The Greedy Dirichlet Process Filter
An Online Clustering Multi-Target Tracker

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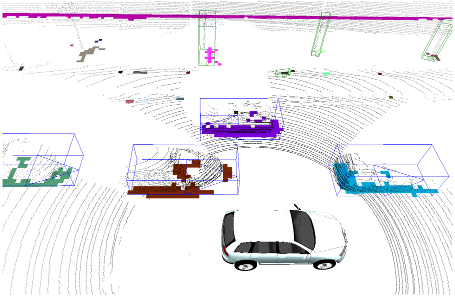
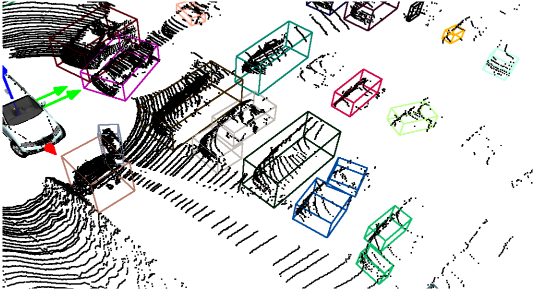
Problem Description

- Goal:
  - Estimate dynamics and dimensions of an unknown number of targets in real-time
  - Associate measurements to correct target
- Challenges:
  - Multiple measurements of one target
  - Unknown number of targets





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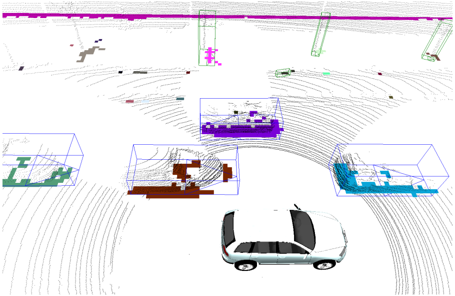

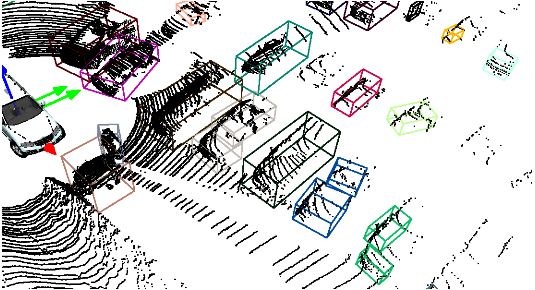

Dirichlet Process

- Bayesian non-parametric model
- It is a distribution over distributions with an infinite amount of mixture components
- Only finite ones are activated by observations
- Defined by a concentration parameter $\alpha \in \mathbb{R}$ and random mixing measure G_0

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Chinese Restaurant Process

- Realization of a DP
- Either: new customer (measurement) chooses new table (component) proportional to α
- Or: joins known table with probability proportional to the number of occupying customers

$\Rightarrow$ Conditional prior, with $n_k(t)$ – the number of assigned measurements to cluster k [1]:

$$CRP(\alpha) = \begin{cases} \frac{n_k(t)}{i-1+\alpha} & k \in K_t, \\ \frac{\alpha}{i-1+\alpha} & \text{else} \end{cases} . \tag{1}$$

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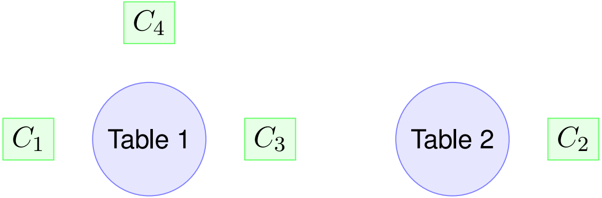
Example of CRP

- $CRP(\alpha)$ for assignment of customer C_5 with $\alpha = 0.1$:
  - Table 1: $\frac{3}{5-1+0.1} = \mathbf{0.73}$
  - Table 2: $\frac{1}{5-1+0.1} = 0.24$
  - New Table: $\frac{0.1}{5-1+0.1} = 0.024$



Distance-Dependent CRP I

- Links customers to customers rather than tables
- Conditional prior, with link assignment $j_i(t)$ and set of previous assignments $\boldsymbol{j}_i$ [2]:

$$p(j_i(t) = l|\boldsymbol{j}_{-i}, \alpha) = \begin{cases} d_{il}(\boldsymbol{y}_i(t), \boldsymbol{y}_l(t)) & i \neq l, \\ \alpha & i = l \end{cases}, \tag{2}$$

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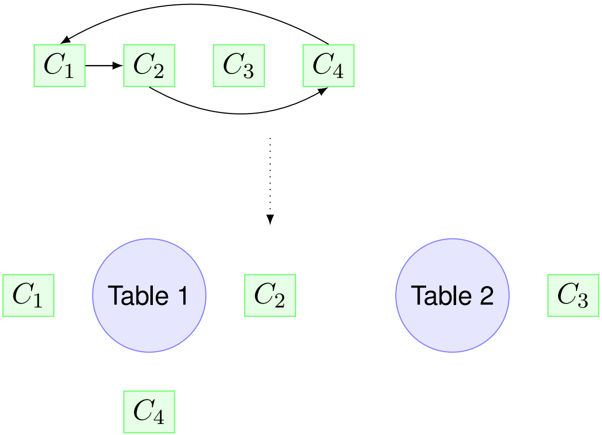
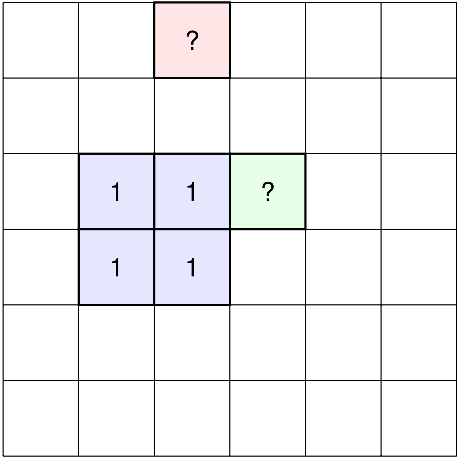
Distance-Dependent CRP II



Grid Example for Distance-Dependent CRP

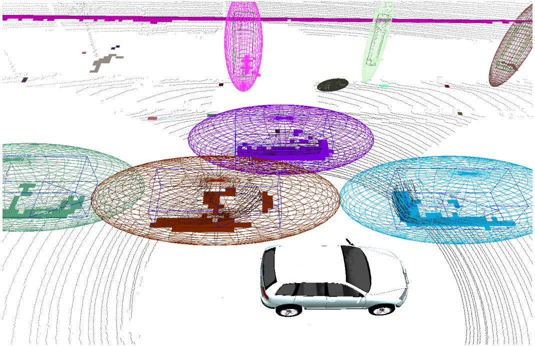


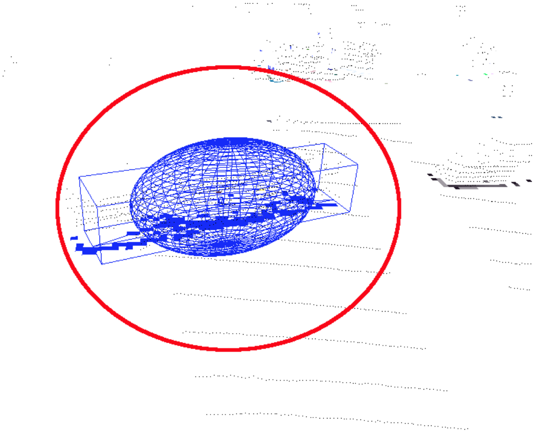
Data to Cluster Assignment

- Can be interpreted as a cluster prior dependent on the current cluster parameters
- Measurement $\boldsymbol{y}_i(t)$ to cluster k assignment [1]:

$$\begin{aligned}\pi_{z_i(t)=k} &= p\left(z_i(t) = k|\boldsymbol{z}(t-1), \boldsymbol{y}_i(t)\right) \\ &= p(\boldsymbol{y}_i(t)|\boldsymbol{\theta}_k(t)) \cdot p\left(z_i(t) = k|\boldsymbol{z}(t-1)\right),\end{aligned} \quad (3)$$

Cluster Prior Examples





Kalman Filtering

- Kalman Filtering is used for the dynamical part $\boldsymbol{x} \in \mathbb{R}^n$ of the mixture components
- Probabilistic Gaussian state space model, with transition matrix $\Phi(t-1) \in \mathbb{R}^{n \times n}$, measurement model matrix $\mathbf{C}(t) \in \mathbb{R}^{m \times n}$, unbiased and Gaussian process noise $\mathbf{Q}(t-1)$ and Gaussian measurement noise $\mathbf{R}(t)$ [3]:

$$p(\boldsymbol{x}(t)|\boldsymbol{x}(t-1)) = \mathcal{N}\left(\boldsymbol{x}(t)|\Phi(t-1)\boldsymbol{x}(t-1), \mathbf{Q}(t-1)\right) \tag{4}$$

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Basic Procedure

Greedy Dirichlet Process Filter (GDPF) consists of the following two main steps:

- Choosing best label for measurement $\boldsymbol{y}_i$
- Update posterior distribution

Choosing the Best Label

- Conditional posterior probability of assigning measurement $\boldsymbol{y}_i(t)$ to cluster k given previous data for measurements $\boldsymbol{Y}^i(t)$ is a combination of distance-dependent CRP and data to cluster assignment
- Conditional posterior probabilities [1]:

$$p\left(z_i(t) = k|\boldsymbol{Y}^i(t), \boldsymbol{j}_{-i}, \boldsymbol{z}(t-1)\right) = \frac{p(j_i = l_k|\boldsymbol{j}_{-i}, \alpha) \cdot \pi_{z_i(t)=k}}{\sum_{m \in K_t} p(j_i = l_m|\boldsymbol{j}_{-i}, \alpha) \cdot \pi_{z_i(t)=m}}. \tag{6}$$

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Posterior Update

- Posterior distribution of cluster parameters consists of:
  - Generation of new components: $G_0(\theta_{z_i(t)=k}(t))$
  - Dynamical part: $p(\boldsymbol{y}_i(t)|\boldsymbol{\theta}_{z_i(t)=k}(t))$
  - Time evolution of cluster parameters: $p(\boldsymbol{\theta}_{z_i(t)=k}(t)|\boldsymbol{\theta}_{z_i(t-1)=k}(t-1))$
- It follows [1]:

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p(\theta_{z_i(t)}|\boldsymbol{y}_{i-1}(t), \boldsymbol{z}(t)) \propto\ & G_0(\theta_{z_i(t)=k}(t)) \\
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Test Scenario

- Real-world scenario in the suburbs near our university
- Passing cars as dynamic objects
- Ground truth obtained with an installed INS-sensor
- Goal: Estimate x- and y-position of our ground-truth object without id-switches
- Tested against:
  - Labeled Multi-Bernoulli Filter (LMB) [4]
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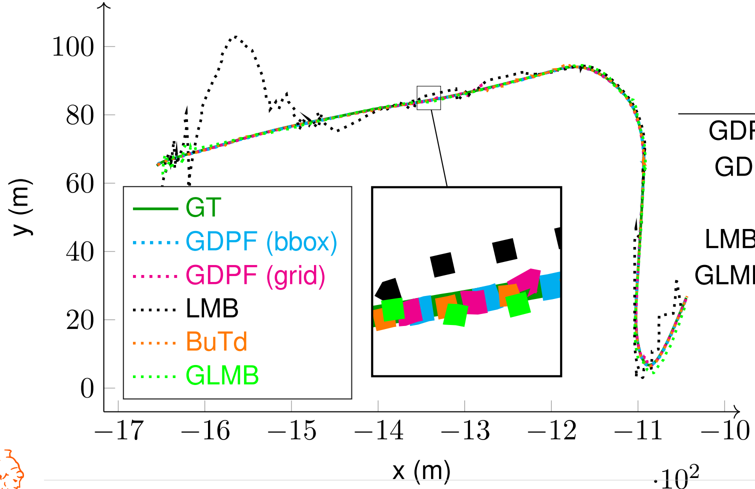
Comparing the Trajectories

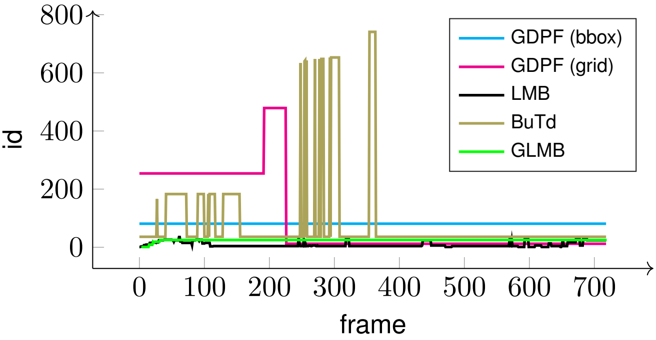


Filter	RMSE	id-switches
GDPF (bbox)	0.63657	0
GDPF (grid)	0.89681	2
LMB	62.729	69
LMB (low det)	3.539	33
GLMB (low det)	2.334	5
BuTd	0.68749	31

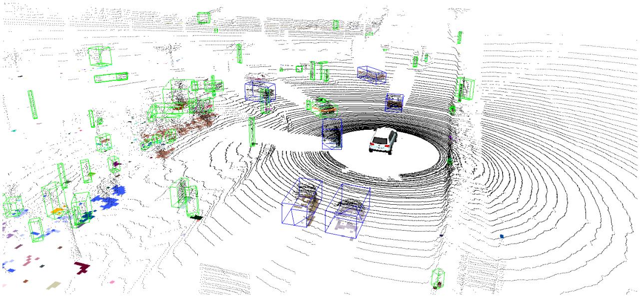
ID-switches and Run-time

Run-times for a mean of 193 objects:

- Grid: 58ms
- Bounding-Boxes: 34ms



Video Footage of the Test-Drive



Conclusions and Future Work

- Proposed GDPF can robustly track an unknown number of targets
- Real-time capable even for a large number of targets
- Probabilistic data association can handle segmentation errors and unclustered data
- We demonstrated improved tracking results compared to popular approaches

Possible Future work:

- Extend to classifying filter to utilize class-specific priors
- Modeling the target appearance by integrating extended object tracking

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Thank you.