A Novel Approach to Joint User Selection and Precoding for Multiuser MISO Downlink Channels

Ashok Bandi, Bhavani Shankar MR, Sina Maleki, Symeon Chatzinotas
Björn Ottersten

Interdisciplinary Centre for Security, Reliability and Trust (SnT),
University of Luxembourg, Luxembourg

Email: {ashok.bandi, bhavani.shankar, sina.maleki, symeon.chatzinotas, bjorn.ottersten}@uni.lu

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Goal: Joint design of user selection and precoding for WSR

System model:

- Single cell MISO system; a BS with M transmit antennas
- Down link scenario; full frequency and time resources reuse
- $N(\geq M)$ single antenna users; Independent data to selected UEs
- Received signal of all users, $\mathbf{y}$, is given by

$$\begin{bmatrix} y_1 \\ y_2 \\ \vdots \\ y_N \end{bmatrix} = \begin{bmatrix} \dots \mathbf{h}_1^H \dots \\ \dots \mathbf{h}_2^H \dots \\ \vdots \\ \dots \mathbf{h}_N^H \dots \end{bmatrix} \begin{bmatrix} \vdots & \vdots & & \vdots \\ \mathbf{w}_1 & \mathbf{w}_2 & \dots & \mathbf{w}_N \\ \vdots & \vdots & & \vdots \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ \vdots \\ x_N \end{bmatrix} + \begin{bmatrix} \vdots \\ \mathbf{n} \\ \vdots \end{bmatrix}$$

where

- y_i, $\mathbf{h}_i$ and $\mathbf{w}_i$ are the received signal, downlink channel and precoding vector of i^{th} user respectively
- $\mathbf{x}$ is the input data vector; $\mathbf{n}$ is noise vector

Design criteria: Maximize weighted sum rate (WSR) s.t. total power constraint

Prior work:

- Decoupled design: User selection and precoding as decoupled problems - usually user selection followed by precoding
- Joint problem formulation with alternative optimization (see ref. [63], [74], [77], [94], [93], [122] in [1])

Scope of improvement: User selection and precoding are coupled $\rightarrow$ the joint solution outperforms aforementioned techniques

Contribution: Joint optimization of joint design problem

[1] E. Castañeda, A. Silva, A. Gameiro, and M. Kountouris, "An overview on resource allocation techniques for multi-user MIMO systems," IEEE Commun. Surveys Tuts., vol. 19, no. 1, pp. 239–284, 1st Quart., 2017

Weighted sum rate maximization

- WSR schedules only the users who contribute to maximum of the SR
- Utmost M users are selected

Problem formulation

$$\max_{\mathbf{w},\mathcal{S}} \quad \sum_{i\in\mathcal{S}} \beta_i \log\left(1+\gamma_i\right)$$

$$\text{subject to } C_1 : \sum_{i=1}^{N} \|\mathbf{w}_i\|_2^2 \leq P_T$$

$$C_2 : |\mathcal{S}| \leq M$$

where β_i and $\gamma_i = \dfrac{|\mathbf{h}_i^H \mathbf{w}_i|^2}{\sigma^2 + \sum_{j\neq i} |\mathbf{h}_i^H \mathbf{w}_j|^2}$ are the weight and SINR of user i respectively, and $\mathcal{S}$ is the set of selected users, and P_T is the total power.

Joint formulations with binary variables in literature

$$\max_{\mathbf{w},\boldsymbol{\eta}} \quad \sum_{i=1}^{N} \eta_i \beta_i \log\left(1+\gamma_i\right) \text{ or } \sum_{i=1}^{N} \beta_i \log\left(1+\eta_i\gamma_i\right)$$

$$\text{subject to } C_1 : \eta_i \in \{0,1\}, \ \forall i,$$

$$C_2 : \sum_{i=1}^{N} \eta_i \ \leq \ M,$$

$$C_3 : \sum_{i=1}^{N} \|\mathbf{w}_i\|_2^2 \ \leq \ P_T$$

Drawbacks

- Coupled formulation of user selection and precoding due to multiplication
- Leads to disjoint update of user selection and precoding variables

Key: User selection through precoding

$$\|\mathbf{w}_i\|_2^2 = \begin{cases} 0; \text{Not selected} \\ \neq 0; \text{selected} \end{cases}$$

Reformulation with binary slack variable

$$\mathcal{P}_1: \max_{\mathbf{W},\mathbf{P},\boldsymbol{\eta}} \sum_{i=1}^{N} \beta_i \log(1+\gamma_i)$$

$$\text{subject to } C_1: \eta_i \in \{0,1\},\ \forall i, \quad C_2: \|\mathbf{w}_i\|_2^2 \le \eta_i P_i, \forall i$$

$$C_3: \sum_{i=1}^{N} \eta_i \le M, \quad C_4: \sum_{i=1}^{N} P_i \le P_T$$

Novelty: Decoupled form. of user selection constraint C_2 (unlike prev. works)

Usefulness: Amenable to joint optimization

Remarks: Non-convex objective and binary constraints $\Longrightarrow$ MINLP

Epigraph reformulation: To address non-convexity

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$$\mathcal{P}_2: \quad \max_{\mathbf{W},\mathbf{P},\eta,\zeta} \quad \sum_{i=1}^{N} \beta_i \log(\zeta_i)$$

$$\text{subject to } C_1, C_2, C_3, C_4$$

$$C_5: \; 1+\gamma_i \;\geq\; \zeta_i, \; \forall i$$

$$C_6: \; \gamma_i \;\geq\; 1, \; \forall i$$

Novel reformulation of C_5 as a DC constraint:

$$1+\gamma_i \geq \zeta_i \quad \Longrightarrow \quad \frac{\sigma^2 + \sum_{j=1}^{N} |\mathbf{h}_i^H \mathbf{w}_j|^2}{\sigma^2 + \sum_{j\neq i} |\mathbf{h}_i^H \mathbf{w}_j|^2} \geq \zeta_i$$

$$\Longrightarrow \quad \underbrace{\frac{\sigma^2 + \sum_{j=1}^{N} |\mathbf{h}_i^H \mathbf{w}_j|^2}{\zeta_i}}_{\textbf{jointly convex in } \mathbf{W},\zeta_i \text{ for } \zeta_i>0} \geq \underbrace{\sigma^2 + \sum_{j\neq i} |\mathbf{h}_i^H \mathbf{w}_j|^2}_{\text{convex}}$$

WSR as a DC problem

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$$\mathcal{P}_3 : \max_{\mathbf{W},\mathbf{P},\boldsymbol{\eta},\boldsymbol{\zeta}} \sum_{i=1}^{N} \beta_i \log(\zeta_i)$$

$$\text{subject to } C_5 : \underbrace{\sigma^2 + \sum_{j \neq i} |\mathbf{h}_i^H \mathbf{w}_j|^2 - \frac{\sigma^2 + \sum_{j=1}^{N} |\mathbf{h}_i^H \mathbf{w}_j|^2}{\zeta_i}}_{\text{difference of convex functions}} \leq 0, \ \forall i$$

$$C_1, C_2, C_3, C_4, C_6$$

Remarks:

- Ignoring or fixing $\boldsymbol{\eta}$ in $\mathcal{P}_3$ yields the DC formulation of classical WSR
- Efficient than SDP based DC formulation: No rank ambiguity and less complex

Binary to continuous: Penalization method

$$\mathcal{P}_4 : \max_{\mathbf{W},\mathbf{P},\boldsymbol{\eta},\boldsymbol{\zeta}} \underbrace{\sum_{i=1}^{N} \left(\beta_i \log\left(\zeta_i\right) + \lambda P\left(\eta_i\right)\right)}_{\text{difference of concave}}$$

$$\text{subject to } C_1 : 0 \leq \eta_i \leq 1, \ \forall i$$

$$C_2, C_3, C_4, C_5, C_6$$

where $p\left(\eta_i\right) \triangleq \eta_i \log \eta_i + \left(1 - \eta_i\right) \log\left(1 - \eta_i\right)$ is a convex penalty function

- Maximizing $p\left(\eta_i\right)$, with appropriate λ, ensures binary nature of η_i.
- $\mathcal{P}_4$ is a DC problem: DC objective s.t DC and convex constraints
- DC problems can be solved efficiently with convex-concave procedure (CCP) which has convergence guarantees to stationary point.

Convex-Concave Procedure

CCP is an iterative algorithm wherein each iteration following two steps are executed until the convergence

- Convexification: The problem is convexified around the previous solution by replacing the convex terms in the objective and concave terms in the constraints by their first-order Taylor approximations
- Optimization: Solve the convexified subproblem globally

[1] Thomas Lipp1 and Stephen Boyd, "Variations and extension of the convex–concave procedure,http://stanford.edu/ boyd/papers/cvx˙ccv.html, 2016

Convexification

- Convexification: Let $\mathbf{W}^{k-1}, \boldsymbol{\eta}^{k-1}, \boldsymbol{\zeta}^{k-1}$ be the estimates of $\mathbf{W}, \boldsymbol{\eta}, \boldsymbol{\zeta}$ in iteration $k-1$ and $\mathcal{G}_i(\mathbf{W}, \zeta_i) = \dfrac{\sigma^2 + \sum_{j=1}^{N} |\mathbf{h}_i^H \mathbf{w}_j|^2}{\zeta_i}$.

$$\tilde{\mathbb{P}}(\eta_i) \triangleq -\lambda \sum_{i=1}^{N} \left(\mathbb{P}\left(\eta_i^{k-1}\right) + \left(\eta_i - \eta_i^{k-1}\right) \nabla \mathbb{P}\left(\eta_i^{k-1}\right)\right),$$

$$\tilde{\mathcal{G}}_i(\mathbf{W}^{k-1}, \zeta_i^{k-1}) \triangleq -\mathcal{G}_i(\mathbf{W}, \zeta_i) - \mathcal{R}\left\{\mathrm{tr}\left\{\nabla^H \mathcal{G}_i(\mathbf{W}^{k-1}, \zeta_i^{k-1}) \begin{bmatrix} \mathbf{W} - \mathbf{W}^{k-1} \\ \zeta i - \zeta_i^{k-1} \end{bmatrix}\right\}\right\}$$

- Optimization: The next update $\left(\mathbf{W}^{k+1}, \boldsymbol{\eta}^{k+1}, \boldsymbol{\zeta}^{k+1}\right)$ is obtained by solving the following convex problem :

$$\mathcal{P}_{\text{WSR}} : \max_{\mathbf{W}, \mathbf{P}, \boldsymbol{\zeta}, \boldsymbol{\eta}} \sum_{i=1}^{N} \left(\beta_i \log(\zeta_i) + \lambda_1 \eta_i \nabla \mathbb{P}\left(\eta_i^{k-1}\right)\right)$$

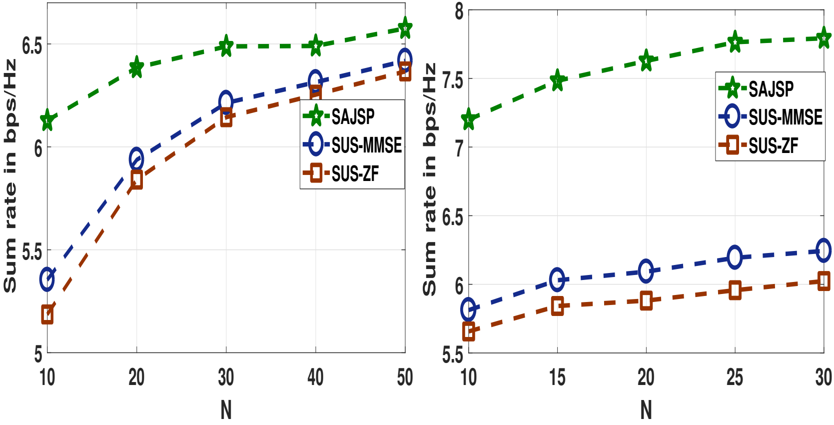
subject to C_2, C_3, C_4, C_5 and C_6

$$C_1 : \sigma^2 + \sum_{j \neq i} |\mathbf{h}_i^H \mathbf{w}_j|^2 - \tilde{\mathcal{G}}_i(\mathbf{W}, \zeta_i) \leq 0, \forall i$$

Simulation setup

- Single cell multiuser MISO system one BS
- BS with M transmit antennas
- BS sends independent data simultaneously to atmost M active users
- Weights $\beta_i = 1, \forall i$
- Tuning parameter $\lambda = 1$, P_T =dB
- Results are averaged over 500 iterations
- **Benchmark Algo**: Channel orthogonality based user selection followed by ZF precoding (SUS-ZF), proposed in [1], is used as benchmark.
- SUS-ZF is used as a initial feasible point

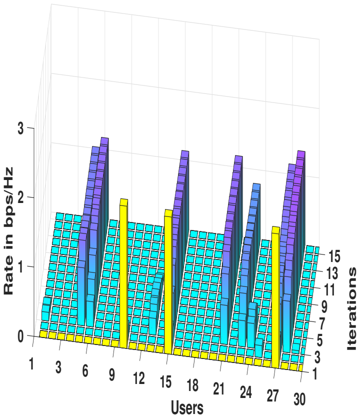
[2] [1] T. Yoo and A. Goldsmith, "On the optimality of multiantenna broadcast user selection using zero-forcing beamforming," IEEE Journal on Selected Areas in Communications, vol. 24, no. 3, pp. 528–541, March 2006.



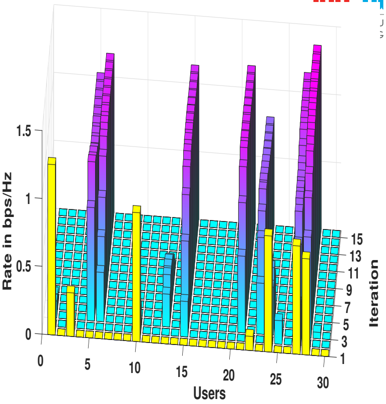
(a) M =4 and N in the range from 10 to 50 insteps of 10

(b) M =8 and N in the range from 10 to 30 insteps of 5.

Figure 1: WSR versus N of a single cell MISO system for P_T=10dB.



(a) SUS-ZF with 3 selected users.



(b) SUS-ZF with 7 selected users.

Figure 2: Rates evolution of users for the following system parameters: $M = 8, N = 30, P_T = 10$dB in a single cell MISO system. Sum rate of proposed method 7.79bps/Hz and SR of SUS-ZF for 3 selected users is 5.97 bps/Hz and for 7 selected users is 4.9932 bps/Hz

Conclusion and Future work

Conclusions:

- Formulated the joint problem for WSR that allows the joint update of user selection and precoding variables
- The joint problem as a DC problem through novel reformulations
- Proposed a CCP based solution to the resulting DC problem
- Shown the efficacy of the solution through Mote-Carlo simulations

Future work:

- Generalize the framework to include other design criteria like max-min fairness and QOS constraints

Q?

Merci !
Thank you :)