Dominant Component Tracking for Empirical Mode Decomposition using a Hidden Markov Model

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Outline

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Motivation

- **Signal:** recording of parakeet calls taken in a social environment
- **Sensor:** wireless microphone in a bird “backpack”
- **Problem:** track the target bird’s (bird with the backpack) call in the presence of environmental noise
- **Assumption:** due to the proximity to the sensor, calls from the target bird are expected to have the much greater energy than noise other birds and noise from the environment

Proposed Approach

In order to track the target bird's call in the presence of environmental noise we propose to:

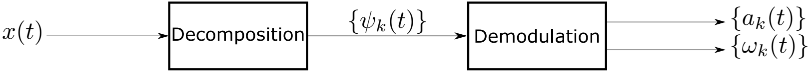
- **Decompose** the signal into a set of component signals using the empirical mode decomposition algorithm.
- **Demodulate** the set of component signals to obtain instantaneous amplitudes and frequencies.
- **Track** the dominant component across the set of component signals using a hidden Markov model.

Outline

Empirical Mode Decomposition

Huang et al. (1998), Sandoval and De Leon (2017)

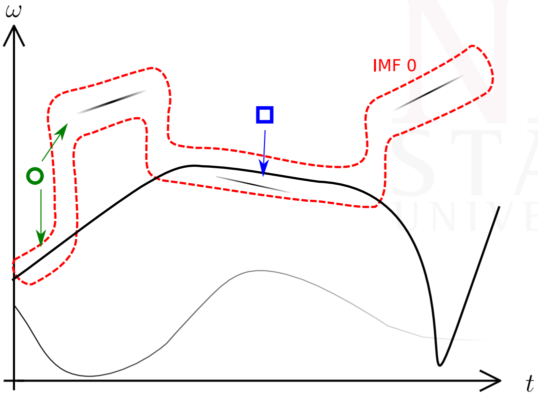
- Empirical Mode Decomposition (EMD) iteratively decomposes a signal into a set of component signals called Intrinsic Mode Functions (IMFs), $\psi_k(t)$.
- At each iteration, the highest frequency component at each time instant is estimated and removed from the signal. The process is then repeated.
- Through demodulation, Instantaneous Amplitudes (IA), $a_k(t)$, and Instantaneous Frequencies (IF), $\omega_k(t)$, associated with each IMF can be estimated.



Empirical Mode Decomposition

Mode Mixing

At each time instant the highest frequency component is estimated and removed.



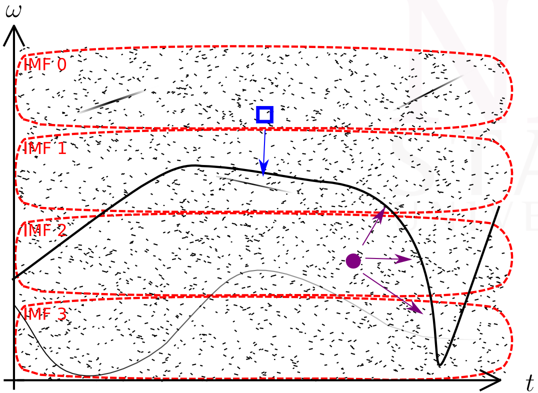
- One problem associated with EMD is termed mode mixing
- Mode mixing is defined as an IMF either consisting of components of disparate scales (○) or components of similar scale (□) residing in the same IMF
- Mode mixing occurs as a consequence of relative component intermittency

Empirical Mode Decomposition

Deering and Kaiser (2005), Wu and Huang (2009)

Masking Signals, Ensemble Averaging, and Component Splitting

Mode mixing is commonly mitigated by using masking signals and ensemble averaging



- Masking signals provide something to track, then vanish in the ensemble
- May resolve some complications (○)
- Neglects some complications (□)
- Introduces a new complication: *component splitting* (●)

Hidden Markov Models

Hidden Markov Models (HMMs) provide a probabilistic approach for relating observations to a hidden state sequence. A typical HMM consisting of K states $\mathbf{Q} = \{q_1 \; q_2 \; \ldots \; q_K\}$ with observation sequences $\mathbf{O}_N = [o_1 \; o_2 \; \cdots \; o_N]$ of length N may be specified by parameter set $\lambda = \{\mathbf{A}, \; \mathbf{B}, \; \mathbf{\Pi}\}$ where

- $\mathbf{A}$ is a transition probability matrix where $(\mathbf{A})_{ij} = \mathrm{a}_{ij}, \; 1 \leq i \leq K, \; 1 \leq j \leq K$ representing the probability of moving from state i to state j
- $\mathbf{B}$ a matrix of observation likelihoods where $(\mathbf{B})_{kn} = b_k(o_n), \; 1 \leq k \leq K, \; 1 \leq n \leq N$ is the likelihood of observation o_n being generated from a state k at time n
- $\mathbf{\Pi} = [\pi_1 \; \pi_2 \; \ldots \; \pi_K]$ an initial probability distribution over states, i.e., π_k is the probability that the Markov model will start in state k

Hidden Markov Models

Time-Varying State Transition Probability Matrix

The state transition probability matrix, $(\mathbf{A})_{ij} = a_{ij}$, is often assumed to be constant.

For this application, we must allow for a time-varying transition matrix

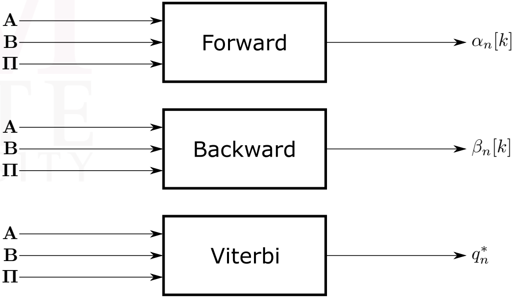
$$(\mathbf{A})_{ij} = a_{ij} \quad \rightarrow \quad (\mathbf{A}[n])_{ij} = a_{ij}[n].$$

Hidden Markov Models

Quantities of Interest

Our analysis using HMMs will require the computation of three quantities:

- $\alpha_n[k]$, the probability of ending up at state q_k at time n, given observations $[o_1 \; o_2 \; \cdots \; o_n]$
- $\beta_n[k]$, the probability of the observations $[o_{n+1} \; o_{n+2} \; \cdots \; o_N]$, given that we are in state q_k at time n
- $q_n^\star$, the most likely state at time n



Outline

Proposed Method

Formulating the Transition Probabilities, Observation Probabilities, and Initial Probability Distribution

- Instantaneous frequencies $\{\hat{\omega}_k(t)\}$ are used to construct a time-varying state transition probability matrix which encourages smoothness in component tracking

$$\mathsf{a}_{ij}[n] = \frac{|\hat{\omega}_i(nT_s) - \hat{\omega}_j([n+1]T_s)|^{-1}}{\sum_{k=1}^{K} |\hat{\omega}_i(nT_s) - \hat{\omega}_k([n+1]T_s)|^{-1}}.$$

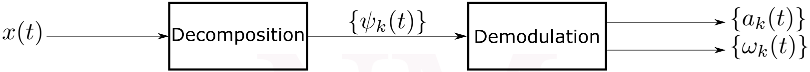
- Instantaneous amplitudes $\{\hat{a}_k(t)\}$ are used to construct a matrix of observation likelihood which encourages high energy tracking

$$b_k(o_n) = \exp(|\hat{a}_k(nT_s)|)$$

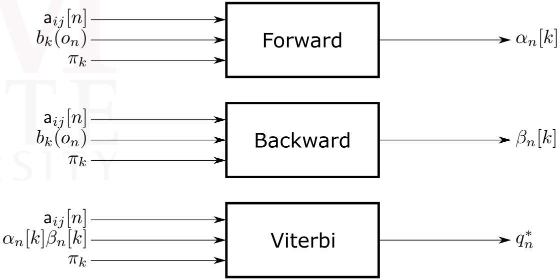
- We assume an equal probability of starting in each state $\pi_k = \dfrac{1}{K}$

Proposed Method

Dominant Component Tracking Algorithm



procedure $\{a^\star[n], \omega^\star[n]\} = \mathrm{DomComp}(\ \{\hat{a}_k(t), \hat{\omega}_k(t)\}\ )$
$\quad a_{ij}[n] \leftarrow \frac{|\hat{\omega}_i(nT_s) - \hat{\omega}_j([n+1]T_s)|^{-1}}{\sum_{k=1}^{K} |\hat{\omega}_i(nT_s) - \hat{\omega}_k([n+1]T_s)|^{-1}}$
$\quad b_k(o_n) \leftarrow \exp(|\hat{a}_k(nT_s)|)$
$\quad \pi_k \leftarrow \frac{1}{K}$
$\quad \alpha_n[k] = \mathrm{forward}(\ \mathbf{A},\ \mathbf{B},\ \mathbf{\Pi}\ )$
$\quad \beta_n[k] = \mathrm{backward}(\ \mathbf{A},\ \mathbf{B},\ \mathbf{\Pi}\ )$
$\quad \beta_n[k] \leftarrow \alpha_n[k]\beta_n[k]$
$\quad [q_n^\star, v^\star] = \mathrm{Viterbi}(\ \mathbf{A},\ \mathbf{B},\ \mathbf{\Pi}\ )$
$\quad a^\star[n] = a_{q_n^\star}(nT_s)$
$\quad \omega^\star[n] = \omega_{q_n^\star}(nT_s)$
end procedure



Control Methods

We compare the proposed approach to two control methods:

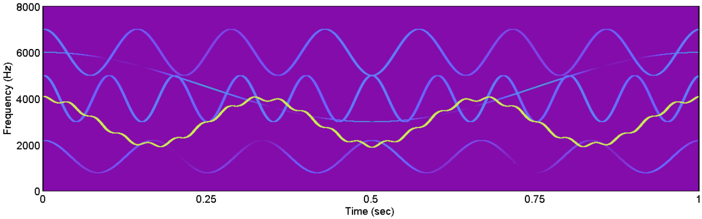
1. Select the Single IMF with Greatest Total Energy
   - The tracking is unable to switch between IMFs to track high energy and thus may not track the dominant component
2. Select the IMF with the Greatest Energy at Each Time Instant
   - The tracking always switches between IMFs to track high energy (making it susceptible to noise) and thus may not track the dominant component

Outline

Experiment #1: Synthetic Signal

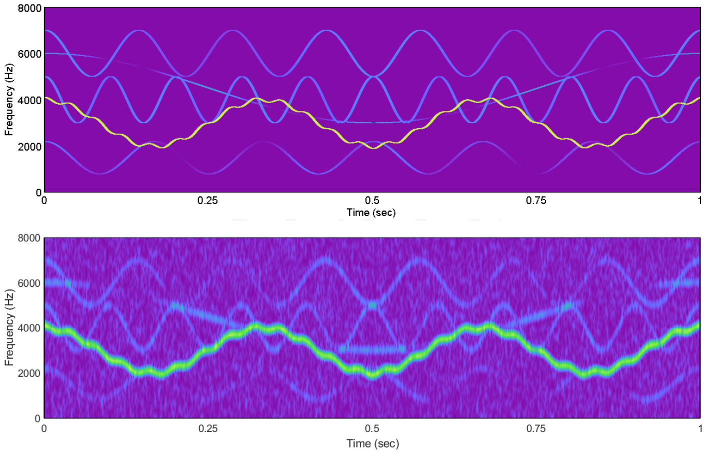
Ground Truth without Noise

- Five known deterministic components—one of which is dominant
- Gaussian noise (not shown below)



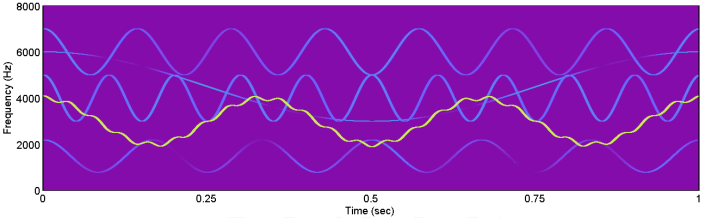
Experiment #1: Synthetic Signal

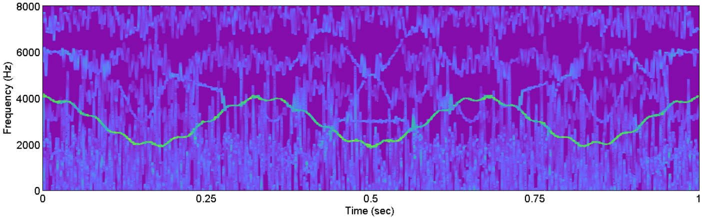
(top) Ground Truth without Noise (bottom) STFT Magnitude for Comparison



Experiment #1: Synthetic Signal

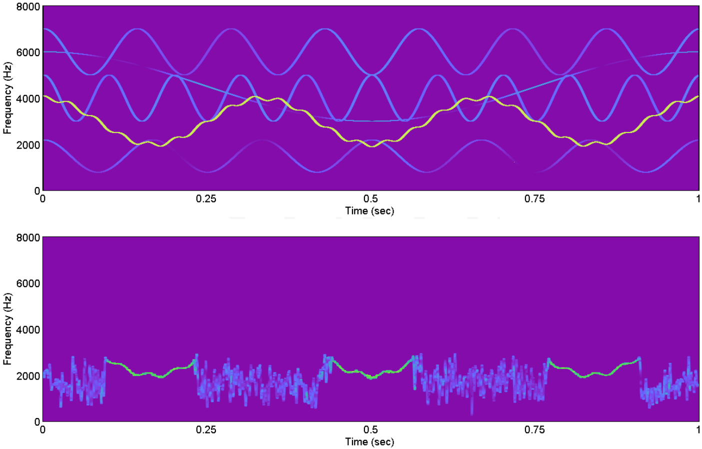
(top) Ground Truth without Noise (bottom) EMD and Demodulation





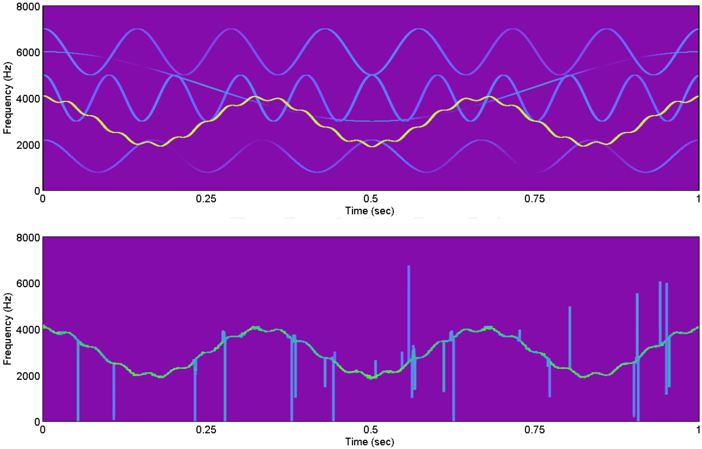
Experiment #1: Synthetic Signal

(top) Ground Truth without Noise (bottom) IMF with Greatest Total Energy (Control Method #1)



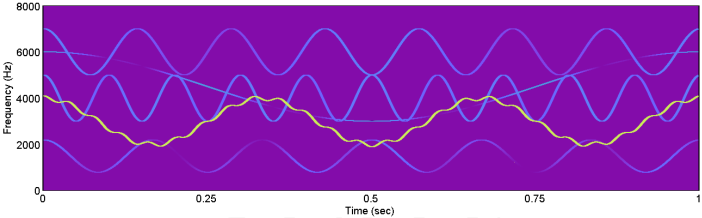
Experiment #1: Synthetic Signal

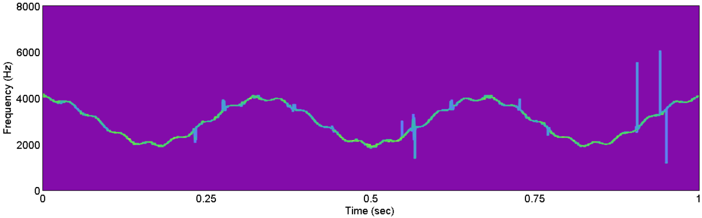
(top) Ground Truth without Noise (bottom) IMF with the Greatest Energy at Each Instant (Control Method #2)



Experiment #1: Synthetic Signal

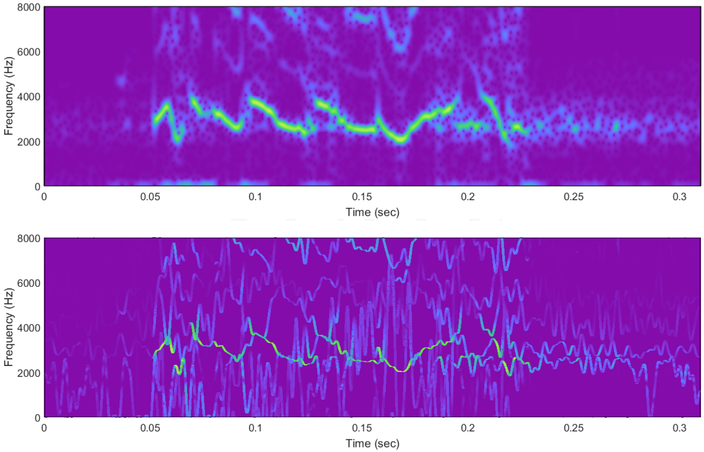
(top) Ground Truth without Noise (bottom) Proposed Method





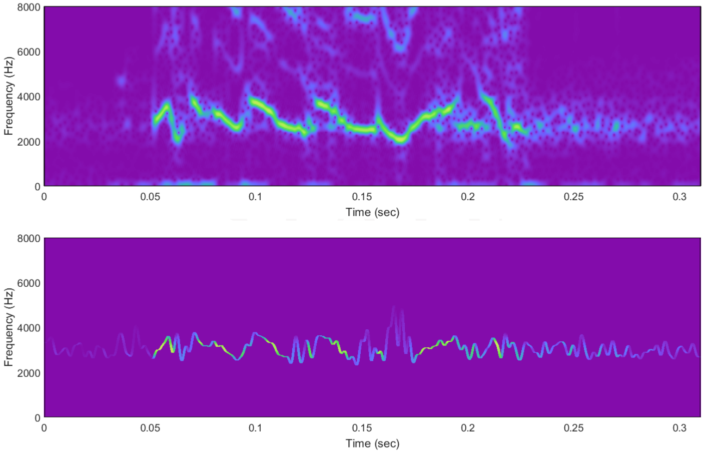
Experiment #2: Parakeet Call

(top) STFT Magnitude (bottom) EMD and Demodulation



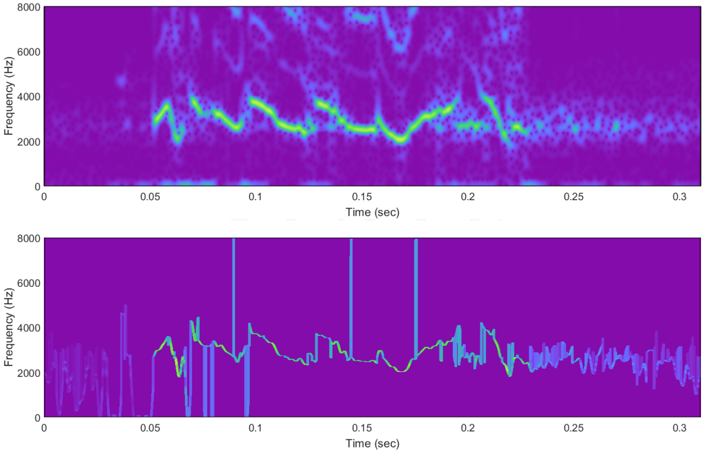
Experiment #2: Parakeet Call

(top) STFT Magnitude (bottom) IMF with Greatest Total Energy (Control Method #1)



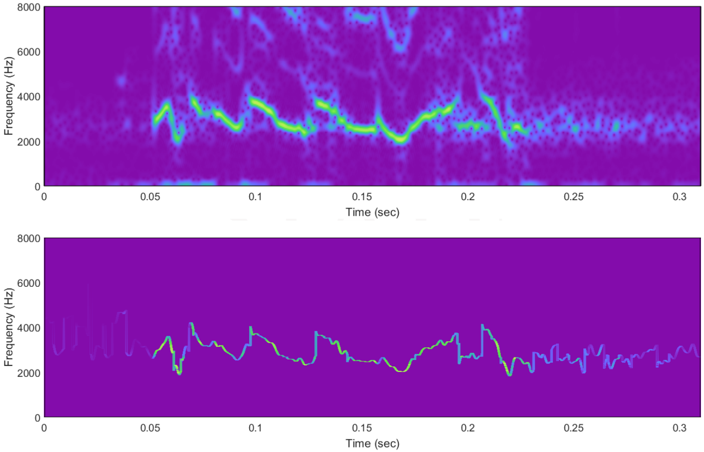
Experiment #2: Parakeet Call

(top) STFT Magnitude (bottom) IMF with the Greatest Energy at Each Instant (Control Method #2)



Experiment #2: Parakeet Call

(top) STFT Magnitude (bottom) Proposed Method



Outline

Concluding Remarks

- We proposed a method of Dominant Component Tracking for Empirical Mode Decomposition using a Hidden Markov Model.
- We evaluated our method using an recording of a parakeet call and compared against two control methods.
- The proposed method led to a dominant component track which yields a compromise between smoothness and energy associated with the track.
- Some items for consideration in future work include:
  1. a more sophisticated formation of the time-varying transition probability matrix to further suppress sporadic jumps, and
  2. introduction of a parameter to control the trade-off between smoothness and energy associated with the track.

Questions?

References I

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IMF

An IMF is defined by two parameters:

1. In the whole signal segment, the number of extrema and the number of zero crossings must be either equal or differ at most by one
2. At any point the mean value of the envelope, defined by the local maxima and the envelope defined by the local minima, is zero.

HMM Forward Algorithm

Algorithm 1 Forward Algorithm

procedure $\alpha_n[k] = \text{forward}(\ \mathbf{A},\ \mathbf{B},\ \mathbf{\Pi}\ )$
    initialize: $\alpha_1[k] = \pi_k b_k(o_1),\ 1 \leq k \leq K$
    for $n = 2, 3, \cdots, N$ **do**
        for $k \in \{1, 2, \cdots, K\}$ **do**
            $\alpha_n[k] = \sum_{i=1}^{K} \alpha_{n-1}[i]\mathsf{a}_{ik}[n]b_k(o_n)$
        end for
    end for
end procedure

HMM Backward Algorithm

Algorithm 2 Backward Algorithm

procedure $\beta_n[k] = \text{backward}(\ \mathbf{A},\ \mathbf{B},\ \mathbf{\Pi}\ )$
    initialize: $\beta_N[k] = 1,\ 1 \leq k \leq K$
    for $n = N-1, N-2, \cdots, 1$ **do**
        for $k \in \{1, 2, \cdots, K\}$ **do**
            $\beta_n[k] = \sum_{j=1}^{K} \mathsf{a}_{kj}[n] b_j(o_{n+1}) \beta_{n+1}[j]$
        end for
    end for
end procedure

Viterbi Algorithm

Algorithm 3 Viterbi Algorithm

procedure $[q_n^\star, v^\star] = \text{Viterbi}(\,\mathbf{A},\ \mathbf{B},\ \mathbf{\Pi}\,)$
  initialize: $v_1[k] = \pi_k b_k(o_1),\ 1 \le k \le K$
  initialize: $p_1[k] = 0,\ 1 \le k \le K$
  for $n = 2, 3, \cdots, N$ **do**
    for $k \in \{1, 2, \cdots, K\}$ **do**
      $v_n[k] = \max_i \ v_{n-1}[i] \mathsf{a}_{ik}[n] b_k(o_n)$
      $p_n[k] = \operatorname{argmax}_i \ v_{n-1}[i] \mathsf{a}_{ik}[n] b_k(o_n)$
    end for
  end for
  $v^\star = \max_i \ v_N[i]$
  $q_N^\star = \operatorname{argmax}_i \ v_N[i]$
  for $n = N-1, N-2, \cdots, 2$ **do**
    $q_n^\star = p_n[q_{n+1}^\star]$
  end for
end procedure