Designing Constrained Projections for Compressed Sensing: Mean Errors and Anomalies with Coherence

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Introduction

A large body of existing work on projection design for compressed sensing aims to minimize a lower bound on metrics like mutual coherence or RIC. Owing to the optimization complexity involved, a relaxation of the metric considered is the average coherence μ_{avg} [1, 2]. This relaxation is a heuristic, and no theoretical bounds exist for CS with μ_{avg}. Further, optimizing on a worst-case bound is not guaranteed to improve the performance on an ensemble.

Designing constrained projections using communications-inspired methods considers energy constraints on rows of the sensing matrix [3, 4]. On the contrary, compressive imagers employing DMD arrays for acquisition impose optical constraints [5] on each element of the sensing matrix. These constraints inhibit the applicability of communications-based methods to image acquisition.

Contributions

In this work, we present

1. Evaluation of an average coherence-based design, with optical constraints, and demonstrate anomalous behavior in mutual coherences and RICs of designed matrices;
2. A novel approach to projection design optimizing on oracular MMSE and validation results on a realistic architecture, using transparent codes with quantization;
3. Comparative results showing the superiority of MMSE-based design over coherence-based design.

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Coherence-based Design

$$\tilde{\Phi} = \underset{\Phi_{ij} \in \mathcal{P}}{\arg\min} \|\Psi^T \Phi^T \Phi \Psi - \boldsymbol{I}\|_F^2$$

(projected gradient descent with multi-start)

Optical Constraints: transparency ($\in [0,1]$) & quantization ($8-\text{bit}$)

Average Coherence: relaxation of the max-norm

$m \rightarrow$	96		128	
ID ↓	Φ_0	$\tilde{\Phi}$	Φ_0	$\tilde{\Phi}$
1	19.85	20.36	19.99	20.41
2	25.95	26.45	26.02	26.49
3	19.33	20.03	19.47	20.10
4	21.42	22.34	21.55	22.47
5	18.44	18.77	18.53	18.86
6	20.33	20.57	20.40	20.63
7	26.33	27.91	26.42	28.06

Table 1: PSNR values from reconstruction of images from BSDS500 at 37.5% and 50% measurements.

m	96	128	150	175	200	250
μ_{avg}	0.082 0.078	0.070 0.067	0.065 0.063	0.060 0.057	0.056 0.053	0.051 0.050
μ_{max}	0.409 0.394	**0.339** **0.371**	0.338 0.326	**0.310** **0.315**	0.270 0.268	0.256 0.253
δ_3	**0.614** **0.701**	0.577 0.546	0.567 0.509	0.495 0.466	0.447 0.423	**0.386** **0.405**
δ_4	$\varnothing$	0.718 0.688	0.719 0.644	0.615 0.576	0.575 0.552	**0.519** **0.525**

Table 2: Simulation results of matrix descriptors for seed (top) and optimized (bottom) sensing matrices. Anomalous behavior in red.

Contrary to the expected behavior, the minimization may increase μ_{max} or RIC δ_s (Table 2, >55% matrices demonstrate anomalies). However, since descending on μ_{avg} even in the above anomalous cases offers better reconstruction (Table 1), we demonstrate examples where a decrease in μ_{max} or δ_s does not guarantee better reconstruction errors, and hence these cannot be reliable metrics for our setup.

MMSE-based Design

- Statistical Compressed Sensing framework for model-based sparsity is used. A learned GM is a good prior on natural image patches [6, 7].
- Decoder: Piecewise-Linear Estimator (PLE) is used: efficient and approximates MAP
- Optimization objective: *MMSE is not tractable!*
  - Use oracular MMSE $\mathcal{M}_\Phi$ instead – tightly approximates MMSE at high SNR [8, 9]

$$\mathcal{M}_\Phi = \sum_{j=1}^{c} \pi_j \cdot \mathbb{E}\left[\|x - \hat{x}\|^2 | \mu_j, \Sigma_j\right] = \sum_{j=1}^{c} \pi_j \mathcal{M}_{\Phi,j}$$

$$tr\left(\Sigma_j^{-1} + \Phi^T \Sigma_{\hat{\eta}} \Phi\right)^{-1}$$

(MMSE for Gaussian component j)

$$\hat{\Phi} = \underset{\Phi_{ij} \in \mathcal{P}}{\arg\min} \sum_{j=1}^{c} \pi_j \mathcal{M}_{\Phi,j}$$

Optical Constraints: transparency ($\in [0,1]$) & quantization ($8-\text{bit}$)

Evaluation

- 25 component GM prior learned on patches from BSDS500; evaluation on *unseen* patches from BSDS and INRIA Holidays
- Image acquisition using non-overlapping 16×16 patches
- ℓ_1 sparsity-based baselines: overcomplete 2D-DCT and 2D-Haar dictionaries, SPGL1 solver
- For results across measurement ratios (12.5% – 50%), noise levels (1% – 5%) and datasets, refer full-text

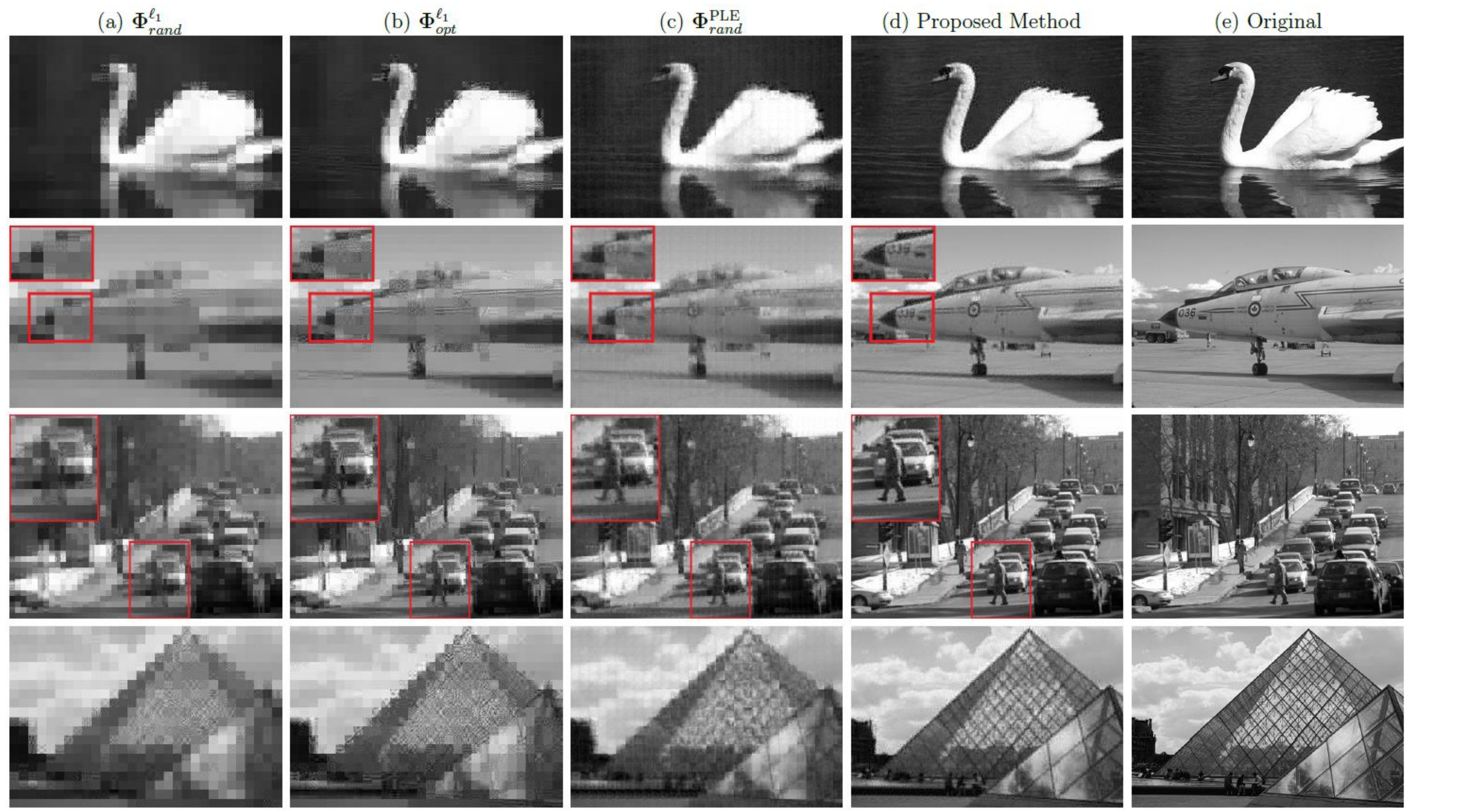

Figure 1: Sample images from BSDS500 and INRIA Holidays datasets reconstructed using 12.5% compressive measurements at 1% noise – (a) random projections, (b) coherence-optimized projections using dictionary-based sparsity; (c) random projections, and (d) oracular MMSE-optimized projections utilizing model-based sparsity.

Image #	$\Phi^{\ell_1}_{\text{rand}}$	$\Phi^{\ell_1}_{\text{opt}}$	$\Phi^{\text{PLE}}_{\text{rand}}$	Proposed
1	18.1798	18.9733	20.1748	**21.1772**
2	25.1973	25.335	26.5923	**27.2622**
3	18.3463	18.6617	19.8185	**20.9138**
4	19.8323	21.0297	21.8075	**23.0925**
5	17.6444	17.6018	18.7195	**19.6204**
6	19.9804	19.8052	20.8265	**21.5922**
7	26.1505	26.6871	27.7679	**29.0994**
8	21.8317	22.0654	23.4717	**24.5041**

Image #	$\Phi^{\ell_1}_{\text{rand}}$	$\Phi^{\ell_1}_{\text{opt}}$	$\Phi^{\text{PLE}}_{\text{rand}}$	Proposed
1	19.0832	19.9328	20.6108	**21.2366**
2	25.4462	26.0214	26.8775	**27.3121**
3	18.7209	19.6227	20.2133	**21.0092**
4	20.768	21.9571	22.296	**23.1672**
5	17.9932	18.3577	19.0837	**19.7392**
6	20.0471	20.2565	21.0641	**21.6476**
7	26.2	27.382	28.1836	**29.223**
8	22.5409	23.2168	23.9792	**24.6183**

Table 3: PSNR values from reconstruction of eight images from BSDS500 using (top) 12.5% ($m = 32$) measurements and (botom) 25% ($m = 64$) measurements. The proposed method offers the best performance across all measurement ratios.

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