Fast Phase-Difference-Based DoA Estimation Using Random Ferns

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1. Motivation and Background

- Direction-of-Arrival (DoA) is important in communications, localization, object tracking and so on.
- Frequency-diversity based methods combine with a search procedure can solve the phase-unwrapping problem in DoA estimation, but the computational time is high.
- A machine learning technique named random ferns can speed up the search procedure.

2.Frequency-Diversity-Based DoA Method

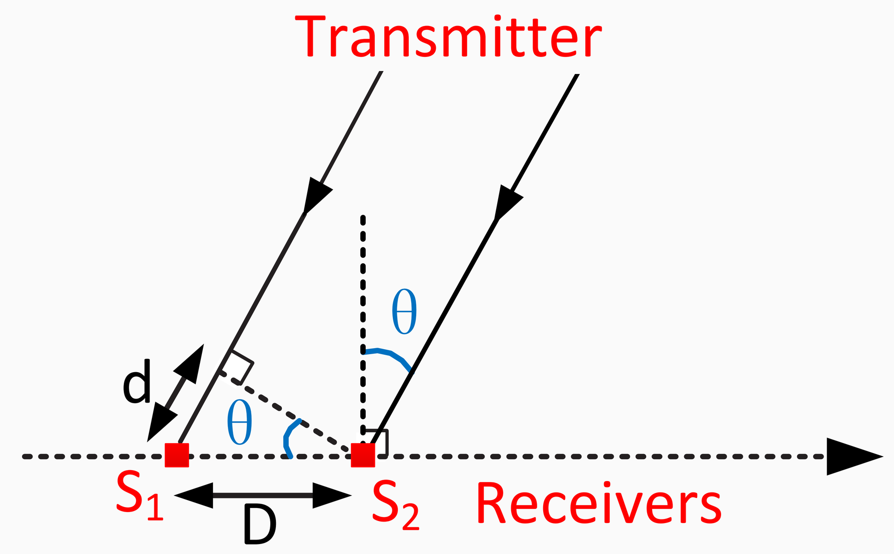


Fig.1. Far-field DoA model.

The noise-free phase difference $\phi_i(\theta)$ at f_i is:

$$\phi_i(\theta) = \frac{2\pi d}{\lambda_i} = \frac{2\pi f_i \sin(\theta) D}{v} \quad (i = 1, ..., N), \tag{1}$$

- f_i: the i-th frequency component.
- v: the speed of signal propagation.

The observed phase-difference $\hat{\psi}_i$ is:

$$\hat{\psi}_i = \text{ang}(Y_1(f_i) \cdot Y_2^*(f_i)) = \hat{\phi}_i - 2\pi N_i, \tag{2}$$

- Y_1: the DFT of the received signals at sensor S_1.
- $(\cdot)^*$: the complex conjugate operation.
- ang$(\cdot)$: the operation returns the angle of a complex number.
- $\hat{\phi}_i$: the noisy unwrapped phase difference.

The phase-difference mismatch function for a hypothesized source direction θ_m is:

$$e(\theta_m) = \sum_{i=1}^{N} |\hat{\psi}_i - \psi_i(\theta_m)| = \sum_{i=1}^{N} |\hat{\psi}_i - \text{wrap}(\phi_i(\theta_m))|, \tag{3}$$

$$\text{wrap}(\alpha) = \left(\frac{\alpha}{2\pi} - \left\lfloor \frac{\alpha}{2\pi} \right\rceil \right) \cdot 2\pi, \tag{4}$$

- $\lfloor . \rceil$: returns the nearest integer.

The DoA estimation can be obtained as:

$$\hat{\theta} = \arg\min_{\theta_m} \{e(\theta_m)\}. \tag{5}$$

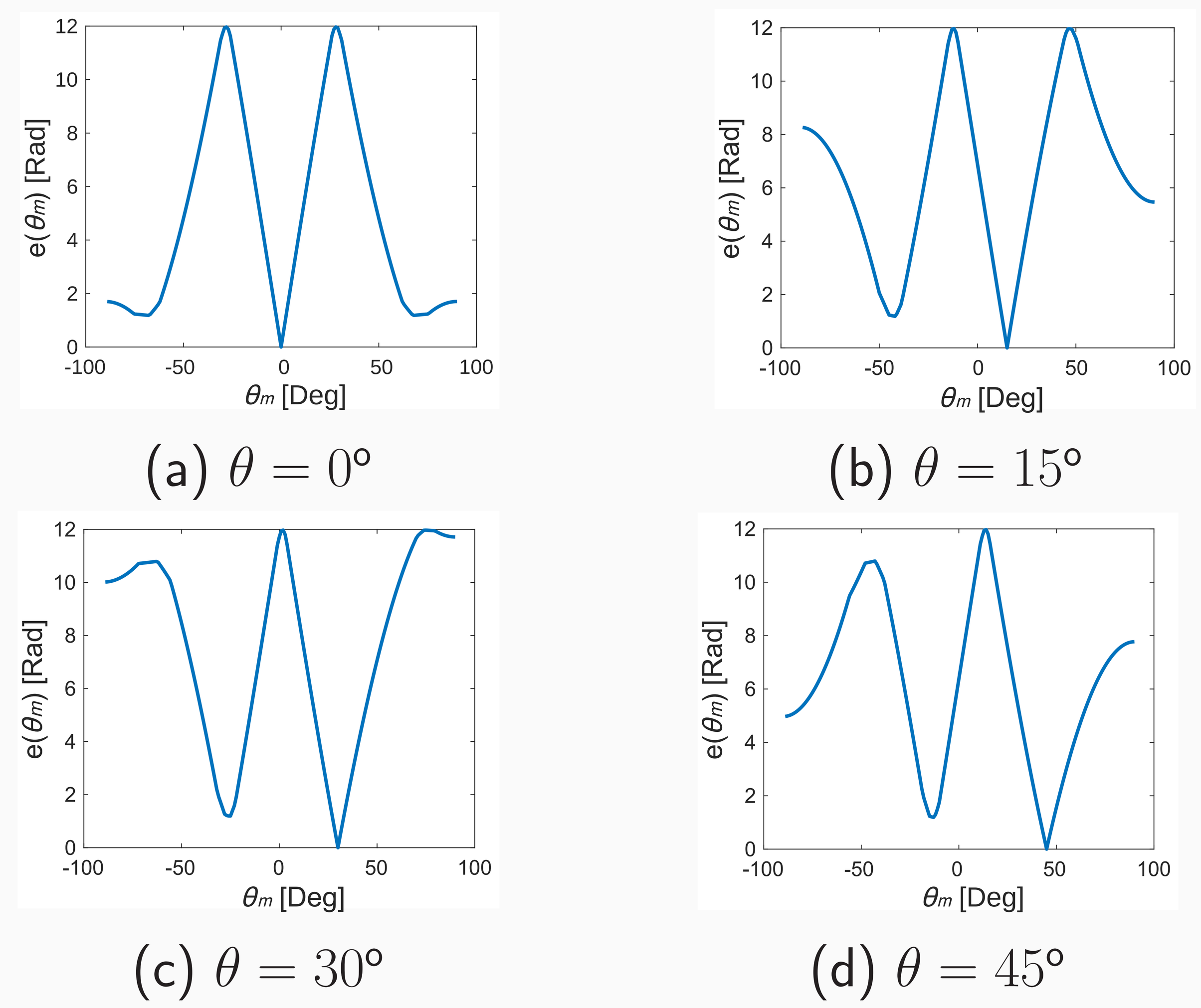


Fig.2. The mismatch function for different source directions.

3. Fern Matrix

Fern matrix cluster the mismatch functions of the candidates into several groups.

Define a error matrix **E** $(M \times M)$ utilizing Equation (3):

$$E_{m1,m2} = \sum_{i=1}^{N} \left|\text{wrap}\left[\tilde{\phi}_i(\theta_{m1}) - \tilde{\phi}_i(\theta_{m2})\right]\right|, \tag{6}$$

By randomly picking k angles from the M candidates as fern points, a set of $K = k(k-1)/2$ (k choose 2) pairs can be created. A fern matrix **R** can be created as:

$$r_{m,i} = \begin{cases} 1, & E_{m,K_{i,1}} \geq E_{m,K_{i,2}} \\ -1, & E_{m,K_{i,1}} < E_{m,K_{i,2}} \end{cases}, \tag{7}$$

- r_m: the fern vector of the m-th candidate.

An example of encoding error pattern into a fern vector at 10°: The fern points are $[-69^\circ, -29^\circ, -4^\circ, 7^\circ, 47^\circ]$. The encoded fern vector for the pair $(7, 47), (-4, 47), ..., (-29, -69)$ is $[-1, -1, ..., 1]^T$ using Equation (7).

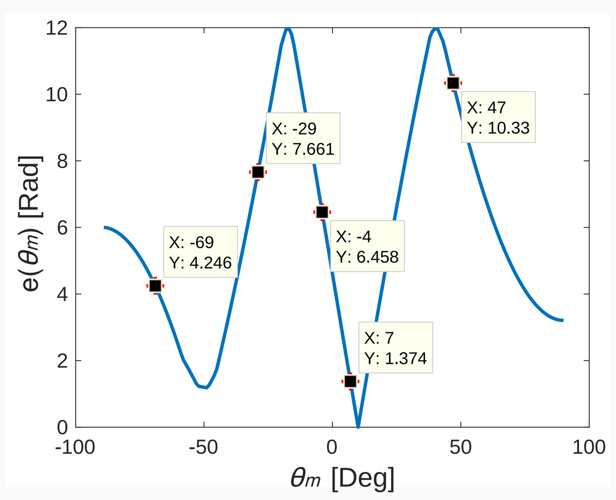


Fig.3. An example of encoding a fern vector.

By encoding the fern vector from binary to decimal, the DoA candidates can be clustered into several groups as shown in Fig.4.

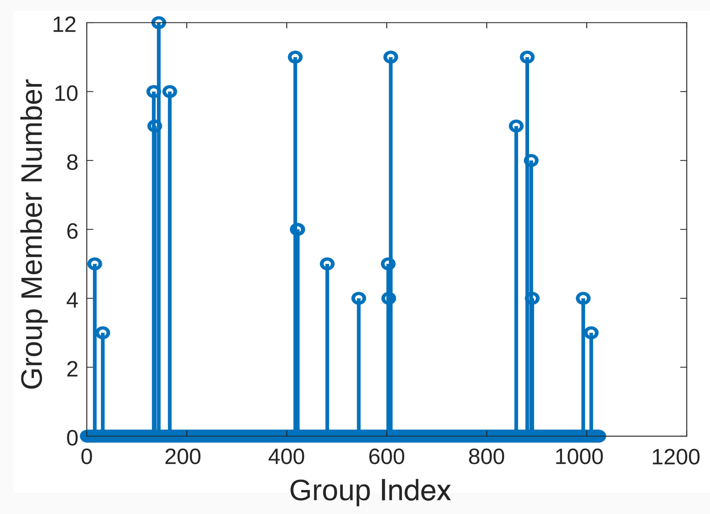


Fig.4. Clustered angle candidates.

We use entropy H as the indicator for evaluating the Fern points selection. For a certain fern vector length, a high entropy value indicates a better fern selection.

$$H = -\sum_{i=1}^{L} p_i \log(p_i), \tag{8}$$

- L: the number of the clustered groups.
- p_i: candidate-number in group i divided by M.

4. Fast DoA Algorithm Using Random Fern

For an observed phase-difference vector, the fern vector $\mathbf{r}_{est}$ can be calculated from Equation (3) and (7). We can then reduce the search area by finding the highest cross-correlation value in **c** given by:

$$\mathbf{c} = \mathbf{R}\mathbf{r}_{est}. \tag{9}$$

The DOA estimation algorithm using random ferns can be summarized as follows:

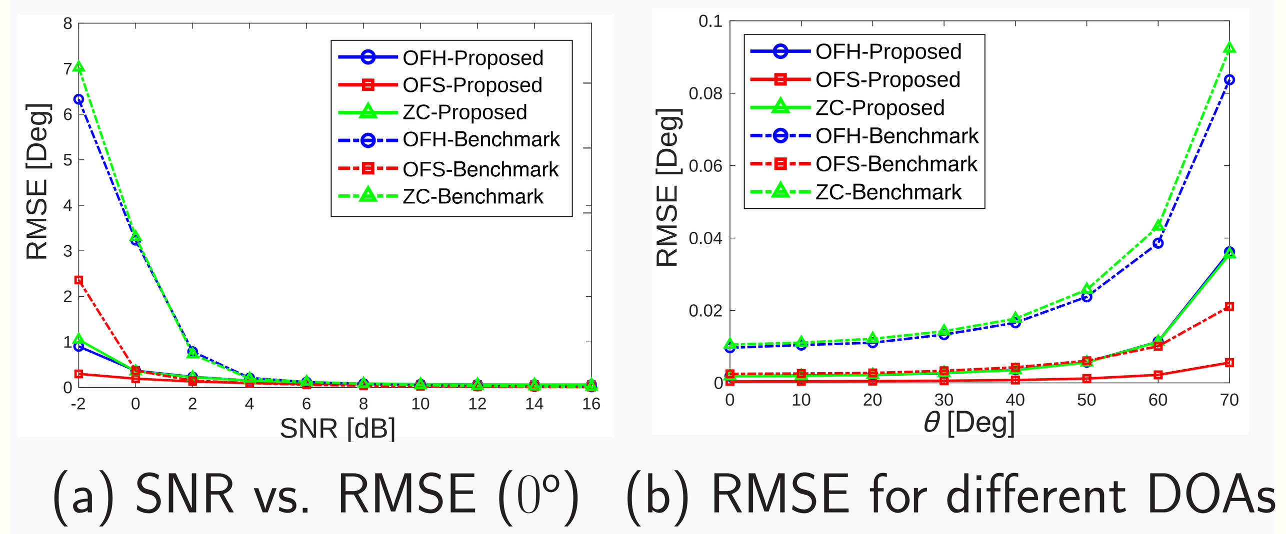
- 1-Calculate the errors at fern points using (3);
- 2-Encode the received signal into fern vector r_{est} using (7);
- 3-Obtain the candidate cross-correlation values using (9).
- 4-Choose the candidates with the highest values (Some candidates may have the same fern vector which results in the same cross-correlation value).
- 5-Find the least phase difference error among the DOA candidates using (5) to obtain the search result.

5. Simulation and Experimental Results

Table 1: Fern Points Selection Summary.

Fern Num	Fern Bits	Max Entropy	Max GS	Avg GS	Processing Time [ms]
2	1	1.000	90	90	0.728
3	3	2.496	47	30	0.301
4	6	3.507	23	12.86	0.203
5	10	4.218	16	6.92	0.174
6	15	4.996	14	4.09	0.180
7	21	5.397	11	3.11	0.189

- GS: Group Size.



(a) SNR vs. RMSE (0°) (b) RMSE for different DOAs

Fig.5. Simulation results.

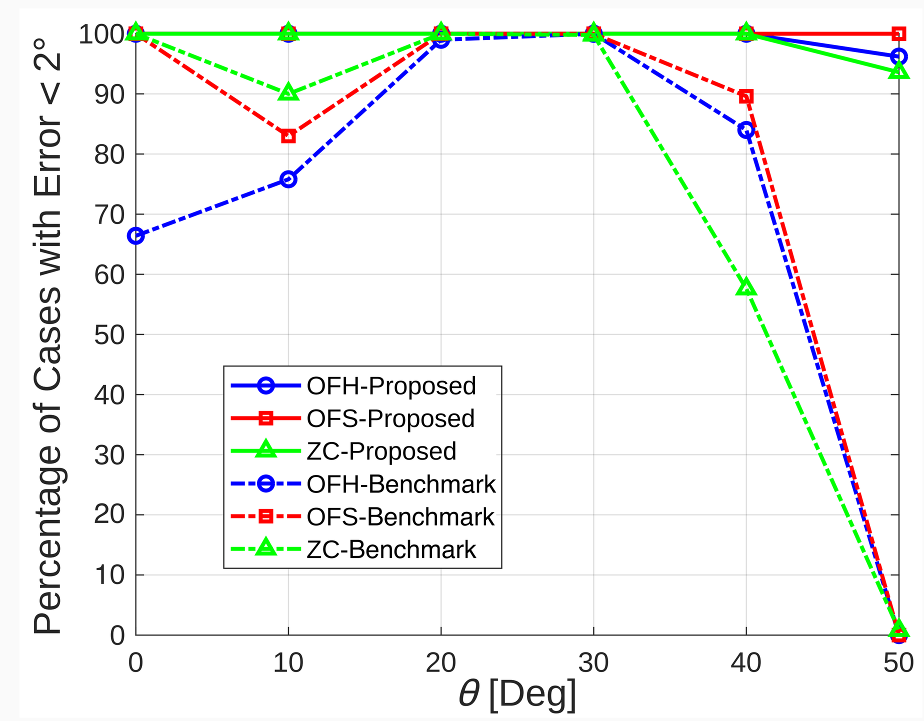


Fig.6. Experimental results for source at different DOAs.

6. Conclusions

- An algorithm for DOA estimation from phase-difference observations is presented. Random fern speeds up this algorithm by 6 times compared to exhaustive search while maintaining the same accuracy.
- Simulation and experimental results confirm the effectiveness of the proposed approach.
- Future works will focus on studying the effect of Doppler, multipath and multi-user interference.

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