 + \lambda_2\|\mathcal{G}\|_F^2 + \frac{\rho_4}{2}\left\|\mathcal{Z} - \mathcal{G}\times_1 V_2 \times_2 V_2 \times_3 U_3 + \frac{\mathcal{P}}{\rho_4}\right\|_F^2 \quad ($$

ground truth of experiment

$256 \times 256 \times 3$

Experimental results (RGB-color image recovery)

Ratio	Metrics	HaLRTC	STDC	LRTC-TV-I	LRTC-TV-II	gHOI	FBCP	Method 1	Method 2
0.4	PSNR	27.32	28.17	28.51	28.80	22.16	25.95	28.57	29.79
	RSE	0.0788	0.0716	0.0682	0.0671	0.1416	0.0927	0.0679	0.0611
0.3	PSNR	25.13	26.87	26.62	26.91	21.37	24.43	26.67	27.45
	RSE	0.1006	0.0838	0.0840	0.0822	0.1551	0.1101	0.0837	0.0783
0.2	PSNR	8.20	24.69	24.44	25.49	20.59	22.68	24.50	25.57
	RSE	0.7949	0.1072	0.1072	0.0956	0.1694	0.1332	0.1065	0.0968
0.1	PSNR	5.29	18.89	21.14	23.22	19.06	20.04	21.34	23.63
	RSE	0.9489	0.2006	0.1557	0.1237	0.2024	0.1802	0.1521	0.1192
0.05	PSNR	5.06	8.30	18.03	21.24	14.37	17.72	18.51	21.89
	RSE	0.9748	0.6715	0.2235	0.1557	0.3382	0.2363	0.2110	0.1431



Compare with recently tensor completion methods: HaLRTC(ICCV2009), gHOI(NIPS2014), FBCP(TPAMI2015), STDC(TPAMI2014), LRTC-TV(AAAI2017)