Sequential Closed-Form Semiblind Receiver for Space-Time Coded Multihop Relaying Systems

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- Introduction
- System model
- Proposed closed-form semiblind receiver
- Simulation assumptions and results
- Conclusions and perspective
- References

• Cooperative relaying are expected to play an important role in 5G systems, e.g., multihop use cases of V2X systems

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- Assuming a simplified KRST coding scheme, the signals received at destination satisfy a (K+3)th-order generalized nested PARAFAC tensor model
- The generalized nested PARAFAC model can be decomposed into K+1 third-order PARAFAC models
- Assuming the coding matrices known, a closed-form semiblind receiver based on rank-one matrix approximations is derived for jointly estimating the information symbols and the individual channels



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- $\tilde{\mathcal{X}} = \mathcal{X} + \mathcal{N}$ is the noisy received signal tensor



Encoded symbols

$$\mathbf{X}_{M_0 \times P_0 N}^{(0)} = \left(\mathbf{G}_0 \diamond \mathbf{S}\right)^T \tag{1}$$

- The symbol matrix $\mathbf{S} \in \mathbb{C}^{N \times M_0}$ containing N data-streams composed of M_0 symbols is multiplexed by M_0 transmit antennas at the source
- Source and relays encode the signals with a KRST coding matrix $\mathbf{G}_k \in \mathbb{C}^{P_k \times M_k}$, chosen as a truncated DFT matrix $\mathbf{G}_k^T \mathbf{G}_k^* = \mathbf{I}_{M_k}$, $(k = 0, \cdots, K)$

First Hop



Received signal at Relay-1

$$\tilde{\mathbf{X}}_{M_{1}\times P_{0}N}^{(1)} = \mathbf{H}_{1} \left(\mathbf{G}_{0} \diamond \mathbf{S} \right)^{T} + \mathbf{N}_{M_{1}\times P_{0}N}^{(1)}$$

- Signals received at relay-1 define a third-order tensor $\tilde{\mathcal{X}}^{(1)} \in \mathbb{C}^{M_1 \times P_0 \times N}$ satisfying a PARAFAC model $\|\mathbf{H}_1, \mathbf{G}_0, \mathbf{S}; M_0\|$
- $ilde{\mathbf{X}}_{M_1 imes P_0 N}^{(1)}$ represents the flat mode-1 unfolding of $ilde{\mathcal{X}}^{(1)}$

Second Hop



Received signal at Relay-2

$$\tilde{\mathbf{X}}_{M_2 \times P_1 P_0 N}^{(2)} = \mathbf{H}_2 \left(\mathbf{G}_1 \diamond \tilde{\mathbf{X}}_{P_0 N \times M_1}^{(1)} \right)^T + \mathbf{H}_2 \left(\mathbf{G}_1 \diamond \mathbf{N}_{P_0 N \times M_1}^{(1)} \right)^T + \mathbf{N}_{M_2 \times P_1 P_0 N}^{(2)} \mathbf{1}$$

- The signals received at relay-2 define a fourth-order tensor $\tilde{\mathcal{X}}^{(2)} \in \mathbb{C}^{M_2 \times P_1 \times P_0 \times N}$
- $ilde{\mathbf{X}}_{M_2 imes P_1 P_0 N}^{(2)}$ represents the flat mode-1 unfolding of $ilde{\mathcal{X}}^{(2)}$

_{5/17} ¹From now on, a noiseless formulation

Third Hop



Received signal at Destination

$$\mathbf{X}_{M_{3}\times P_{2}P_{1}P_{0}N}^{(3)}=\mathbf{H}_{3}\left(\mathbf{G}_{2}\diamond\mathbf{X}_{P_{1}P_{0}N\times M_{2}}^{(2)}\right)^{T}$$

- The signals received at the destination define a fifth-order tensor $\mathcal{X}^{(3)} \in \mathbb{C}^{M_3 \times P_2 \times P_1 \times P_0 \times N}$
- $\mathbf{X}_{M_3 \times P_2 P_1 P_0 N}^{(3)}$ is the flat mode-1 unfolding of tensor $\mathcal{X}^{(3)}$ following a PARAFAC decomposition

Third Hop



Received Signal at Destination

• Replacing $\mathbf{X}_{P_1P_0N \times M_2}^{(2)}$ and then $\mathbf{X}_{P_0N \times M_1}^{(1)}$, tensor $\mathcal{X}^{(3)}$ also satisfies a generalized nested PARAFAC decomposition

$$\mathbf{X}_{M_{3}\times P_{2}P_{1}P_{0}N}^{(3)} = \mathbf{H}_{3} \left[\mathbf{G}_{2} \diamond \qquad \left(\mathbf{G}_{1} \diamond \left(\mathbf{G}_{0} \diamond \mathbf{S} \right) \mathbf{H}_{1}^{T} \right) \mathbf{H}_{2}^{T} \right]^{T}$$

Received Signal at Destination



• Another unfolding of the PARAFAC model $\mathcal{X}^{(3)} \in \mathbb{C}^{M_3 imes P_2 imes P_1 imes P_0 imes N}$ is

$$\mathbf{X}_{M_3P_2 \times P_1P_0N}^{(3)} = (\mathbf{H}_3 \diamond \mathbf{G}_2) \, \mathbf{X}_{M_2 \times P_1P_0N}^{(2)}$$

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• Replacing $\mathbf{X}_{M_2 \times P_1 P_0 N}^{(2)} = \mathbf{H}_2 \left(\mathbf{G}_1 \diamond \mathbf{X}_{P_0 N \times M_1}^{(1)} \right)^T$ leads to

$$\mathbf{X}_{M_{3}P_{2}\times P_{1}P_{0}N}^{(3)} = \left(\mathbf{H}_{3} \diamond \mathbf{G}_{2}\right) \mathbf{H}_{2} \left(\mathbf{G}_{1} \diamond \mathbf{X}_{P_{0}N \times M_{1}}^{(1)}\right)^{T}$$

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$$\mathbf{X}_{M_{3}P_{2}\times P_{1}P_{0}N}^{(3)} = \underbrace{(\mathbf{H}_{3}\diamond\mathbf{G}_{2})\mathbf{H}_{2}}_{\mathbf{H}_{M_{3}P_{2}\times M_{1}}^{(1\mapsto3)}} \begin{pmatrix} \mathbf{G}_{1}\diamond\mathbf{X}_{P_{0}N\times M_{1}}^{(1)} \end{pmatrix}^{T}$$

• $\mathbf{H}_{M_3P_2 \times M_1}^{(1 \mapsto 3)}$ is a unfolding of the third-order effective channel tensor $\mathcal{H}^{(1 \mapsto 3)} \in \mathbb{C}^{M_3 \times P_2 \times M_1}$ linking the relay-1 and the destination (node-3)

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• The fifth-order nested PARAFAC model is decomposed into three third-order PARAFAC models from $\mathcal{X}^{(3)},\,\mathcal{H}^{(0\mapsto3)}$, and $\mathcal{H}^{(1\mapsto3)}$ as

$$\mathbf{X}_{NM_{3}P_{2}P_{1}\times P_{0}}^{(3)} = \left(\mathbf{S} \diamond \mathbf{H}_{M_{3}P_{2}P_{1}\times M_{0}}^{(0\mapsto3)}\right) \mathbf{G}_{0}^{T} = \mathbf{R}\mathbf{G}_{0}^{T}$$

$$\mathbf{H}_{M_{3}P_{2}M_{0}\times P_{1}}^{(0\mapsto3)} = \left(\mathbf{H}_{M_{3}P_{2}\times M_{1}}^{(1\mapsto3)} \diamond \mathbf{H}_{1}^{T}\right) \mathbf{G}_{1}^{T} = \mathbf{Q}\mathbf{G}_{1}^{T}$$

$$\mathbf{H}_{M_{3}M_{1}\times P_{2}}^{(1\mapsto3)} = \left(\mathbf{H}_{3} \diamond \mathbf{H}_{2}^{T}\right) \mathbf{G}_{2}^{T} = \mathbf{Z}\mathbf{G}_{2}^{T}$$

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As G^T_kG^{*}_k = I_{M_k} for k = 0, · · · , K allows us to derive a three-step closed-form semiblind receiver to estimate the symbol matrix S and the channel matrices H_k, k = 1, 2, 3

• Using the column orthonormality of \mathbf{G}_k , k = 0, 1, 2, the least squares (LS) estimates of $(\mathbf{R}, \mathbf{Q}, \mathbf{Z})$ can be successively calculated as

$$\hat{\mathbf{R}} = \mathbf{X}_{NM_3P_2P_1 \times P_0}^{(3)} \mathbf{G}_0^* \cong \left(\mathbf{S} \diamond \mathbf{H}_{M_3P_2P_1 \times M_0}^{(0 \mapsto 3)} \right),$$
(2)

$$\hat{\mathbf{Q}} = \hat{\mathbf{H}}_{M_3P_2M_0 \times P_1}^{(0 \mapsto 3)} \mathbf{G}_1^* \cong \left(\mathbf{H}_{M_3P_2 \times M_1}^{(1 \mapsto 3)} \diamond \mathbf{H}_1^T \right), \qquad (3)$$

$$\hat{\mathbf{Z}} = \hat{\mathbf{H}}_{M_3M_1 \times P_2}^{(1 \mapsto 3)} \mathbf{G}_2^* \cong \left(\mathbf{H}_3 \diamond \mathbf{H}_2^T\right), \tag{4}$$

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$$\hat{\mathbf{Q}} = \hat{\mathbf{H}}_{M_3 P_2 M_0 \times P_1}^{(0 \mapsto 3)} \mathbf{G}_1^* \cong \left(\mathbf{H}_{M_3 P_2 \times M_1}^{(1 \mapsto 3)} \diamond \mathbf{H}_1^T \right), \tag{3}$$

$$\hat{\mathbf{Z}} = \hat{\mathbf{H}}_{M_3M_1 \times P_2}^{(1 \mapsto 3)} \mathbf{G}_2^* \cong \left(\mathbf{H}_3 \diamond \mathbf{H}_2^T\right), \tag{4}$$

• From the LS estimate $\hat{\mathbf{R}}$, the factors S and $\mathbf{H}_{M_3P_2P_1 \times M_0}^{(0 \mapsto 3)}$ are found by the Khatri-Rao factorization (KRF) algorithm

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- From the LS estimate $\hat{\mathbf{R}}$, the factors \mathbf{S} and $\mathbf{H}_{M_3P_2P_1 \times M_0}^{(0 \mapsto 3)}$ are found by the Khatri-Rao factorization (KRF) algorithm
- Then, the estimate $\hat{\mathbf{H}}_{M_3P_2P_1 \times M_0}^{(0 \mapsto 3)}$ is reshaped as $\hat{\mathbf{H}}_{M_3P_2M_0 \times P_1}^{(0 \mapsto 3)}$ to compute $\hat{\mathbf{Q}}$, from which the factors \mathbf{H}_1 and $\mathbf{H}_{M_3P_2 \times M_1}^{(1 \mapsto 3)}$ are extracted by applying the KRF algorithm

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- Finally, $\hat{\mathbf{H}}_{M_3P_2 \times M_1}^{(1 \mapsto 3)}$ is reshaped as $\hat{\mathbf{H}}_{M_3M_1 \times P_2}^{(1 \mapsto 3)}$ to compute $\hat{\mathbf{Z}}$, from which the channels $(\mathbf{H}_2, \mathbf{H}_3)$ are extracted using again the KRF algorithm

General Case K relays

• The signals received at the destination defines a (K+3)th-order tensor $\tilde{\mathcal{X}}^{(K+1)} \in \mathbb{C}^{M_{K+1} \times P_K \times P_{K-1} \times \cdots \times P_0 \times N}$, with flat mode-1 unfolding as

 $\mathbf{X}_{M_{K+1} \times P_K P_{K-1} \cdots P_0 N}^{(K+1)} = \mathbf{H}_{K+1} \left(\mathbf{G}_K \diamond \mathbf{X}_{P_{K-1} \cdots P_0 N \times M_K}^{(K)} \right)^T$

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• The equivalent third-order PARAFAC model $\|\mathbf{H}_{M_{K+1}P_{K}\cdots P_{1}\times M_{0}}^{(0\mapsto K+1)}, \mathbf{G}_{0}, \mathbf{S}; M_{0}\|$ for $\mathcal{X}^{(K+1)}$ is

$$\mathbf{X}_{NM_{K+1}P_{K}\cdots P_{1}\times P_{0}}^{(K+1)} = \left(\mathbf{S} \diamond \mathbf{H}_{M_{K+1}P_{K}\cdots P_{1}\times M_{0}}^{(0\mapsto K+1)}\right) \mathbf{G}_{0}^{T}$$

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• From $\mathbf{H}_{M_{K+1}P_K\cdots P_1 \times M_0}^{(0 \mapsto K+1)}$ we can define other K third-order PARAFAC models to estimate the individual channel hops as

 $\mathbf{H}_{M_{K+1}P_{K}\cdots P_{k+2}M_{k}\times P_{k+1}}^{(k\mapsto K+1)} = (\mathbf{H}_{M_{K+1}P_{K}\cdots P_{k+2}\times M_{k+1}}^{(k\mapsto K+1)} \diamond \mathbf{H}_{k+1}^{T})\mathbf{G}_{k+1}^{T}$

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- The performance criteria are the SER and the NMSE of the estimated channels averaged over 4×10^4 Monte Carlo runs.
- Each run corresponds to a realization of all channel and symbol matrices, and noise tensors
- The transmitted symbols are randomly drawn from a unit energy QAM symbol alphabet
- We assume \mathbf{H}_{k+1} , $k = 0, \cdots, K$, i.i.d. zero-mean circularly-symmetric complex Gaussian entries with variances given by $1/\eta^{\beta}M_k$, where $\eta = d/d_0 = 1/(K+1)$ and $\beta = 3$, d denoting the distance between two consecutive nodes, and d_0 the distance between the source and the destination

Simulation Results

SER: 4-QAM, K = 1:3 relays



Figure 2: SER for three different numbers of relays and two system configurations.

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Simulation Results

NMSE: 4-QAM, K = 2 relays, $P_k = M_k = N = 3$



Figure 3: NMSE of the individual channels with K = 2.

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- Diversity gains are obtained as the number of relays increases from K = 1 to K = 3 due to the KRST coding at each relay. Increasing the values of N, P_k and M_k from 2 to 4 also leads to better SER performance
- Extensions of this work include the exploitation of the noise structure to develop a tensor-based receiver using a minimum-mean-square-error (MMSE) algorithm, and the case of orthogonal frequency division multiplexing (OFDM) relay systems

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Thank you for your attention.