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Rumour Source Detection in Social Networks using Partial Observations

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2018 6th IEEE Global Conference on Signal and Information Processing, Anaheim, California, USA

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Content

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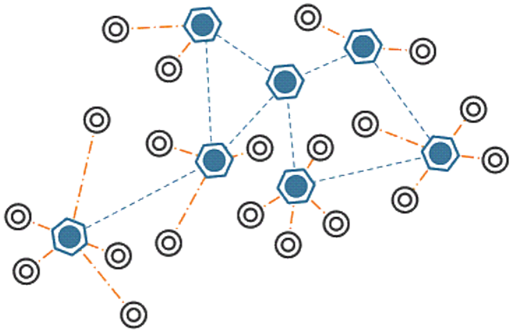
Motivation



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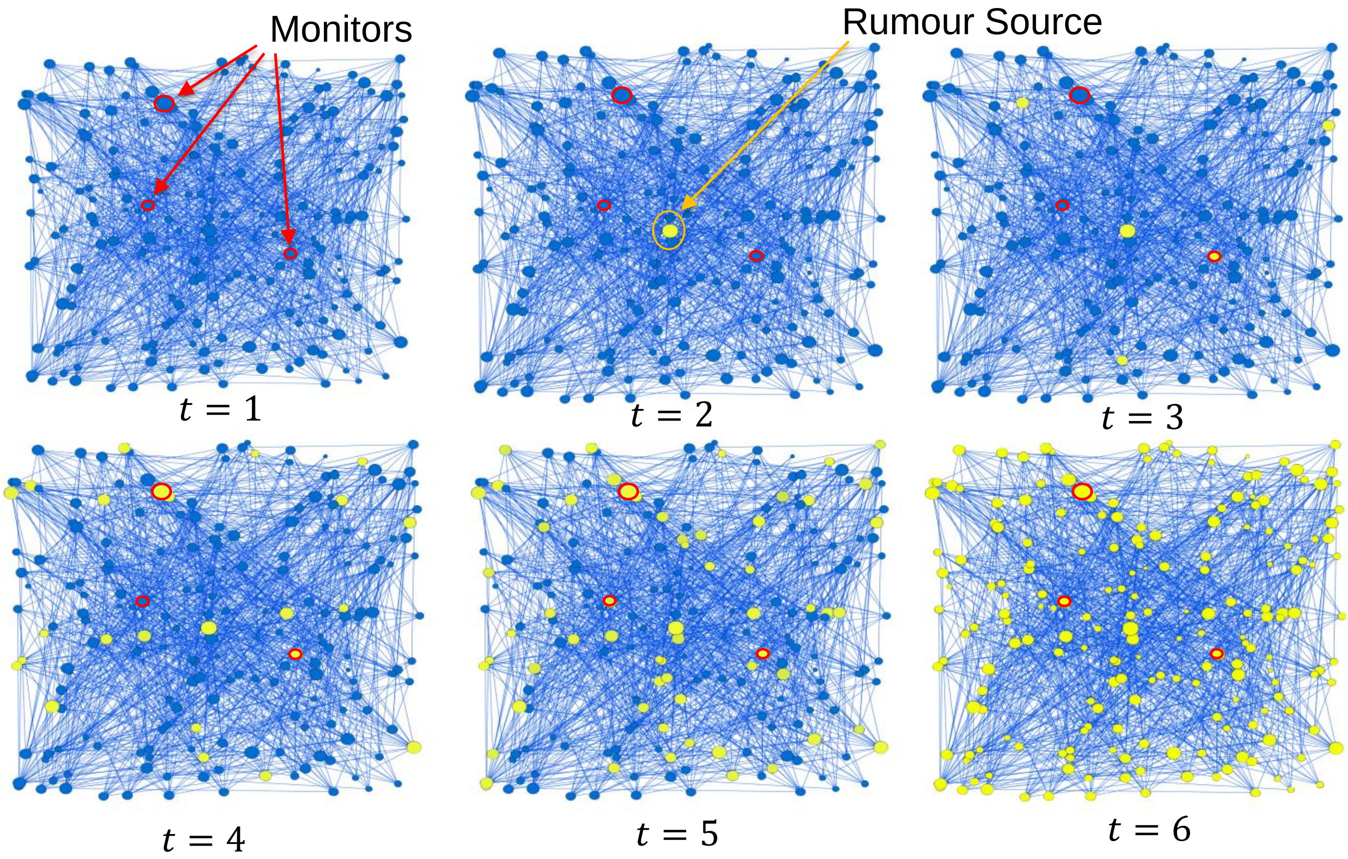
Problem Statement



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Problem Statement and Assumptions

Network topology

- General graph with small-world property.

Epidemic model

- Discrete-time version of susceptible-infected model.
- Constant transmission rate within the network.

Observation model

- Known graph topology.
- Monitoring of a small fraction of nodes.

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Problem Statement and Assumptions

Source localisation problem

- A source emits R rumours, at $t_0 = 0$.

- We observe some monitors, at discrete times $\mathrm{t} \in \{0, 1, \dots, T\}$.

- The probability of infection of a monitor i at time t is given by:

$$\tilde{F}_i(t) = \frac{R_i(t)}{R},$$

  where $R_i(t)$ is the number of rumours which have reached i by time t.

- We aim to leverage the divergence of the monitor measurements from an analytical probability of infection.

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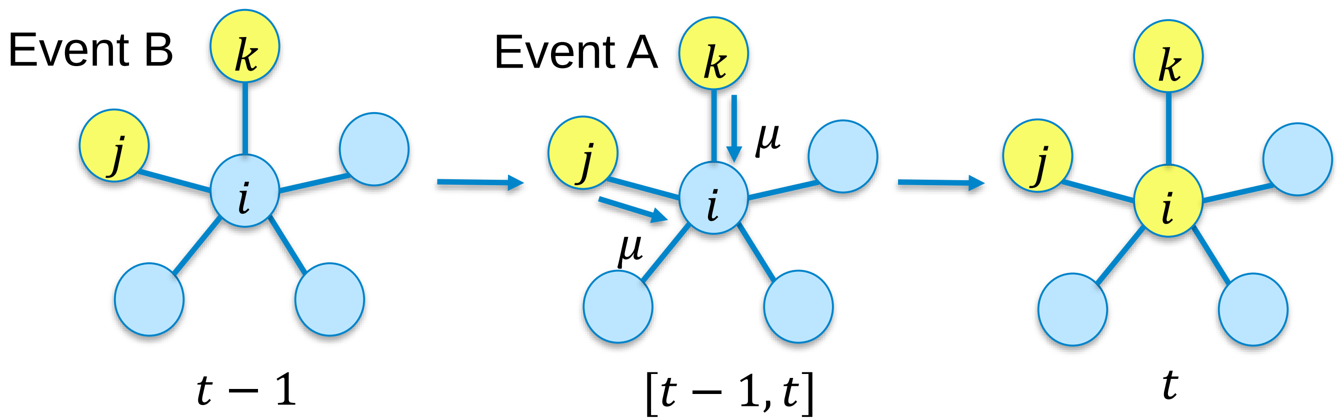
Approach I to Model Diffusion in a Network

What is the probability a node i gets first infected at time t, $f_i(t)$?



$$f_i(t) = P(A \cap B) = P(A|B)P(B)$$

μ is the constant transmission rate

Derivation in spirit with the methods presented in:

[1] M. Gomez-Rodriguez, D. Balduzzi, B. Schölkopf. *Uncovering the Temporal Dynamics of Diffusion Networks.*

[2] A. Lokhov, M. Mézard, H. Ohta, L. Zdeborová. *Inferring the origin of an epidemic with a dynamic message-passing algorithm.*

[3] N. Ruhi, H. Ahn, B. Hassibi. *Analysis of Exact and Approximated Epidemic Models over Complex Networks.*

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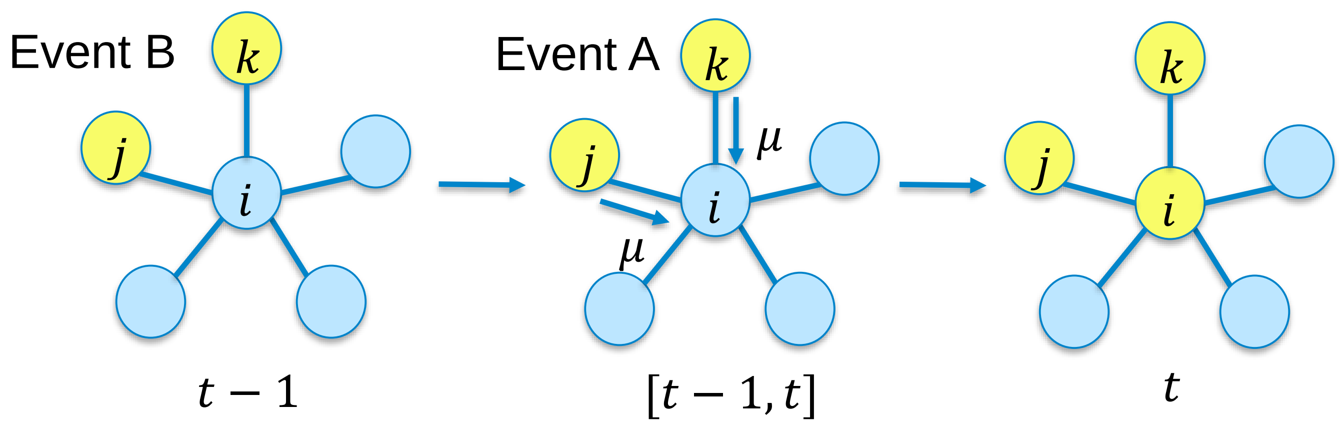
Approach I to Model Diffusion in a Network

What is the probability a node i gets first infected at time t, $f_i(t)$?



B is the event of node i being in a susceptible state at time $t-1$:

$$P(B) = \prod_{\tau=1}^{t-1} (1 - f_i(\tau))$$

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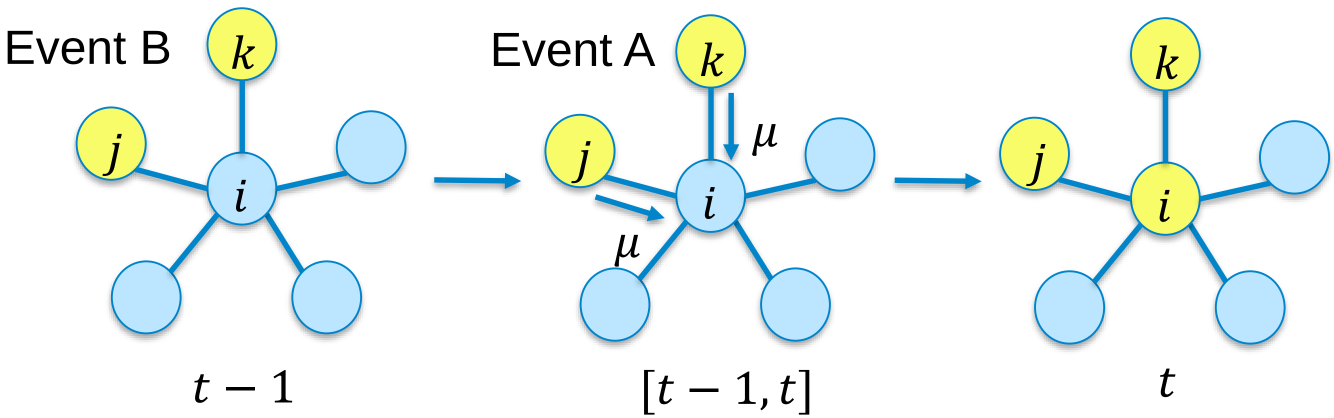
Approach I to Model Diffusion in a Network



$$P(A) = 1 - \prod_{j \in N_i} [1 - \mu \times F(x_j(t-1) = 1)]$$

neighbour j infected

neighbour j does not transmit

none of neighbours transmit

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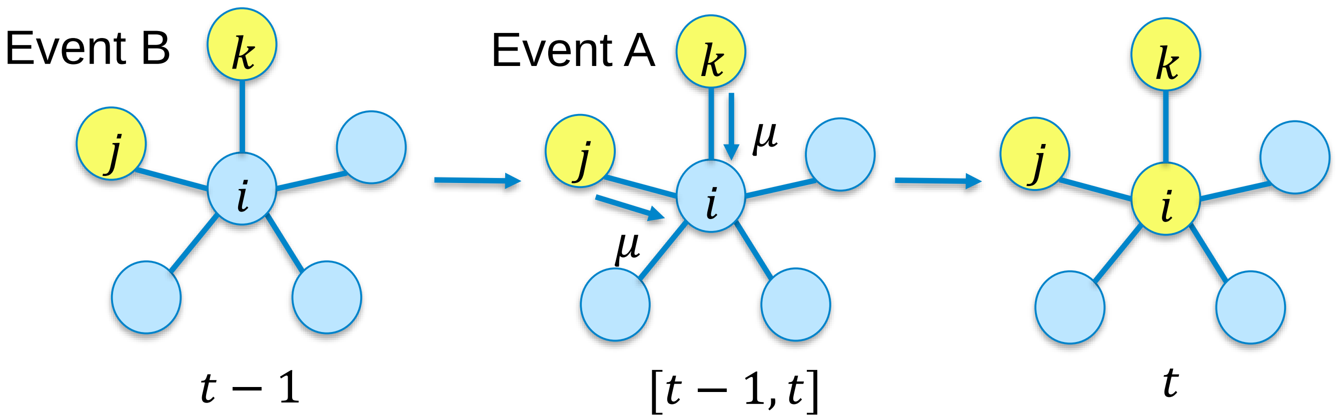
Approach I to Model Diffusion in a Network



$$P(A) = 1 - \prod_{j \in N_i} [1 - \mu \times F(x_j(t-1) = 1)]$$

The probability i gets the rumour from at least one neighbour, given i was previously in a susceptible state is:

$$P(A|B) = 1 - \prod_{j \in N_i} [1 - \mu \times F(x_j(t-1) = 1 | \boldsymbol{x_i(t-1) = 0})]$$

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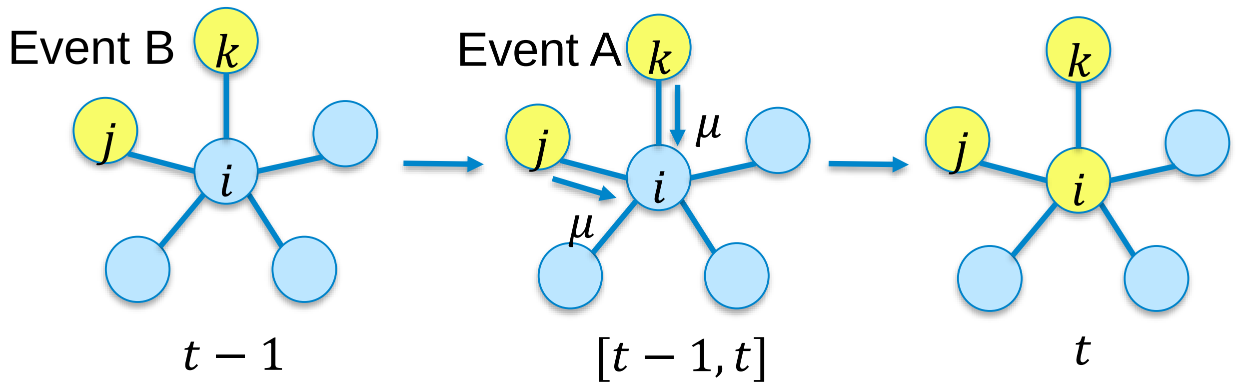
Approach I to Model Diffusion in a Network



The probability a node i gets first infected at time t, $f_i(t)$ is:

$$f_i(t) = \underbrace{\left[1 - \prod_{j \in N_i} \left(1 - \mu \times F\big(x_j(t-1) = 1 \,|\, x_i(t-1) = 0\big)\right)\right]}_{P(A|B)} \times \underbrace{\prod_{\tau=1}^{t-1} \left(1 - f_i(\tau)\right)}_{P(B)}$$

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Approach I to Model Diffusion in a Network

- The probability a node i gets first infected at time t is:

$$f_i(t) = \underbrace{[1 - \prod_{j \in N_i} (1 - \mu \times F(x_j(t-1) = 1 | x_i(t-1) = 0))]}_{P(A|B)} \times \underbrace{\prod_{\tau=1}^{t-1} (1 - f_i(\tau))}_{P(B)}$$

- We make the approximation:

$$f_i(t) \approx [1 - \prod_{j \in N_i} (1 - \mu \times F(x_j(t-1) = 1))] \times \prod_{\tau=1}^{t-1} (1 - f_i(\tau))$$

- The approximate probability a node i is infected at time t is:

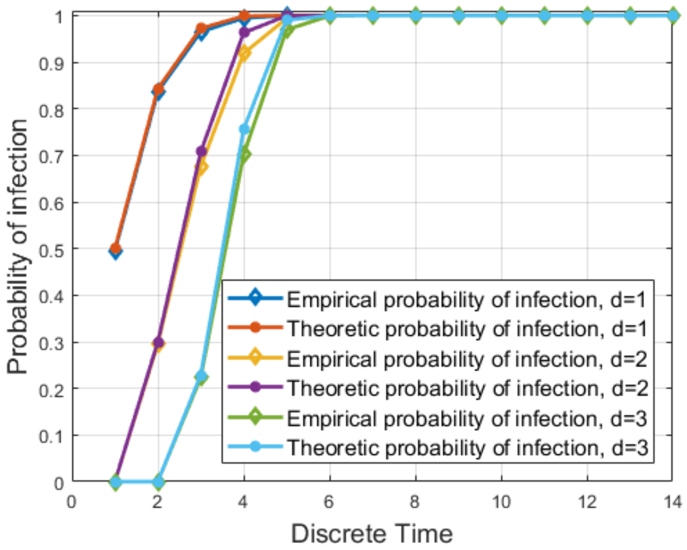
$$F_i(\tau) \approx \sum_{t=1}^{\tau} [1 - \prod_{j \in N_i} (1 - \mu \times F(x_j(t-1) = 1))] \times \prod_{\theta=1}^{t-1} (1 - f_i(\theta))$$

Approach I to Model Diffusion in a Network

- The approximate probability a node i is infected at time t is:

$$F_i(\tau) \approx \sum_{t=1}^{\tau} [1 - \prod_{j \in N_i} (1 - \mu \times F(x_j(t-1) = 1))] \times \prod_{\theta=1}^{t-1} (1 - f_i(\theta))$$

- Spreading of 1000 Rumors, small-world network, 200 Nodes, for distances 1, 2, and 3 from the source:



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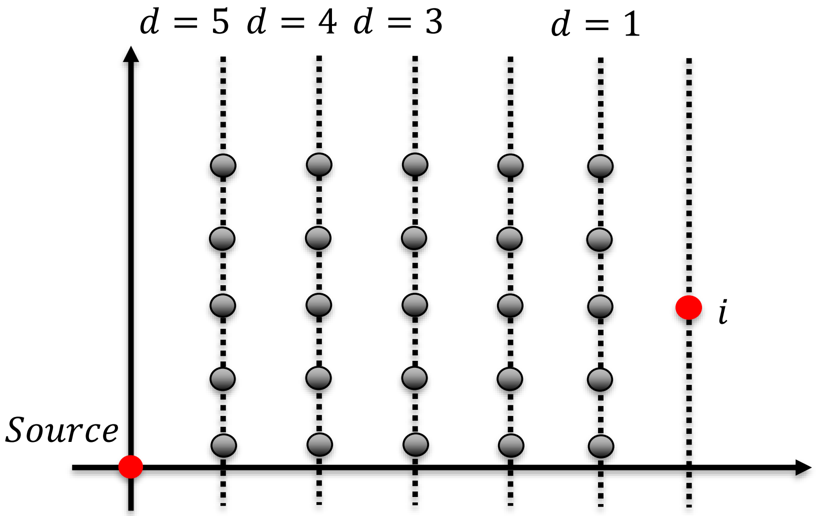
Approach II to Model Diffusion in a Network

- Probability of infection based on the shortest distance to the source.
- Arrange the nodes according to the shortest distance to the destination.
- What is the probability of first infection of a node i at distance d, at time t?



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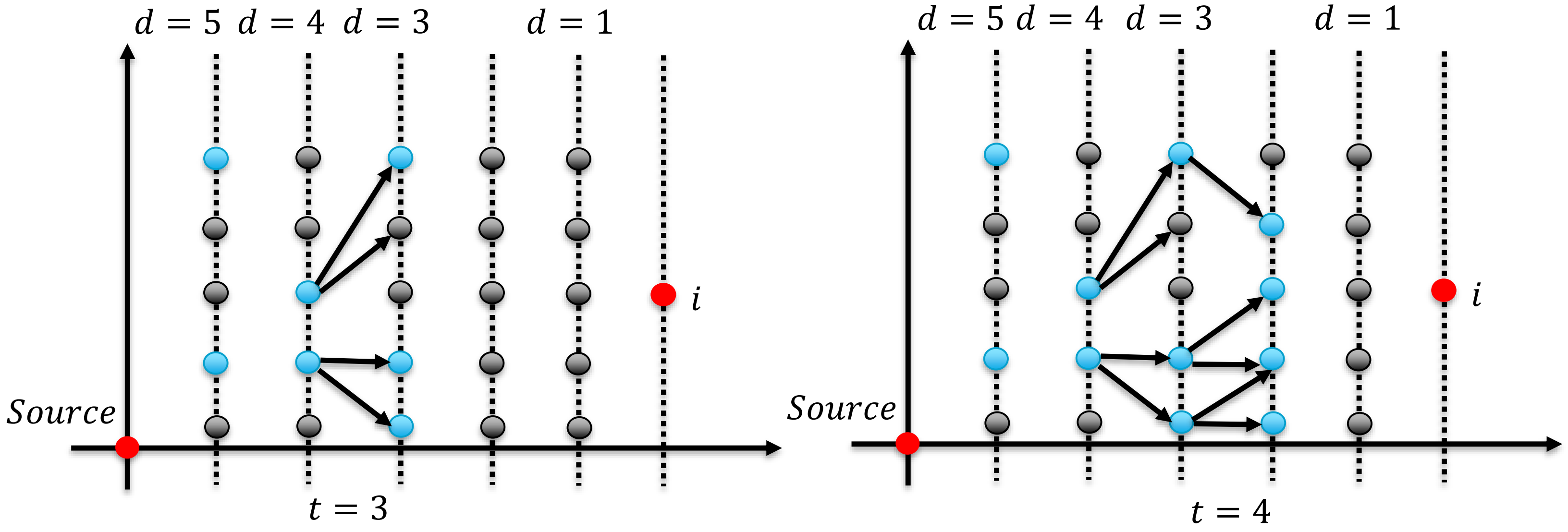
Approach II to Model Diffusion in a Network

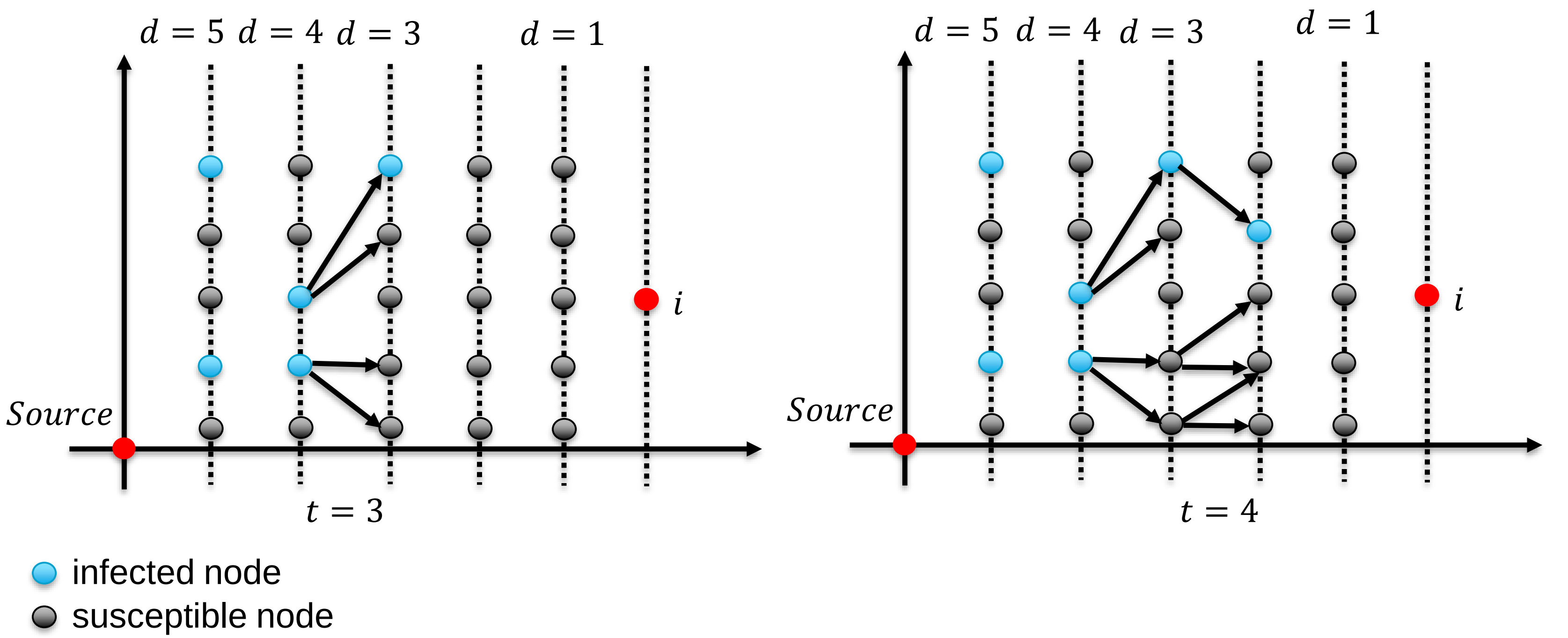
- What is the probability of first infection of a node i at distance d, at time t?
- Success: move closer to node i.



infected node
susceptible node

Approach II to Model Diffusion in a Network

- What is the probability of infection of a node i at distance d, at time t?
- Failure: not spreading the rumour to a sufficient number of nodes closer to the destination.



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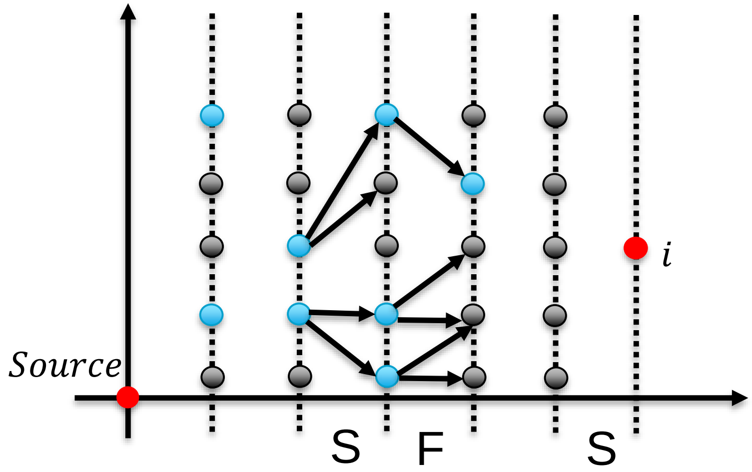
Approach II to Model Diffusion in a Network

- Number of paths is the number of ways to choose $d-1$ *success* steps of $t-1$ time steps: $\binom{t-1}{d-1}$.
- Probability of success: p_S.
- Probability of each path: $p_S^d \times (1-p_S)^{t-d}$.
- Approximate $p_S = \alpha_d \mu$, where μ is the constant transmission rate in the graph.



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Approach II to Model Diffusion in a Network

- Number of paths is: $\binom{t-1}{d-1}$.
- Probability of each path: $p_S^d \times (1-p_S)^{t-d}$.
- Set $p_S = \alpha_d \mu$, where μ is the constant transmission rate in the graph.
- The probability of first infection is:

$$f_d(t) = \underbrace{(\alpha_d \mu)^d}_{p_S} \times (1-\alpha_d\mu)^{t-d} \times \underbrace{\binom{t-1}{d-1}}_{\text{\# of paths from source to destination}}$$

- The probability of infection of a node at distance d from the source at time τ is:

$$F_d(\tau) \approx \sum_{t=d}^{\tau} (\alpha_d\mu)^d \times (1-\alpha_d\mu)^{t-d} \times \binom{t-1}{d-1}$$

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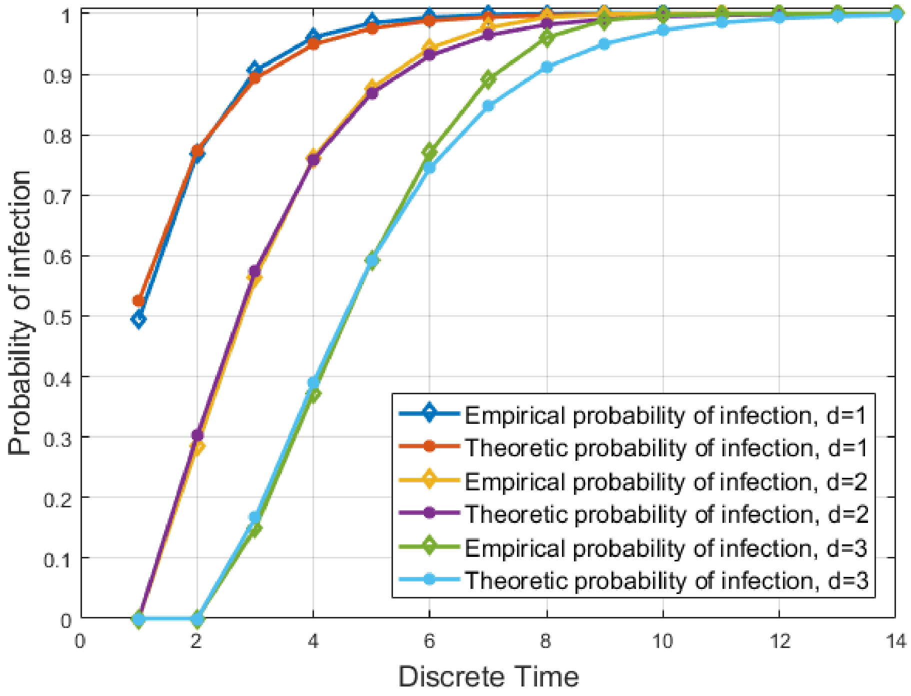
Approach II to Model Diffusion in a Network

- 1000 Rumors, small-world network, 200 Nodes:



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Single Diffusion Source Detection Algorithm

- **Estimate the distances** between each monitor i and the potential source, by computing the dissimilarity between the observed $\tilde{F}_i(t)$ and the theoretical $F_d(t)$.

- Create a set of potential sources using **triangulation**.

- Select the most likely rumour origin, using the approximate model of infection, given a rumour source s:

$$F(x_i(\tau) = 1|s) \approx \sum_{t=1}^{\tau}[1 - \prod_{j \in N_i}(1 - \mu \times F(x_j(t-1) = 1))] \times \prod_{\theta=1}^{t-1}(1 - f_i(\theta))$$

- For each potential source s, compute the dissimilarity between empirical $\tilde{F}_i(t)$ and analytical $F(x_i(T) = 1|s)$. The **most likely rumour origin** is the node with the **lowest dissimilarity.**

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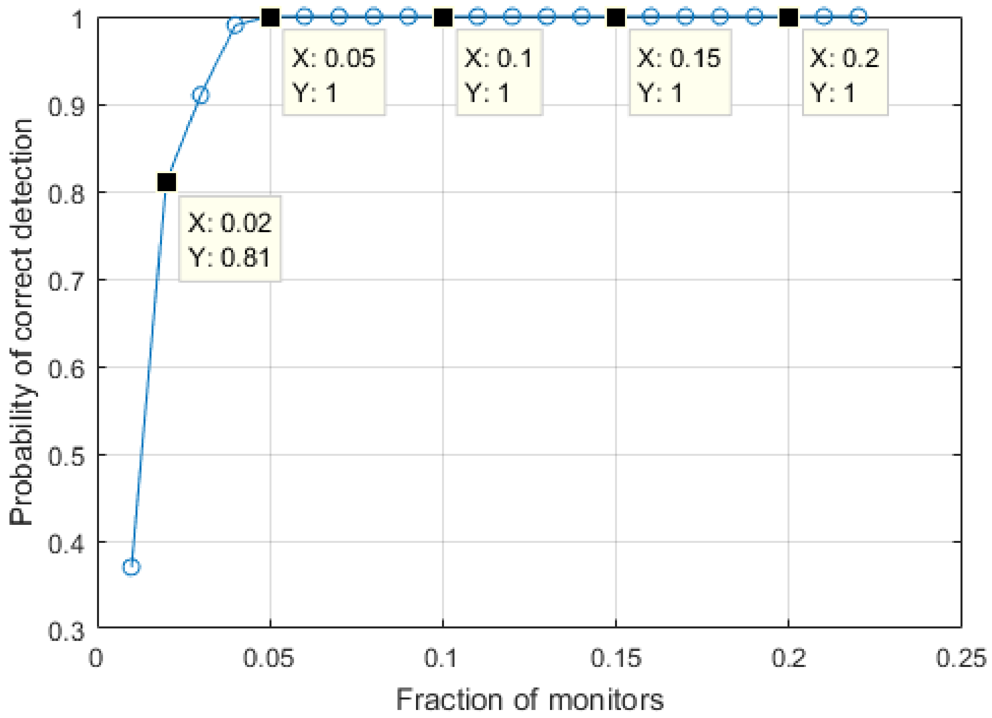
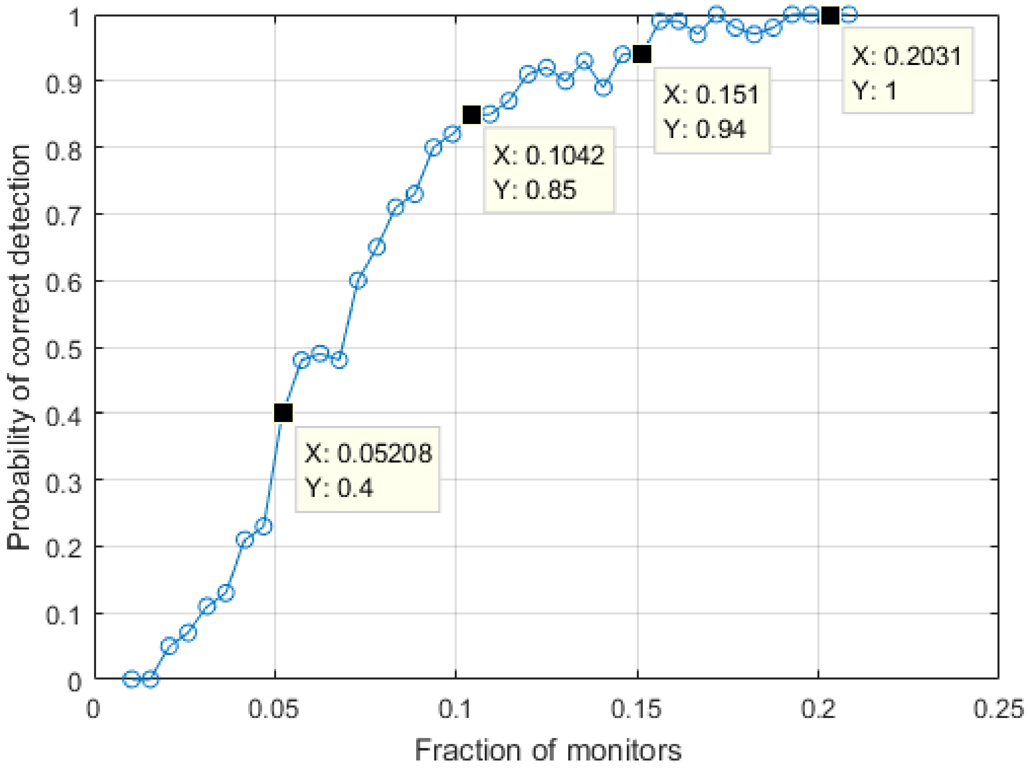
Simulations

- 10 Rumors, small-world network, 1000 Nodes, $\mu = 0.5$, 100 experiments.



Simulations

- 10 Rumors, Facebook network, 192 Nodes, $\mu = 0.5$, 100 experiments.



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Conclusion

- Mathematical models of information propagation, which accurately capture the diffusion process.
- Source detection algorithm, which assumes:
  - Single source, which emits multiple rumours.
  - All rumours start at the same time, which is known.
  - A finite set of monitor nodes is observed at discrete times.
- Future extensions:
  - Source detection with unknown start time.
  - Multiple source detection algorithm.

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Thank you for listening!

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How do we find the optimal parameters α_d in the distance-dependent probabilities?

- The distance-dependent probability of infection for a node at distance d, at time t is:

$$F_d(t) = \sum_{\tau=d}^{t} (\mu \times \alpha_d)^d \times (1 - \mu \times \alpha_d)^{\tau-d} \times \binom{\tau-1}{d-1}$$

- Artificially spread a number of rumours from a random node in the network, and obtain the empirical probabilities $\tilde{F}_i(t)$.
- The optimal parameter α_d minimizes the dissimilarity between $F_d(t)$ and $\tilde{F}_i(t)$ for a particular distance d:

$$\alpha_d^{opt} = argmin_{\alpha_d} \sum_{i \in N_d} \sum_{t=0}^{T} ||F_d(t) - \tilde{F}_i(t)||^2 ,$$

where N_d is the set of nodes at shortest distance d from the source.

How do we estimate the shortest distances between monitor nodes and the source?

- We find the dissimilarity between the distance-dependent analytical probability of infection $F_d(t)$, and the observed infection probability at a node i, using mean-squared error.

- Then, the optimal distance for a monitor i is:

$$d_{i,s} = argmin_d \sum_{t=0}^{T} ||F_d(t) - \tilde{F}_i(t)||^2$$

- We select as potential sources all the nodes at distance $d_{i,s}$ from node i.

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How do we select the most likely rumour origin?

- Select the most likely rumour origin, using the approximate model of infection, given a rumour source s:

$$F(x_i(T) = 1|s) = \sum_{t=1}^{T}[1 - \prod_{j \in N_i} 1 - \mu \times F\left(x_j(t-1) = 1\right)] \times \prod_{\tau=1}^{t-1}(1 - f_i(\tau))$$

- For each potential source s, compute the dissimilarity between the observed infection probabilities of all monitors, and the theoretic model of infection:

$$\bar{C}(s) = \sum_{i}\sum_{t=0}^{T} ||F(x_i(t) = 1|s) - \tilde{F}_i(t)||^2$$

- The most likely rumour origin is the node with the lowest dissimilarity.

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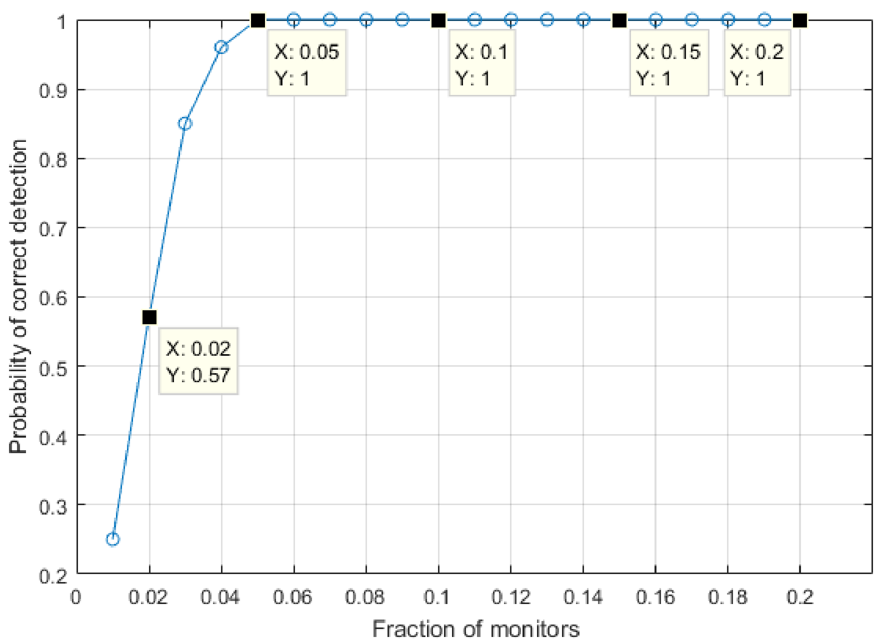
More simulation results

- 10 Rumors, small-world network, 1000 Nodes, *varying* spreading probability



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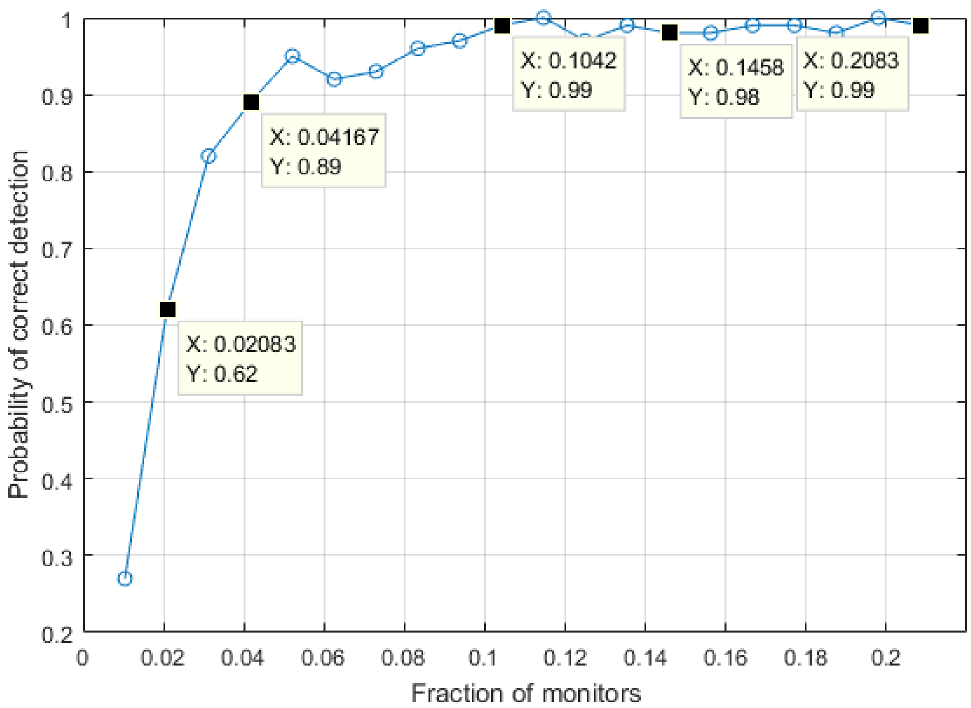
More simulation results

- 10 Rumors, Facebook network, 192 Nodes, *varying* spreading probability



Probability of infection

- A node has the infection at time t if it got initially infected at any of the times before, $\tau = 1,2, \dots, t$.

- The events of a node getting the initial infection at different times are mutually disjoint.

- Hence, the probability of infection is given by the sum of the likelihoods of first infection at different discrete times:

$$F_i(t) = \sum_{\tau=1}^{t} f_i(\tau)$$

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Probability of being susceptible

- A node is susceptible at time t if it didn't get infected at any of the times before, $\tau = 1,2, \ldots, t$.
- The events of a node not getting the initial infection at different times are mutually disjoint.
- Hence:

$$\bar{F}_i(t) = \prod_{\tau=1}^{T} 1 - f_i(\tau)$$

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Distance-dependent probability of infection

- Number of paths is: $\binom{t-1}{d-1}$.
- Probability of each path: $p_S^d \times (1 - p_S)^{t-d}$.
- A node at distance d gets infected if *any* succession of d success steps, and $t - d$ failure steps happens.
- Different successions of S and F events are mutually disjoint.
- Hence, the probability of first infection is:

$$f_d(t) = \underbrace{(\alpha_d \mu)}_{p_S}{}^d \times (1 - \alpha_d \mu)^{t-d} \times \underbrace{\binom{t-1}{d-1}}_{\text{\# of paths from source to destination}}$$

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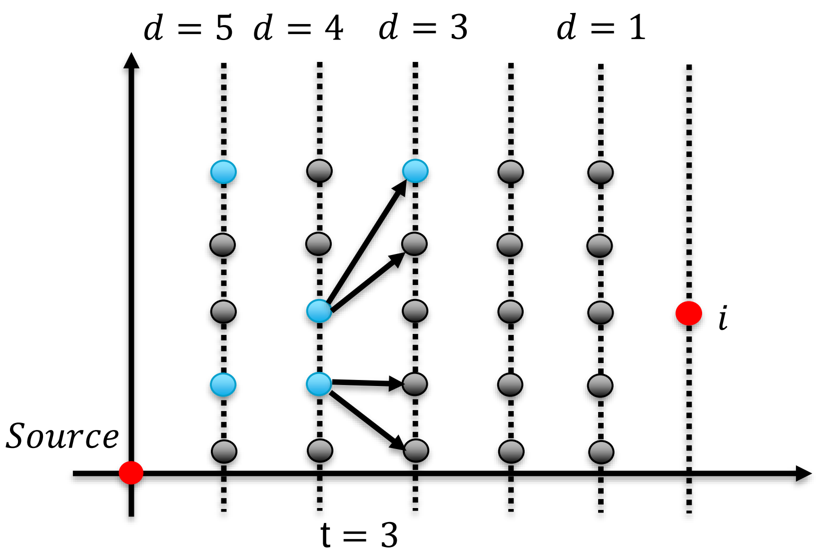
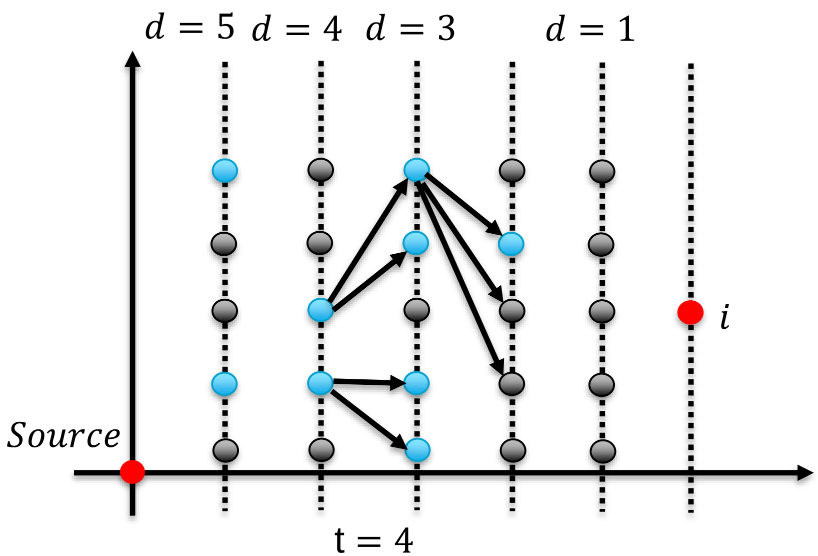
Distance-dependent probability of infection

- A node at distance d gets infected if *any* succession of d success steps, and $t-d$ failure steps happens.
- There can by a success following a failure, at the next time step.





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Comparison to existing methods

- The authors in [1] propose a Monte Carlo method for single source estimation, with unknown infection time. In a **random geometric graph**, the probability of the origin to be within the first **10% ranked nodes** is around **0.5** when **observing 5%** of the network, increasing to **0.9** when **observing the full network**.

- In a **small-world network**, our method achieves correct detection probability of **0.75** when **observing 5%** of the network, and **1** when **observing the full network**. The number of **rumours is 2**, and the rumour **start time is known.**

[1] A. Agaskar and Y. M. Lu. *A fast Monte Carlo algorithm for source localization on graphs.*