Optimal Local Thresholds for Distributed Detection in Energy Harvesting Wireless Sensor Networks

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Outline

- Introduction
- System Model and problem statement
- Optimizing local decision thresholds
- Simulation results
- Conclusions

Introduction

- The designs of wireless sensor networks to perform the task of distributed detection are often based on the conventional battery-powered sensors, leading into designs with a short lifetime, due to battery depletion.
- Energy harvesting, which can collect energy from renewable resources of environment (e.g., solar, wind, and geothermal energy) promises a self-sustainable system with a lifetime.

System Model

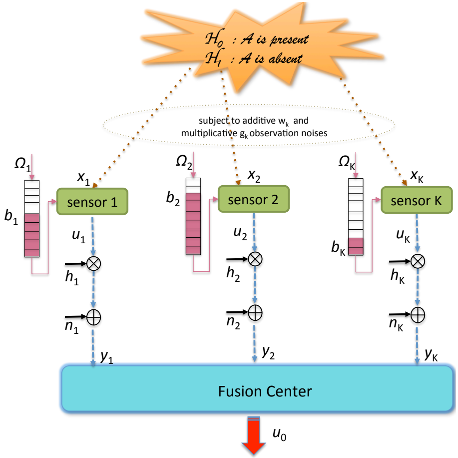


Figure 1: Our System model during one observation period.

System Model

Let x_k denote the local observation at sensor k:

$$x_k = \begin{cases} g_k \mathcal{A} + w_k & \mathcal{H}_1 \\ w_k & \mathcal{H}_0 \end{cases} \tag{1}$$

- $\mathcal{A}$ is a known scalar signal
- $w_k \sim \mathcal{N}(0, \sigma^2_{w_k}) \rightarrow$ Additive noise
- $g_k \sim \mathcal{N}(0, \gamma_{g_k}) \rightarrow$ Multiplicative noise
- All observation noises are independent over time and among K sensors.

During each observation period, sensor k takes N samples of x_k to measure the received signal energy and applies an energy detector to make a binary decision, i.e., sensor k decides whether or not signal $\mathcal{A}$ is present.

$$\Lambda_k = \frac{1}{N}\sum_{n=1}^{N} |x_{k,n}|^2 \underset{d_k=0}{\overset{d_k=1}{\gtrless}} \theta_k \tag{2}$$

- $P_{f_k} = \Pr(\Lambda_k > \theta_k|\mathcal{H}_0) = \frac{\Gamma\left(N/2, \frac{N\theta_k}{\sigma_{w_k}^2}\right)}{\Gamma(N/2)}$
- $P_{d_k} = \Pr(\Lambda_k > \theta_k|\mathcal{H}_1) = Q_{N/2}\left(\frac{\sqrt{\eta_k}}{\sigma_{w_k}}, \frac{\sqrt{N\theta_k}}{\sigma_{w_k}}\right)$
- **Our goal is optimize the local decision threshold θ_k**

System Model

Assumptions:

- Each sensor is able to harvest energy from the environment and stores it in a battery with the capacity $\mathcal{K}$ units of energy.
- The sensors communicate with the FC through orthogonal fading channels with channel gains $|h_k|$'s with parameters γ_{h_k}.
- The sensors employ on-off keying signaling.
- We use the channel-inversion power, the number of energy units spent to convey a decision is inversely proportional to $|h_k|$.
- To avoid the battery depletion when $|h_k|$ is too small, we impose an extra constraint for channel quality.

System Model

Let $u_{k,t}$ represent the sensor output corresponding to the observation period t.

$$u_{k,t} = \begin{cases} \lceil \frac{\lambda}{|h_k|} \rceil & \Lambda_k > \theta_k,\ b_{k,t} > \lceil \frac{\lambda}{|h_k|} \rceil,\ |h_k|^2 > \zeta_k \\ 0 & \text{Otherwise} \end{cases} \quad (3)$$

- $b_{k,t}$ denote the battery state of sensor k
- $|h_k|$ is channel gain
- ζ_k is threshold of the channel quality
- λ is a power regulation constant

System Model

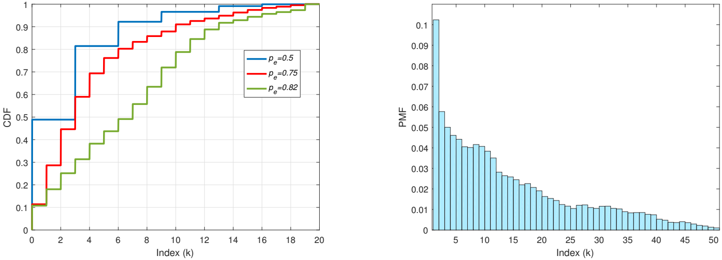
We model $b_{k,t}$ in (3) as the following

$$b_{k,t} = \min\Big\{ b_{k,t-1} - \lceil \frac{\lambda}{|h_k|} \rceil I_{u_{k,t-1}} + \Omega_{k,t} \ , \ \mathcal{K} \Big\} \tag{4}$$

- $\Omega_{k,t} \in \{0,1\}$ indicates units of harvesting energy and it is a Bernoulli random variable, with $\Pr(\Omega_{k,t}=1)=p_e$
- $I_{u_{k,t-1}} = \begin{cases} 1 & u_{k,t-1} > 0 \\ 0 & \text{Otherwise} \end{cases}$

Battery Model

- Assuming b_k in (4) is a stationary random process, one can compute the CDF and the pmf of b_k in terms of $\mathcal{K}, p_e, \gamma_{h_k}$. Further, we use pmf of b_k for our numerical results.



(a) (b)

Figure 2: (a) CDF of b_k for $\mathcal{K}=20$ and $p_e=0.5, 0.75, 0.82$, (b) pmf of b_k for $\mathcal{K}=50$ and $p_e=0.8$.

Optimizing local decision thresholds

We consider two detection performance metrics to find the optimal θ_k's:

- The detection probability at the FC, assuming that the FC utilizes the optimal fusion rule based on Neyman-Pearson optimality criterion.

- the KL distance between the two distributions of the received signals at the FC conditioned on hypothesis $\mathcal{H}_0, \mathcal{H}_1$

Optimal LRT Fusion Rule and P_D, P_F Expressions

The received signal at the FC from sensor k is $y_k = h_k u_k + n_k$, where the additive communication channel noise $n_k \sim \mathcal{N}\left(0, \sigma_{n_k}^2\right)$. The likelihood ratio at the FC is

$$\Delta_{\mathrm{LRT}} = \sum_{k=1}^{K} \log\left(\frac{\sum_{u_k} f\left(y_k | u_k, \mathcal{H}_1\right) \Pr\left(u_k | \mathcal{H}_1\right)}{\sum_{u_k} f\left(y_k | u_k, \mathcal{H}_0\right) \Pr\left(u_k | \mathcal{H}_0\right)}\right) \tag{5}$$

Given u_k, y_k is Gaussian, i.e., $y_k|_{u_k=0} \sim \mathcal{N}\left(0, \sigma_{n_k}^2\right)$ and $y_k|_{u_k=\lceil \frac{\lambda}{|h_k|} \rceil} \sim \mathcal{N}\left(\lceil \frac{\lambda}{|h_k|} \rceil h_k, \sigma_{n_k}^2\right)$.

Optimal LRT Fusion Rule and P_D, P_F Expressions

The probabilities $\Pr(u_k|\mathcal{H}_1)$, $\Pr(u_k|\mathcal{H}_0)$ in (5) are

- $\Pr\left(u_k = \lceil\frac{\lambda}{|h_k|}\rceil|\mathcal{H}_1\right) = P_{d_k}\rho_k q_k = \alpha_k$
- $\Pr\left(u_k = \lceil\frac{\lambda}{|h_k|}\rceil|\mathcal{H}_0\right) = P_{f_k}\rho_k q_k = \beta_k$

where $\rho_k = \Pr(b_k > \lceil\frac{\lambda}{|h_k|}\rceil)$ and $q_k = \Pr(|h_k|^2 > \zeta_k)$.
Given a threshold τ, the optimal likelihood ratio test (LRT) is $\Delta_{\text{LRT}} \gtrless_{\mathcal{H}_0}^{\mathcal{H}_1} \tau$. The P_F, P_D at the FC

$$P_F = \Pr\left(\Delta_{\text{LRT}} > \tau|\mathcal{H}_0\right) = Q\left(\frac{\tau - \mu_{\Delta|\mathcal{H}_0}}{\sigma_{\Delta|\mathcal{H}_0}}\right) \quad (6)$$

$$\begin{aligned} P_D &= \Pr\left(\Delta_{\text{LRT}} > \tau|\mathcal{H}_1\right) \\ &= Q\left(\frac{Q^{-1}(a)\sigma_{\Delta|\mathcal{H}_0} + \mu_{\Delta|\mathcal{H}_0} - \mu_{\Delta|\mathcal{H}_1}}{\sigma_{\Delta|\mathcal{H}_1}}\right) \end{aligned} \quad (7)$$

KL Expression

Kullback-Leibler distance (KL) between the two distributions of the received signals at the FC

$$KL_k = \int_{y_k} f(y_k|\mathcal{H}_1) \log \left(\frac{f(y_k|\mathcal{H}_1)}{f(y_k|\mathcal{H}_0)} \right) dy_k \tag{8}$$

One can approximate KL_k in (8) by the KL distance of two Gaussian distributions

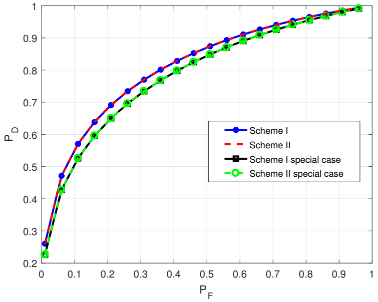
$$KL_k \approx \frac{1}{2} \log\left(\frac{\sigma^2_{y_k|\mathcal{H}_0}}{\sigma^2_{y_k|\mathcal{H}_1}}\right) + \frac{\sigma^2_{y_k|\mathcal{H}_1} - \sigma^2_{y_k|\mathcal{H}_0} + (\mu_{y_k|\mathcal{H}_1} - \mu_{y_k|\mathcal{H}_0})^2}{2\sigma^2_{y_k|\mathcal{H}_0}} \tag{9}$$

Simulation Results

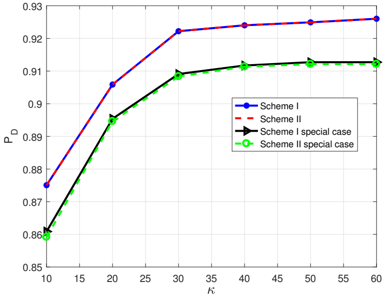
In this section, we consider:

- Scheme I: Numerically find θ_k's which maximize P_D in (7) $\rightarrow$ K-dimensional search is required $\rightarrow$ computational complexity!
- Scheme II: Finding θ_k's which maximize $KL_{tot} = \sum_{k=1}^{K} KL_k$, using the KL_k approximation in (9) $\rightarrow$ Only one dimensional search $\rightarrow$ computationally efficient.
- Special case: Assume all sensors employ the same local threshold $\theta_k = \theta$ and compare schemes I and II.

We then compare P_D evaluated at the θ_k's obtained from mentioned schemes.



(a)



(b)

Figure 3: (a) P_D vs. P_F
(b) P_D vs. $\mathcal{K}$

Conclusion

- We studied a distributed detection problem in a wireless network with K heterogeneous energy harvesting sensors and investigated the optimal local decision thresholds for given transmission and battery state models.
- Our numerical results indicate that the thresholds obtained from maximizing the KL distance are near-optimal and computationally very efficient, as it requires only K one-dimensional searches, as opposed to a K-dimensional search required to find the thresholds that maximize the detection probability.
- The performance gap between each scheme and its corresponding special case indicates that when sensors are heterogeneous, it is advantageous to use different local thresholds according to sensors' statistics.

Thank You

Questions?