A Performance Analysis on the Optimal Number of Measurements for Coded Compressive Imaging

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Motivation

- Practical coded compressive imaging settings
  - Focal Plane Array (FPA) imaging
    - Gathers noisy undersampled measurements of spatially modulated light intensity from a scene
    - Spatial modulation can be performed at sub-pixel level using a DMD
    - Reconstruction using sparse recovery algorithm
  - Magnetic Particle Imaging (MPI)
    - Allows fast imaging of magnetic nanoparticle (MNP) samples in a FOV
    - System matrix (SM) calibration is done using coded scenes with MNP samples at multiple positions
    - SM reconstruction using compressive sensing
- Investigation of the trade-off between input pSNR, number of measurements, and image quality

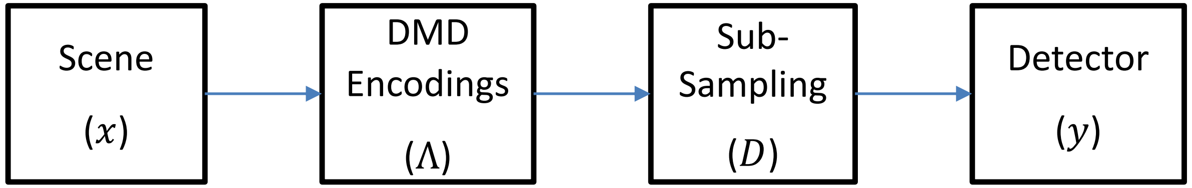
Previous Work

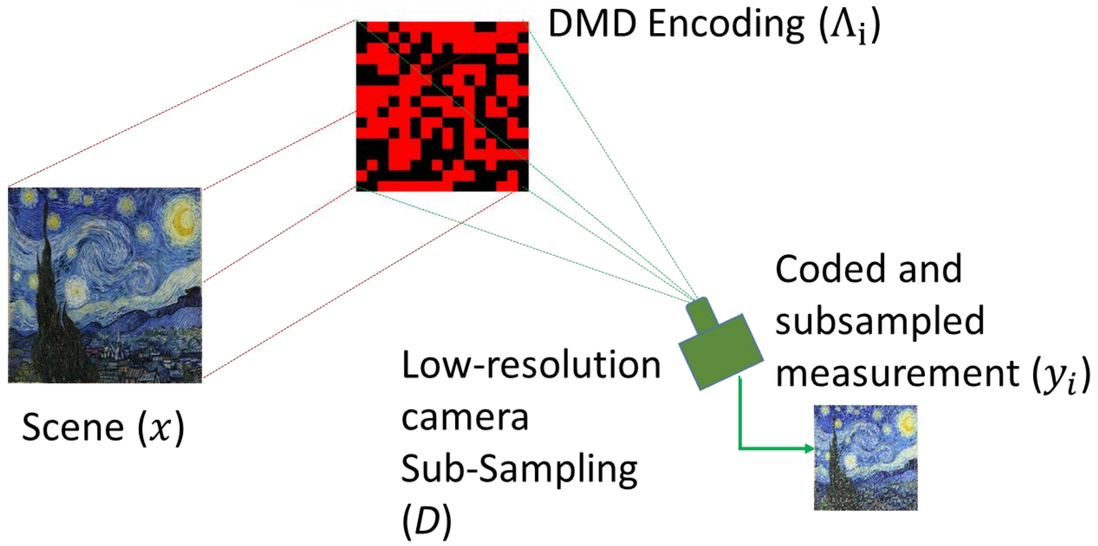
- Practical signal transmission in radar/sonar with a fixed power budget (Yang et al., 2017)
  - Measurement matrix with Gaussian iid entries
  - Gaussian, Bernoulli-Gaussian, and least favourite distributions for signal models
  - Sparsity level should be known
  - Based on state evolution technique proposed for approximate message passing algorithm

Assumptions

- FPA Imaging
  - Constant integration time for measurements
    - Per-frame integration time is divided among different spatial modulations
    - k different modulations -> Input SNR scales by $1/k$
  - Signal energy per pixel increases with pixel size
- MPI
  - Signal energy decays linearly with number of coded scenes

Real Domain : FPA Imaging





- k different DMD encodings
- $x \epsilon R^N$, $y \ \epsilon R^{nk}$ (FPA with n pixels) $(N > n)$
- Super-resolution factor $d = \frac{N}{n}$ & Compression ratio $m = \frac{k}{d}$
- Forward model: $y = \tau A x + n$ where $n \ \epsilon R^{nk}$ is AWGN and $A = D\Lambda$
- $\tau = \frac{d}{k}$ reflects the effects of increased pixel size and decreased integration time per DMD mask, given constant noise level

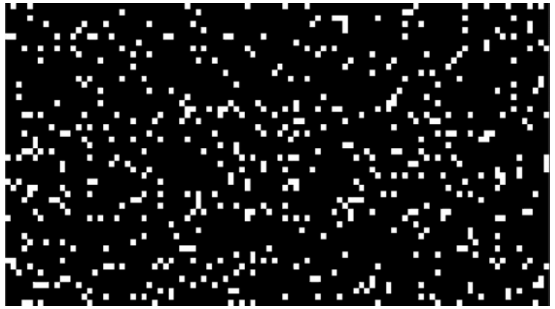
FPA Imaging – Reconstruction

$$y = \tau A x + n$$

$$\min_{x} \alpha_1 \|Fx\|_1 + \alpha_2 TV(x) \quad subject\ to \quad \left\| Ax - \frac{y}{\tau} \right\|_2 \leq \epsilon/\tau\,, x[i] \geq 0\ \forall i$$

- F : Sparsifying transform such as the Fourier
- $TV(.)$: Total variation operator
- ϵ : Bound on the noise
- Weighted sum is due to superior performance
- ADMM based reconstruction algorithm (Kar et al., 2018)

$$y = Xp + n$$



- $y \epsilon C^{M}$: measurements, $p \epsilon R^{N}$: Calibration Scene
- $X \epsilon C^{M \times N}$: System matrix (SM), $\mathrm{n} \epsilon C^{N}$: Complex AWGN

- Taking multiple measurements using different p, a single row of X, i.e. $x^{(i)}$:

$$y^{(i)} = P^{T} x^{(i)} + n$$

  - P : Binary coding scene, $y^{(i)}$: i-th row of SM sensed with P

Magnetic Particle Imaging – Reconstruction

$$y^{(i)} = P^T x^{(i)} + n$$

$$\min_x \left\| D x^{(i)} \right\|_1 \quad subject\ to \quad \left\| P^T x^{(i)} - y^{(i)} \right\|_2 \leq \epsilon_i$$

- D: Sparsifying transform such as the DCT
- Entries of P are drawn from a symmetric Bernoulli distribution
- ADMM based reconstruction algorithm (Ilbey et al., 2018)
- Can be considered as a special case of the FPA-imaging problem

Compressive Sensing Perspective

- Theorem 1.2 (Candes, 2008)

$$\|\hat{x} - x\|_2 \leq C_0 s^{-0.5} \|x - x_s\|_1 + C_1 \epsilon \qquad (1)$$

$$C_0 = 2\frac{1-(1-\sqrt{2})\delta_{2s}}{1-(1+\sqrt{2})\delta_{2s}}, \; C_1 = 4\frac{\sqrt{1+\delta_{2s}}}{1-(1+\sqrt{2})\delta_{2s}}$$

  - $\hat{x}$: Estimate of x
  - x_s : $s-$ sparse version of x
  - $\epsilon = m\sigma^2$: Bound on the noise
  - δ_{2s}: Restricted isometry constant

- Increasing m increases the second term in (1)
- δ_{2s} is monotonically decreasing function of m
- C_0 and C_1 calculations are NP-hard, thus the bound is an NP-hard problem
- There exists an optimal number of measurements for a given problem, but its solution is impractical

ADMM Based Reconstruction

- ADMM
- Problem formulation:

$$\begin{array}{ll} \underset{\mathbf{x},\mathbf{z}}{\text{minimize}} & f_1(\mathbf{x}) + f_2(\mathbf{z}) \\ \text{subject to} & \mathbf{G}\mathbf{x} + \mathbf{Q}\mathbf{z} - \mathbf{r} = 0 \end{array}$$

- $f_1(.)$ and $f_2(.)$ separable convex functions
- Two small problems instead of one large problem
- Updates x and z alternatingly

Solved Problem in ADMM Form

Solved Problem

$$\begin{array}{ll} \underset{\mathbf{x}}{\text{minimize}} & \alpha_1 \|\mathbf{F}\mathbf{x}\|_1 + \alpha_2 TV(\mathbf{x}) \\ \text{subject to} & \|\mathbf{B}\mathbf{x} - \mathbf{y}\|_2 \leq \epsilon, \; x[i] \geq 0 \; \forall i \end{array}$$

ADMM form

$$\begin{array}{ll} \underset{\mathbf{x},\mathbf{z}}{\text{minimize}} & f_1(\mathbf{x}) + f_2(\mathbf{z}) \\ \text{subject to} & \mathbf{G}\mathbf{x} + \mathbf{Q}\mathbf{z} - \mathbf{r} = 0 \end{array}$$

$$f_1(\mathbf{x}) = \iota_{(\mathbf{x} \geq 0)}(\mathbf{x}), \; f_2(\mathbf{z}) = \alpha_1 \|\mathbf{F}\mathbf{z}^{(1)}\|_1 + \alpha_2 TV(\mathbf{z}^{(2)}) + \iota_{(\|\mathbf{B}\mathbf{z}^{(3)} - \mathbf{y}\|_2 \leq \epsilon)}(\mathbf{z}^{(3)})$$

$$\mathbf{G} = [\mathbf{I} \;\; \mathbf{I} \;\; \mathbf{B}]^T, \; \mathbf{Q} = -\mathbf{I}, \; \mathbf{r} = 0, \; \mathbf{z} = [\mathbf{z}^{(1)} \;\; \mathbf{z}^{(2)} \;\; \mathbf{z}^{(3)}]^T$$

- Efficient solutions of ADMM steps (Kar et al., 2018)

Results : FPA imaging

- Image size : 360×360
- FPA size : 30×30, 60×60, 90×90
  - Super resolution ratios (d) : 144, 36, 16 respectively
- Input pSNR levels (for full integration time) : 40 dB, 50 dB, 60 dB
- Compression ratios (m) : 0.05, 0.10, 0.15, …, 0.80
- Each experiment is repeated 10 times with different noise & mask realizations

Lena

Parrot

Peppers

Results : FPA imaging

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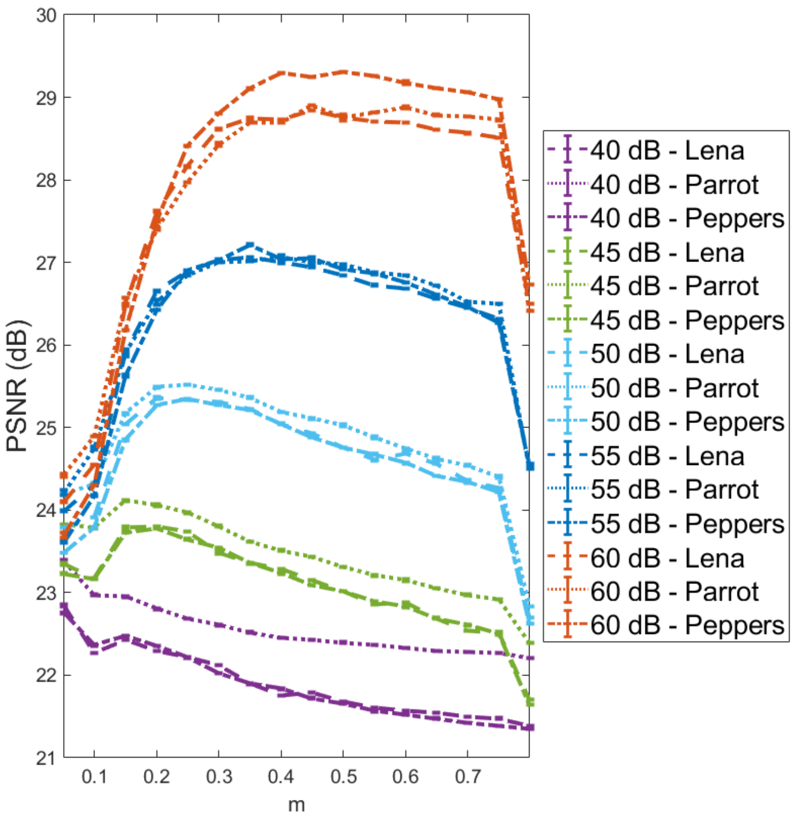

30×30 FPA

Results : FPA imaging

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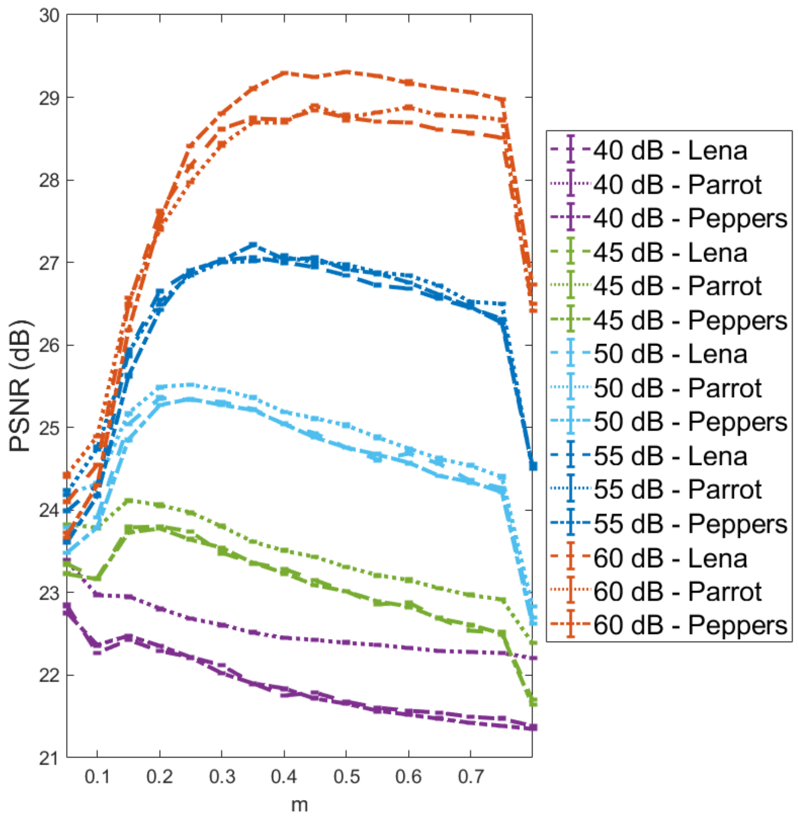
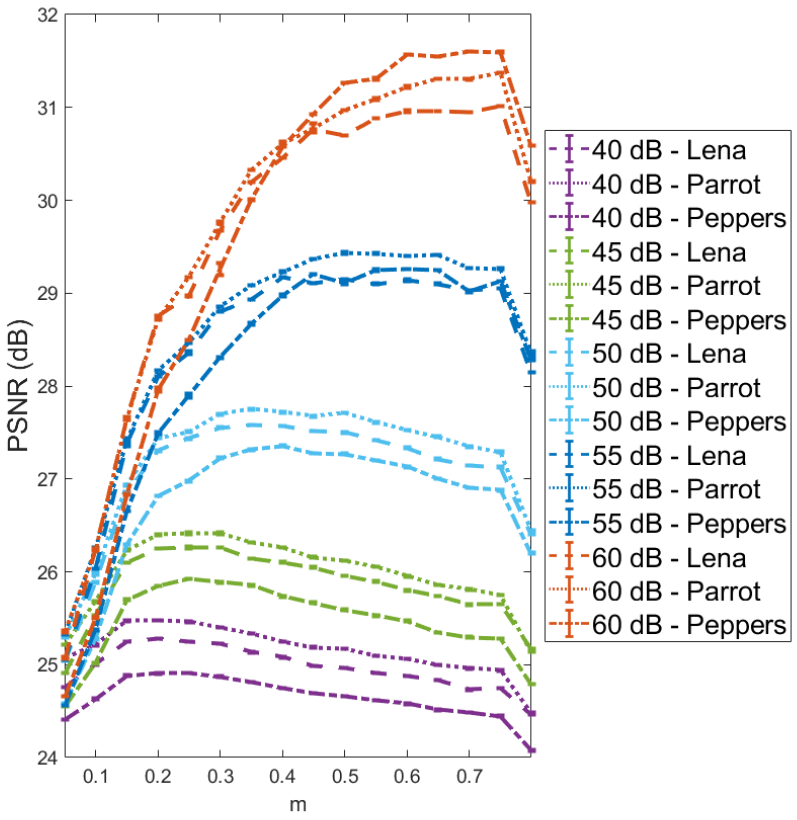
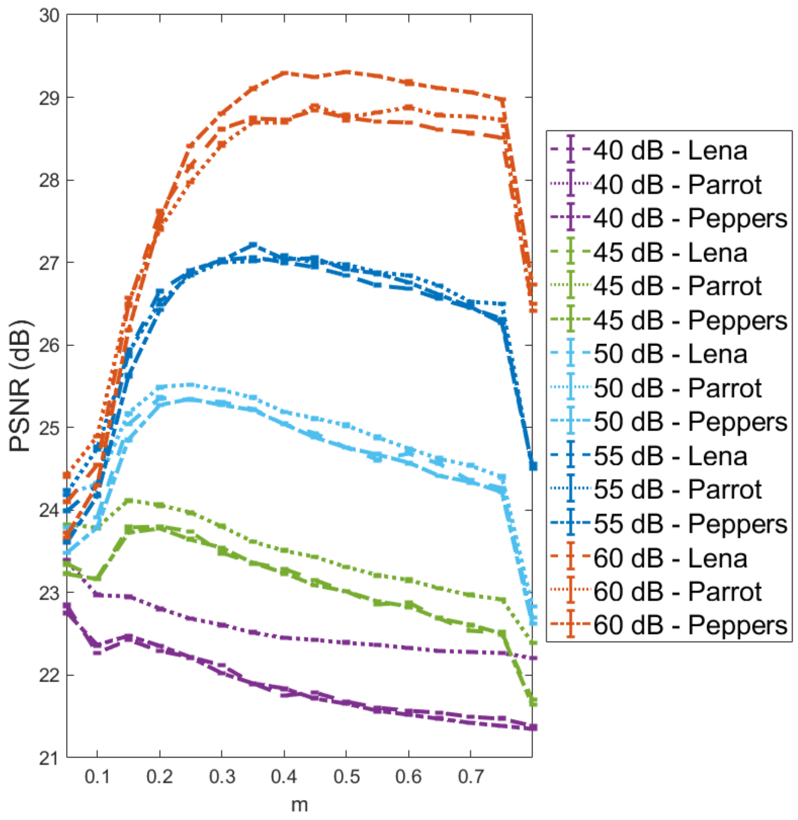


30×30 FPA

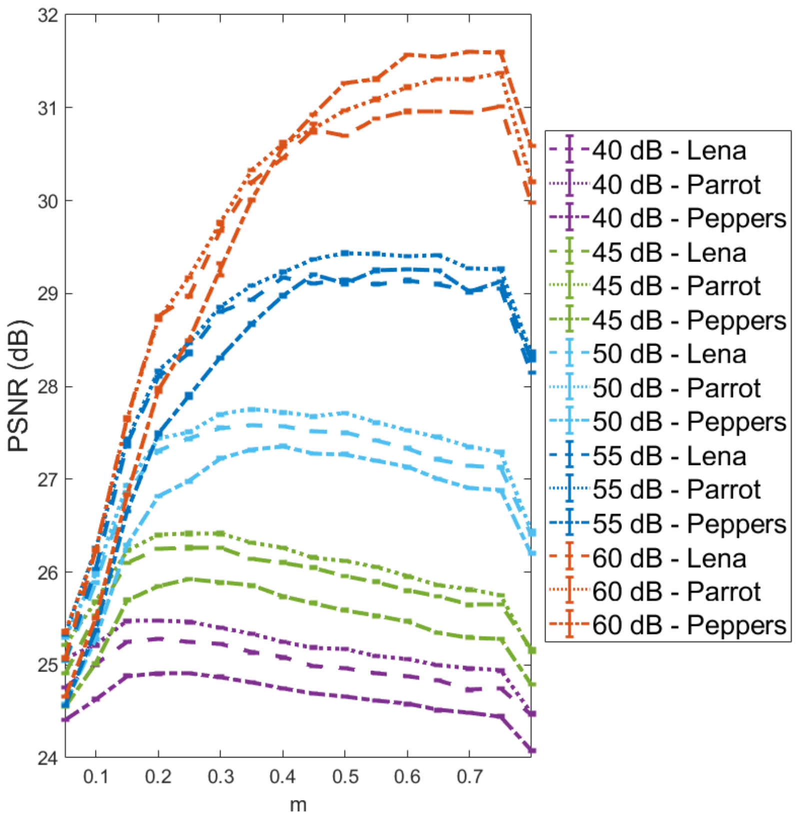


60×60 FPA

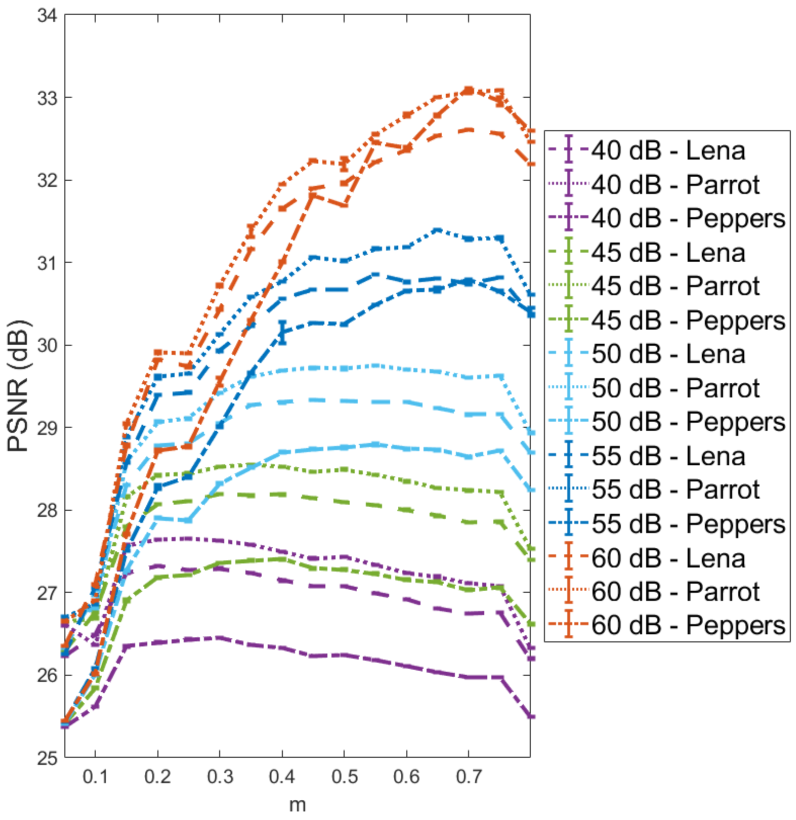
Results : FPA imaging



30 × 30 FPA



60 × 60 FPA

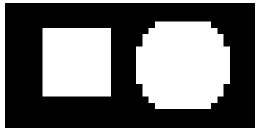


90 × 90 FPA

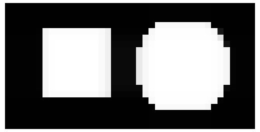
- Reconstruction improves up to some measurement level, and decrease afterwards
- As the noise level decreases, optimal number of measurements favors more measurements
- All three images result in similar performance and optimal number of measurements
- Reconstruction performance decreases with lower FPA resolution

Results : MPI

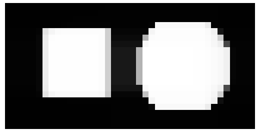
- Image size : 40×20
- Input pSNR levels : 0 dB, 10 dB,…, 40 dB
- Compression ratios (m) : 0.05, 0.10, 0.15, …, 0.80



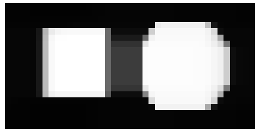
Reference phantom



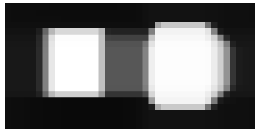
40 dB reconst. (m*=0.40)



30 dB reconst. (m*=0.35)

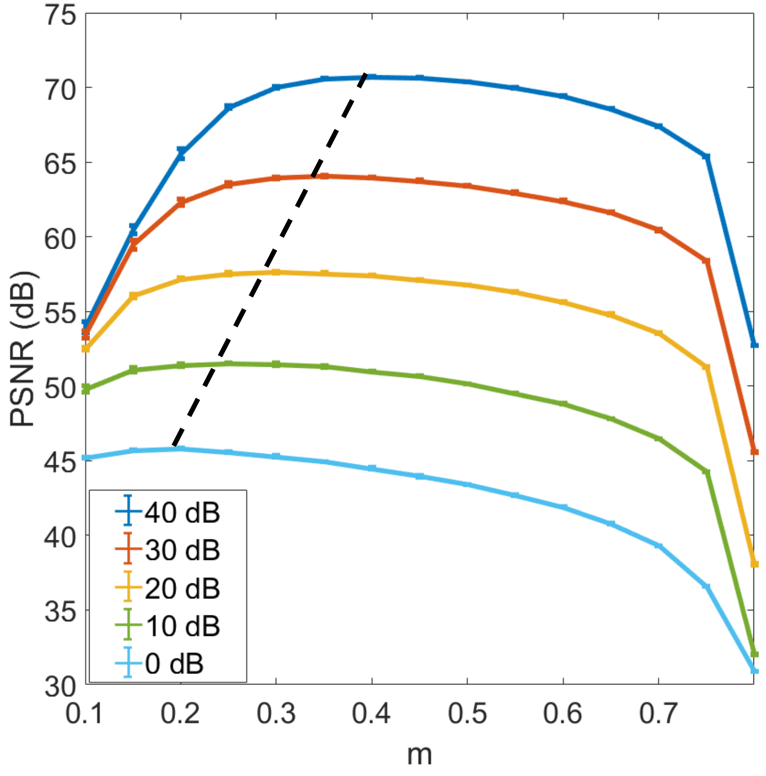


20 dB reconst. (m*=0.30)



10 dB reconst. (m*=0.25)

0 dB reconst. (m*=0.20)



Conclusions & Future Work

- Practical analysis of two coded compressive imaging techniques
  - FPA imaging and MPI
  - Under different noise, super resolution, compression ratio settings
- Optimal number of measurements favor higher number of measurements as the input pSNR increases, and vice versa
- Finding it analytically requires knowledge of sparsity level which is impractical

- Shortcomings
  - Linear scaling in signals
  - Additional non-idealities such as photon noise

THANK YOU

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