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# A Low-Complexity LS Turbo Channel Estimation Technique for MU-MIMO Systems

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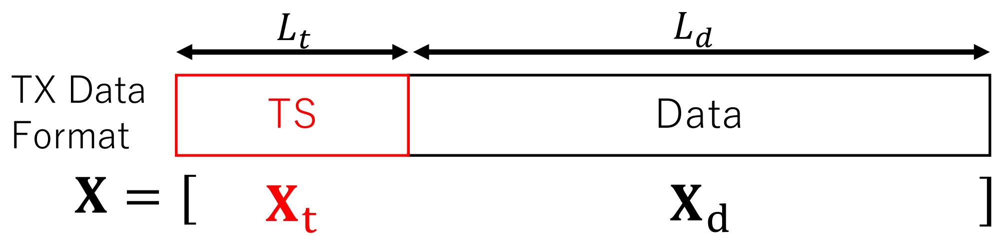
# Summary

- Turbo Receiver improves the MUI problem in MU-MIMO sys.
- However, the complexity expands as Num. of ANT increases
- Because, in order to deal Spatial matrix $\boldsymbol{\Gamma}$ correctly

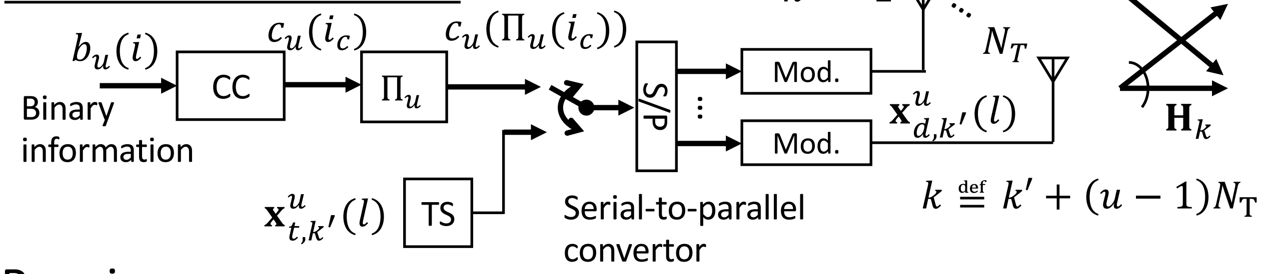
**NEW: Low complexity LS algo. for Turbo receiver**

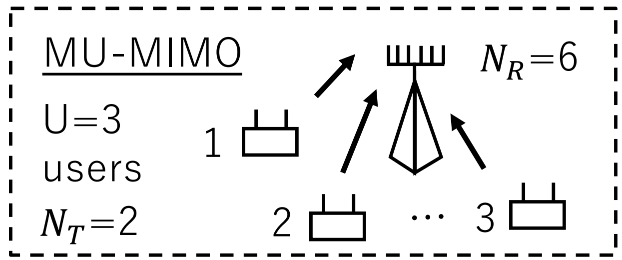
- Independent of Rx Ants.: $\boldsymbol{\mathcal{O}}\left(\boldsymbol{\kappa}^3 \boldsymbol{N}_T^3\right) \leftarrow \boldsymbol{\mathcal{O}}\left(\boldsymbol{\kappa}^3 \boldsymbol{N}_T^3 \boldsymbol{N}_R^3\right)$
- **No Accuracy Deterioration !**
- Algebraic property of Cov. Matrix $\mathfrak{R}_{\mathrm{XX}}$ is utilized

# System Model

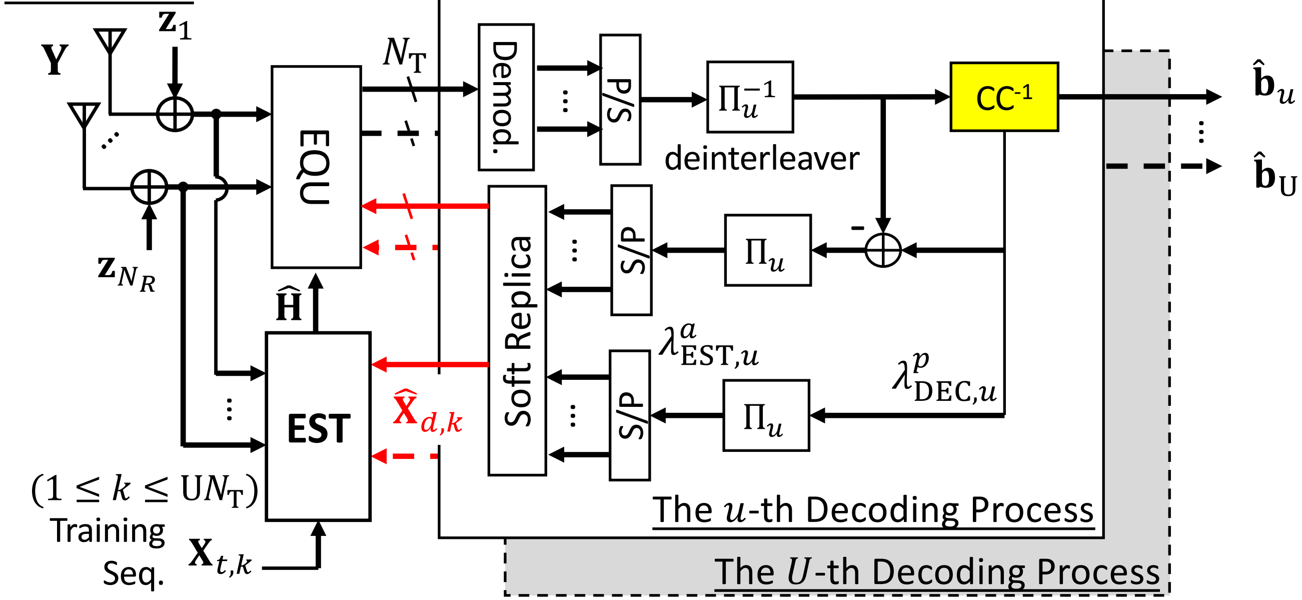



## The $u$-th Transmitter




## Receiver



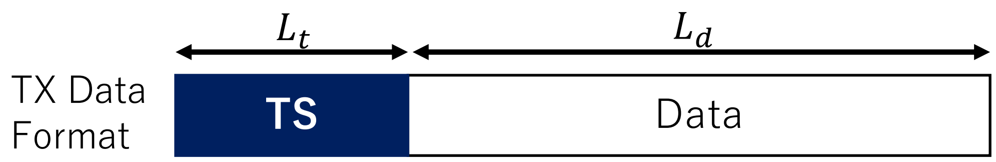

## Turbo Receiver

$\hat{\mathbf{X}}_{d,k}$ : Replica of TX Data can be obtained from Decoder's feedback info.

# LS
# Channel Estimation

$$\hat{\mathbf{H}} = \mathrm{argmin}_{\mathbf{H}} \mathcal{L}_t(\mathbf{H})$$
$$= \mathbf{Y}_\mathrm{t}\mathbf{X}_\mathrm{t}^{+} = \mathbf{Y}_\mathrm{t}\mathbf{X}_\mathrm{t}^{\mathrm{H}}\left(\mathbf{X}_\mathrm{t}\mathbf{X}_\mathrm{t}^{\mathrm{H}}\right)^{-1}$$

$$\mathcal{L}_\mathrm{t}(\mathbf{H}) = \frac{1}{\sigma_Z^2}\|\mathbf{Y}_\mathrm{t} - \mathbf{H}\mathbf{X}_\mathrm{t}\|^2$$




$\mathbf{X} = [\ \mathbf{X}_\mathrm{t}\ \ \mathbf{X}_\mathrm{d}\ ]$ (Known, Unknown)

Rx Signal: $\mathbf{Y} = \mathbf{H}\mathbf{X} + \mathbf{Z}$

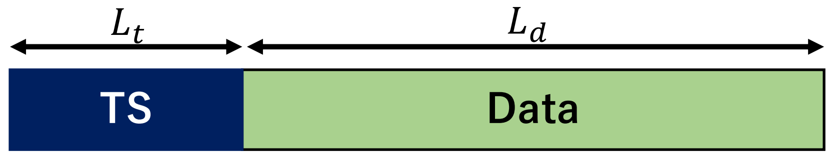
$\mathbf{H}$: Channel

$\mathbf{X}$: TX Signal

$\mathbf{Z}$: AWGN $\sim \mathcal{CN}(0, \sigma_Z^2)$

# LS Turbo Channel Estimation

TX Data Format




$\mathbf{X} = [\ \mathbf{X}_{\mathrm{t}}\ \ \mathbf{X}_{\mathrm{d}}\ ]$

Known — Unknown

$$\hat{\mathbf{H}} = \mathrm{argmin}_{\mathbf{H}}\, \mathcal{L}_{td}(\mathbf{H}), \text{ where } \mathcal{L}_{td}(\mathbf{H}) = \mathcal{L}_{t}(\mathbf{H}) + \mathcal{L}_{d}(\mathbf{H})$$

**Joint** Log-Likelihood Function

$$\begin{cases} \mathcal{L}_{\mathrm{d}}(\mathbf{H}) = \dfrac{1}{\sigma_z^2}\left\|\mathbf{Y}_{\mathrm{t}} - \mathbf{H}\hat{\mathbf{X}}_{\mathrm{d}}\right\|_{\boldsymbol{\Gamma}}^{2} \\ \qquad = \dfrac{1}{\sigma_z^2}\mathrm{tr}\left\{\left(\mathbf{Y}_{\mathrm{d}} - \mathbf{H}\hat{\mathbf{X}}_{\mathrm{d}}\right)^{\mathrm{H}}\boldsymbol{\Gamma}\left(\mathbf{Y}_{\mathrm{d}} - \mathbf{H}\hat{\mathbf{X}}_{\mathrm{d}}\right)\right\} \\ \boldsymbol{\Gamma} = \left(\mathbf{I}_{N_{\mathrm{R}}} + \displaystyle\sum_{u}\frac{\Delta\sigma_d^2}{\sigma_z^2}\mathbf{R}_{\mathbf{H},u}\right) \end{cases}$$

**Spatial Weight** Matrix

**LS Solution:**

$$\mathrm{vec}\{\widehat{\mathbf{H}}\} = \mathfrak{R}_{\mathbf{XX}}^{-1} \cdot \mathrm{vec}\{\mathbf{R}_{\mathbf{YX}}\}$$

Gaussian Elimination?
$\mathcal{O}\left((WUN_TN_R)^3\right)$

$WUN_TN_R \times WUN_TN_R$ Matrix

$$\mathfrak{R}_{\mathbf{XX}} = \mathbf{R}_{\mathbf{XX}_\mathrm{t}}^{\mathrm{T}} \otimes \mathbf{I}_{N_R} + \widehat{\mathbf{R}}_{\mathbf{XX}_\mathrm{d}}^{\mathrm{T}} \otimes \mathbf{\Gamma}$$

$$\mathbf{R}_{\mathbf{YX}} = \mathbf{R}_{\mathbf{YX}_\mathrm{t}} + \mathbf{\Gamma}\widehat{\mathbf{R}}_{\mathbf{YX}_\mathrm{d}}$$

$$\mathbf{R}_{\mathbf{XX}_\mathrm{t}} = \mathbf{X}_\mathrm{t}\mathbf{X}_\mathrm{t}^{\mathrm{H}} \qquad \mathbf{R}_{\mathbf{YX}_\mathrm{t}} = \mathbf{Y}_\mathrm{t}\mathbf{X}_\mathrm{t}^{\mathrm{H}}$$

$$\widehat{\mathbf{R}}_{\mathbf{XX}_\mathrm{d}} = \widehat{\mathbf{X}}_\mathrm{d}\widehat{\mathbf{X}}_\mathrm{d}^{\mathrm{H}} \qquad \mathbf{R}_{\mathbf{YX}_\mathrm{d}} = \mathbf{Y}_\mathrm{d}\widehat{\mathbf{X}}_\mathrm{d}^{\mathrm{H}}$$

$\mathcal{O}\left((WUN_T)^2L_{td}\right)$ $\qquad$ $\mathcal{O}(N_R^2L_{td})$

$L_{td} = L_t + L_d$

Matrix size
- $\mathbf{X}_\mathrm{t}$ : $WUN_T \times L_t$
- $\widehat{\mathbf{X}}_\mathrm{d}$ : $WUN_T \times L_d$
- $\mathbf{Y}_\mathrm{t}$ : $N_R \times L_t$
- $\mathbf{Y}_\mathrm{d}$ : $N_R \times L_d$

System Size
- $W$ : CIR length
- $U$ : Num. of Users
- $N_T, N_R$ : Tx, Rx Ants.
- $L_t$, $L_d$: TS, Data Len.

$$\mathfrak{R}_{\mathbf{XX}} = \mathbf{R}_{\mathbf{XX}_\mathrm{t}}^{\mathrm{T}} \otimes \mathbf{I}_{N_R} + \widehat{\mathbf{R}}_{\mathbf{XX}_\mathrm{d}}^{\mathrm{T}} \otimes \mathbf{\Gamma}$$

$$= \left(\mathbf{R}_{\mathbf{XX}_\mathrm{t}}^{\mathrm{T}/2} \otimes \mathbf{I}_{N_R}\right) \mathbf{J} \left(\mathbf{R}_{\mathbf{XX}_\mathrm{t}}^{\mathrm{T}/2} \otimes \mathbf{I}_{N_R}\right)^{\mathrm{H}}$$

$$\mathbf{J} = \mathbf{I}_{WUN_T} \otimes \mathbf{I}_{N_R} + \mathbf{R}_{\mathbf{XX}_\mathrm{t}}^{-\mathrm{H}/2} \widehat{\mathbf{R}}_{\mathbf{XX}_\mathrm{d}}^{\mathrm{T}} \mathbf{R}_{\mathbf{XX}_\mathrm{t}}^{-1/2} \otimes \mathbf{\Gamma}$$

$$= (\mathbf{U}_\mathbf{Q} \otimes \mathbf{U}_\mathbf{\Gamma}) \mathbf{\Sigma}_\mathbf{J} (\mathbf{U}_\mathbf{Q} \otimes \mathbf{U}_\mathbf{\Gamma})^{\mathrm{H}}$$

SVD: $\mathcal{O}(N_R^3)$

$$\mathbf{Q} = \mathbf{U}_\mathbf{Q} \mathbf{\Sigma}_\mathbf{Q} \mathbf{U}_\mathbf{Q}^{\mathrm{H}}$$

$$\mathbf{\Gamma} = \mathbf{U}_\mathbf{\Gamma} \mathbf{\Sigma}_\mathbf{\Gamma} \mathbf{U}_\mathbf{\Gamma}^{\mathrm{H}}$$

$$\mathbf{\Sigma}_\mathbf{J} = \mathbf{I}_{WUN_TN_R} + \mathbf{\Sigma}_\mathbf{Q} \otimes \mathbf{\Sigma}_\mathbf{\Gamma}$$

$$\widetilde{\mathbf{U}}_\mathbf{Q} = \mathbf{R}_{\mathbf{XX}_\mathrm{t}}^{*/2} \mathbf{U}_\mathbf{Q}$$

$\mathcal{O}(WUN_TN_R)$

$$\mathfrak{R}_{\mathbf{XX}}^{-1} = (\widetilde{\mathbf{U}}_\mathbf{Q} \otimes \mathbf{U}_\mathbf{\Gamma}) \mathbf{\Sigma}_\mathbf{J}^{-1} (\widetilde{\mathbf{U}}_\mathbf{Q} \otimes \mathbf{U}_\mathbf{\Gamma})^{\mathrm{H}}$$

Unitary Diagonal Unitary

$$\mathrm{vec}\{\hat{\mathbf{H}}\} = \mathfrak{R}_{\mathbf{XX}}^{-1} \cdot \mathrm{vec}\{\mathbf{R}_{\mathbf{YX}}\}$$

Multiplications of **Huge** Matrices ? $\mathcal{O}\left((WUN_TN_R)^3\right)$

$$\mathfrak{R}_{\mathbf{XX}}^{-1} = \left(\tilde{\mathbf{U}}_{\mathbf{Q}} \otimes \mathbf{U}_{\mathbf{\Gamma}}\right) \cdot \mathbf{\Sigma}_{\mathbf{J}}^{-1} \cdot \left(\tilde{\mathbf{U}}_{\mathbf{Q}} \otimes \mathbf{U}_{\mathbf{\Gamma}}\right)^{\mathrm{H}}$$

$$\hat{\mathbf{H}} = \mathrm{mat}_{\mathrm{N_R}}\left[\mathrm{vec}\{\hat{\mathbf{H}}\}\right]$$

Revert **vec**-operation
$(\mathbf{C}^{\mathrm{T}} \otimes \mathbf{A})\ \mathrm{vec}\{\mathbf{B}\} = \mathrm{vec}\{\mathbf{ABC}\}$

$$= \mathbf{U}_{\mathbf{\Gamma}}\ \mathrm{mat}_{\mathrm{N_R}}[\mathbf{v}]\ \tilde{\mathbf{U}}_{\mathbf{Q}}^{\mathrm{T}}$$

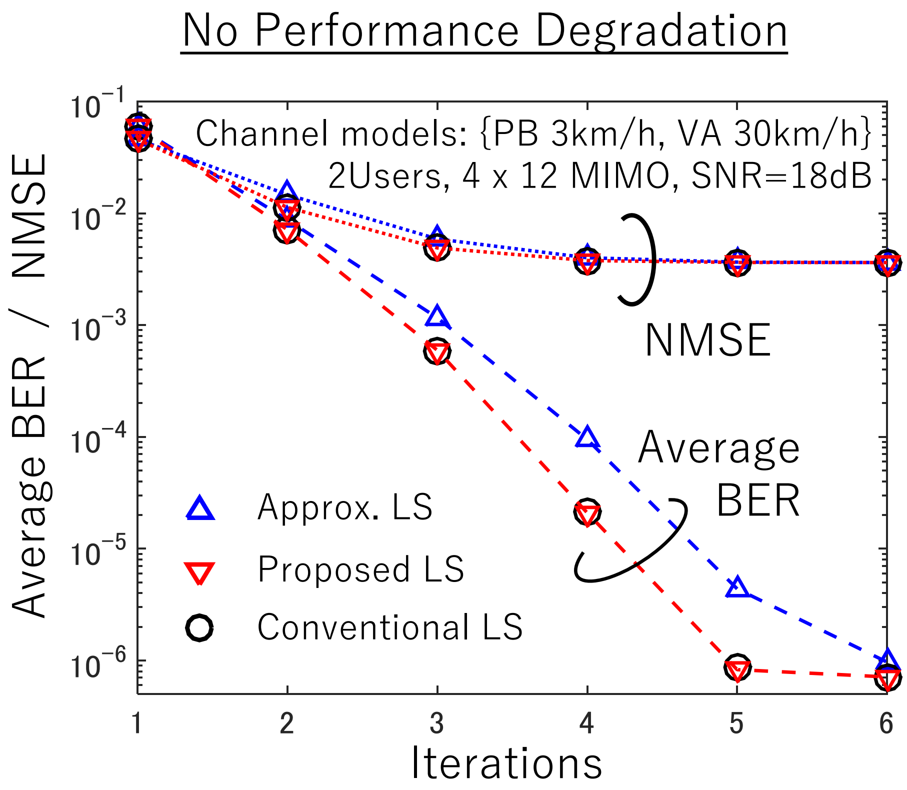
**Small** Matrices $\mathcal{O}(N_R^3)$

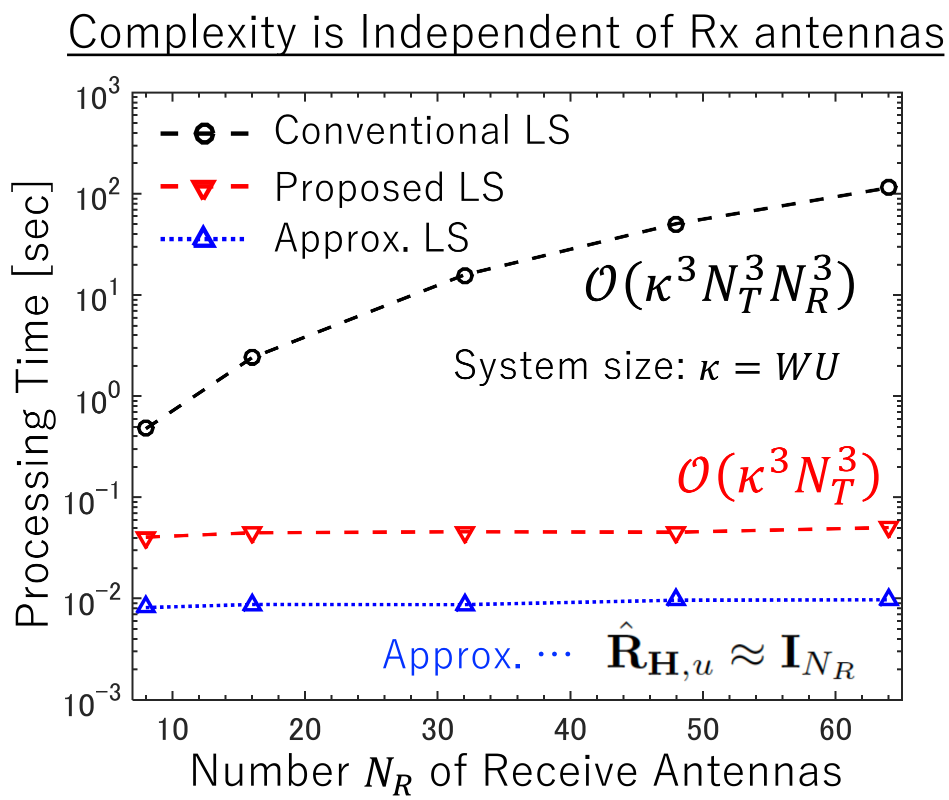
$$\mathbf{v} = \mathrm{diag}\{\mathbf{\Sigma}_{\mathbf{J}}^{-1}\} \odot \mathrm{vec}\{\tilde{\mathbf{U}}_{\mathbf{Q}}^{\mathrm{H}} \mathbf{R}_{\mathbf{YX}} \mathbf{U}_{\mathbf{\Gamma}}^{*}\}$$

Consequently, $\mathcal{O}\left((WUN_T)^3\right)$ when $WUN_T \ll N_R$

$\leftarrow \mathcal{O}\left((WUN_T)^3 + N_R^3 + (WUN_T)^2 L_{td}\right)$

# Numerical Results





[1] Y. Takano, H. J. Su, "A low-complexity LS turbo channel estimation technique for MU-MIMO systems," *IEEE Signal Proc. Lett.*, vol. 25, no. 5, pp. 710-714, May 2018.