Efficient RFI Detection in Radio Astronomy Based on Compressive Statistical Sensing

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Everyone is a genius. *But if you judge a fish by its ability to climb a tree, it will live its whole life believing that it is stupid.*

- Albert Einstein

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Scientific measurements in the radio spectrum are critically affected by external RFI

- Signal Regulatory spectrum management is increasingly unable to prevent spectral leakage into frequency bands designated for radio astronomy (RAS) and earth science (EESS)

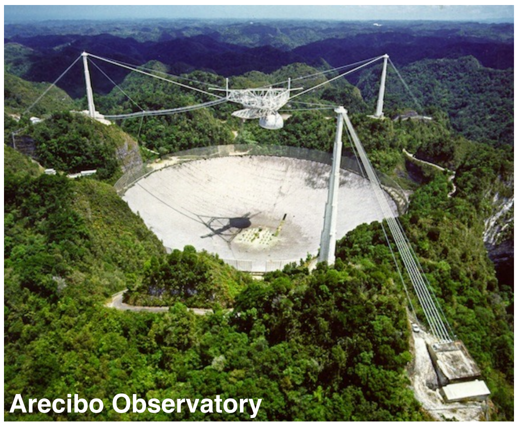
- Scientific interest beyond protected bands is growing with advances in wide bandwidth receivers.

- Effects of RFI contamination on scientific data can be disproportionate:
  - signals of interest are typically faint and require large signal-to-noise ratio to detect
  - RFI can mimic natural signals

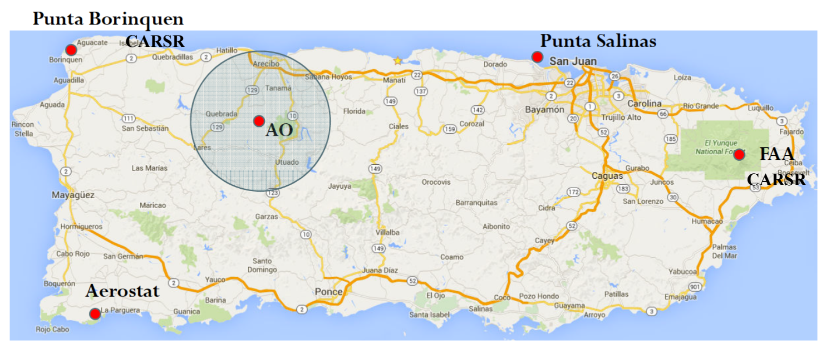
- Narrow-band RFI most affects spectral observations (such as elemental emission lines or plasma resonances), while broad-band RFI can reduce sensitivity, bias calibration, and/or saturate detectors

Quasi-periodic RFI significantly affects passive observations in the L-band for radio astronomy

- This talk will focus on the detection of quasi-periodic RFI from the single dish telescope in Arecibo Radio Observatory, where both passive and active (radar) observations can be strongly contaminated by these external signals.



Ground radars seen in L-band by Arecibo



- Each Common Air Route Surveillance Radar (CARSR) station transmits at 8 distinct frequencies with a 10% duty cycle and multiple IPPs.
- Punta Salinas is frequency-agile with 100 dual-frequency channels.
- Memos of Understanding for directional blanking and transmission over restricted frequencies are in place, but not always honored.

Compressive cyclic analysis as a mean for feature detection such as **cyclo-stationarity** of radio-astronomy signals.

Signal Model

(1) $x(t) = \boxed{\sum_{i=1}^{I} x_i(t)} + \boxed{w(t)}$

RFI

Radio-astronomy data

Cyclo-stationarity

(2) $R_x(\alpha, \tau) = \int R_x(t, \tau) e^{-j2\pi\alpha t} dt$

(3) $S_x(\alpha, f) = \int R_x(\alpha, \tau) e^{-j2\pi f \tau} d\tau,$

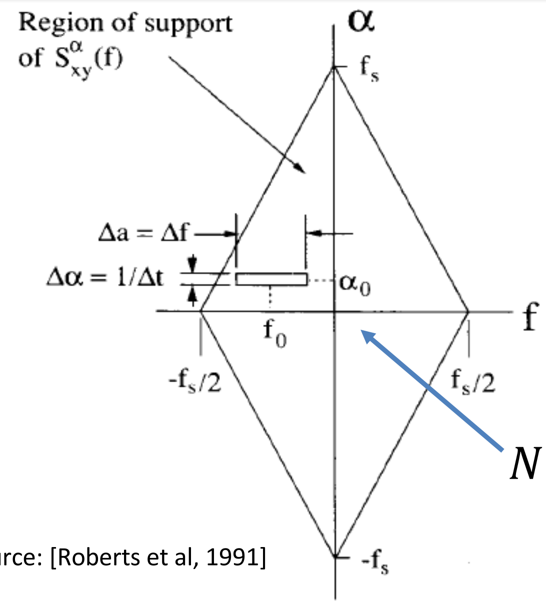


Figure source: [Roberts et al, 1991]

Sparse Cyclic Spectrum Recovery

(4) $\mathbf{z}_t = \mathbf{A}\mathbf{x}_t \qquad z \in \mathbb{R}^{M \times L},\ A \in \mathbb{R}^{M \times N},\ \boldsymbol{x} \in \mathbb{R}^{N \times L}, M/N$

(5) $\text{vec}\{\mathbf{R}_z\} = \text{vec}\{\mathbf{A}\mathbf{R}_x\mathbf{A}^H\} = (\mathbf{A}^* \otimes \mathbf{A})\, \text{vec}\{\mathbf{R}_x\}$

(6) $\text{vec}\{\mathbf{R}_x\} = \text{vec}\{\mathbf{F}^{-1}\mathbf{S}_x\mathbf{F}^{-H}\} = \left(\left(\mathbf{F}^{-H}\right)^T \otimes \mathbf{F}^{-1}\right) \text{vec}\{\mathbf{S}_x\}$

(7) $\mathbf{r}_z = \mathbf{T}\mathbf{s}_x$

Where:

$\mathbf{R}_x = E\{\mathbf{x}_t\mathbf{x}_t^H\} \quad \mathbf{R}_z = E\{\mathbf{z}_t\mathbf{z}_t^H\}$

$\mathbf{r}_z = \text{vec}\{\mathbf{R}_z\} \quad \mathbf{s}_x = \text{vec}\{\mathbf{S}_x\}$

$\mathbf{T} = (\mathbf{A}^* \otimes \mathbf{A})\left((\mathbf{F}^{-H})^T \otimes \mathbf{F}^{-1}\right)$

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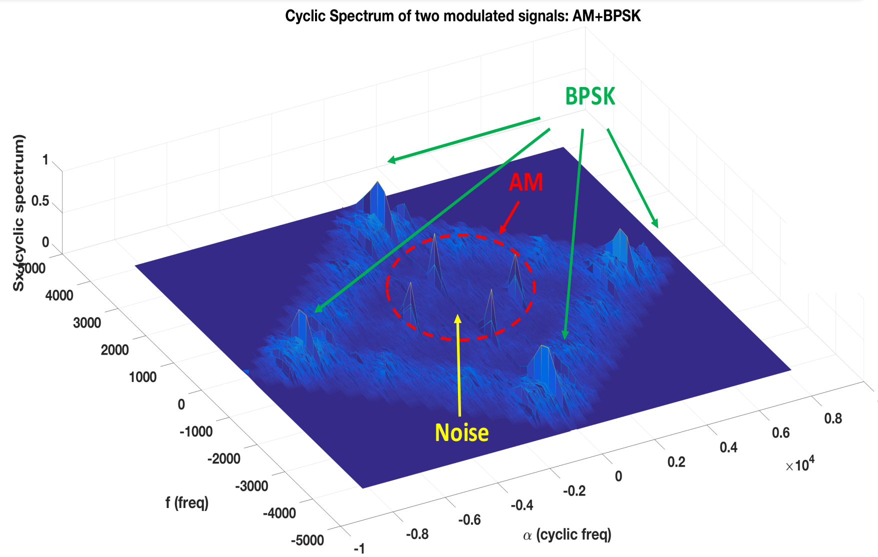
RFI

Radio-astronomy data



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Compressive cyclic analysis as a mean for feature detection such as **cyclo-stationarity** of radio-astronomy signals.

- **Cyclic-spectrum S_x estimation based on $L1$-regularization**

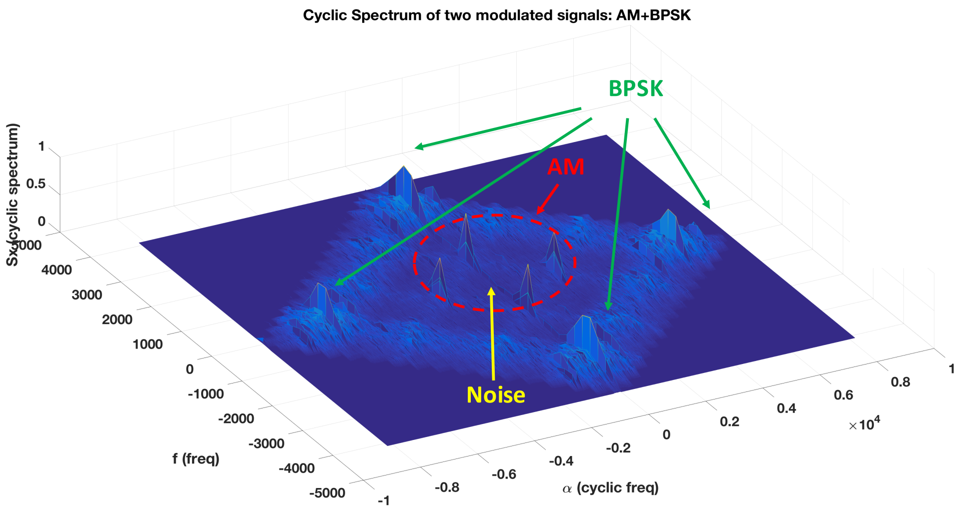
$$\mathbf{r}_z = \mathbf{T}\mathbf{s}_x$$

$$\boxed{\hat{\mathbf{s}}_x = \arg\min_{\mathbf{s}_x} \|\mathbf{r}_z - \mathbf{T}\mathbf{s}_x\|_2^2 + \lambda\|\mathbf{s}_x\|_1}$$

$$\tilde{\mathbf{r}}_z = \mathrm{vec}\{(1/L)\sum_{l=1}^{L} \mathbf{z}_t(l)\mathbf{z}_t^H(l)\}$$

We use "**CVX**" MATLAB-based solver.

- **Example:** Detection of communication signals (AM and BPSK)



Periodic signals manifest as a diamond pattern (enhanced strength at the four vertices)

Compressive cyclic analysis as a mean for feature detection such as **cyclo-stationarity** of radio-astronomy signals.

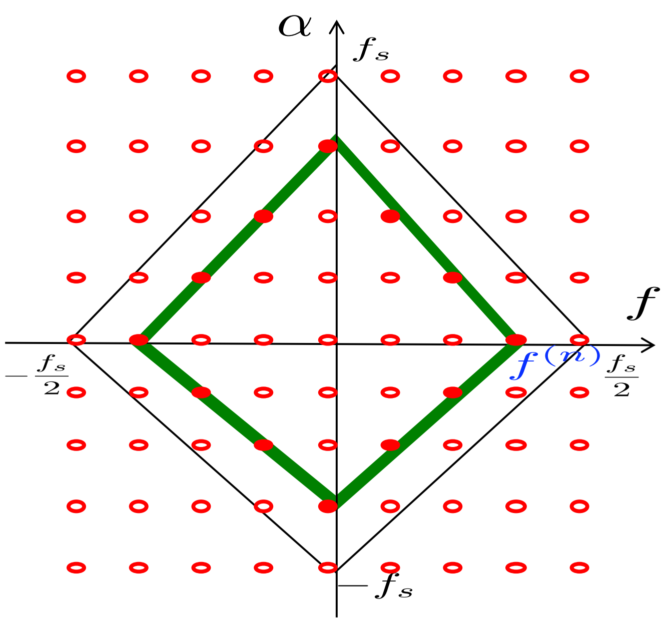
- **Band-by-band detection:** Is $f^{(n)}$ occupied by RFI or not?

Collect all discrete points related to into:

$$\hat{\mathbf{c}}^{(n)} : \{\hat{S}_x(\alpha_i, f_i)\}_i$$

which are along a diamond shape satisfying:

$$(\alpha, f) = \begin{cases} f + \frac{\alpha}{2} = f^{(n)} \\ |f| + \frac{|\alpha|}{2} \le f_{max} \end{cases}$$



Compressive cyclic analysis as a mean for feature detection such as **cyclo-stationarity** of radio-astronomy signals.

- **RFI detection based on Mul GLRT:**

Binary hypothesis test on band n:
$$\begin{cases} H_1 : \hat{\mathbf{c}}^{(n)} = \mathbf{c}^{(n)} + \epsilon \\ H_0 : \hat{\mathbf{c}}^{(n)} = \epsilon \end{cases}$$

$\mathbf{c}^{(n)} : \{\hat{S}_x(\alpha_i, f_i)\}_i$: unknown true SCD over multiple cyclic frequencies

$\epsilon : \mathcal{N}(0, \Sigma_\epsilon)$: noise statistics determined mainly by finite-sample effects or non-periodic signals

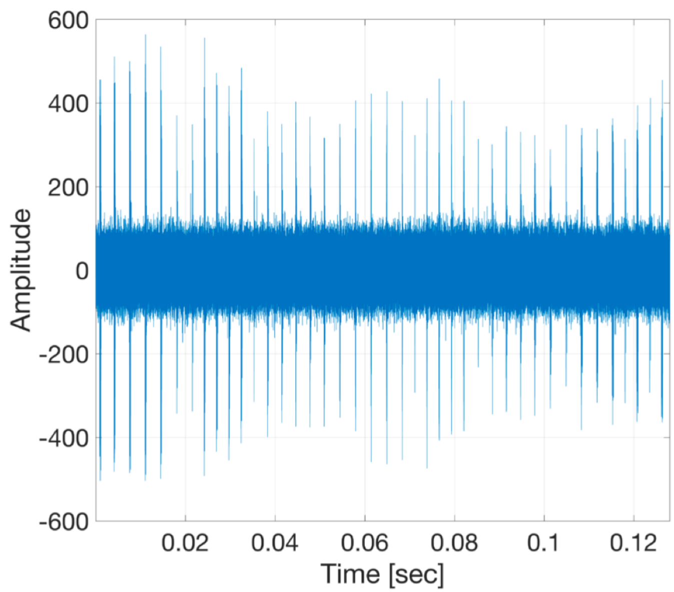
Generalized Likelihood Ratio Test (GLRT):

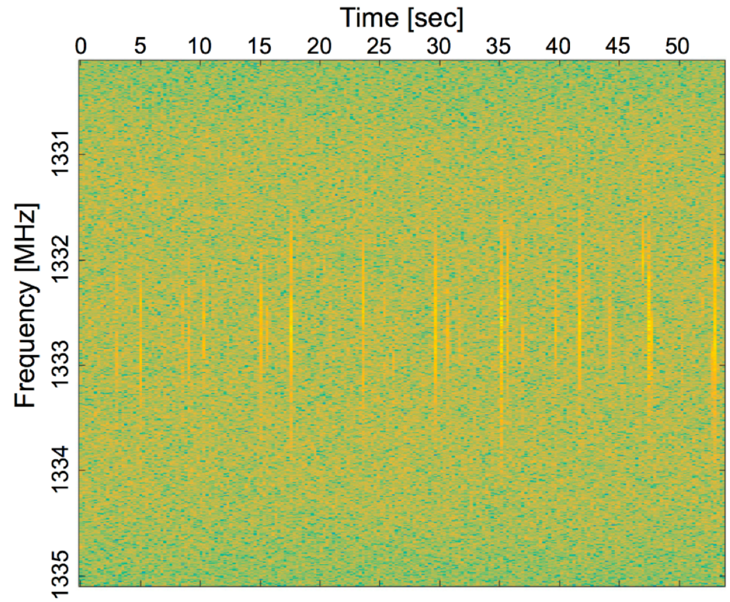
Test statistics: $$\mathcal{T}^{(n)} = (\hat{\mathbf{c}}^{(n)})^H (\hat{\mathbf{\Sigma}}^{(n)})^{-1} \hat{\mathbf{c}}^{(n)}$$

Binary decision by thresholding

L-band passive measurements from Arecibo Radio Observatory are adversely affected by active radars.

- **L-band data from Arecibo Observatory**





- Raw data acquired from the AO L-Wide band antenna (dual polarization) centered at 1332.6 MHz with a 5 MHz bandwidth near noon on May 17, 2018.

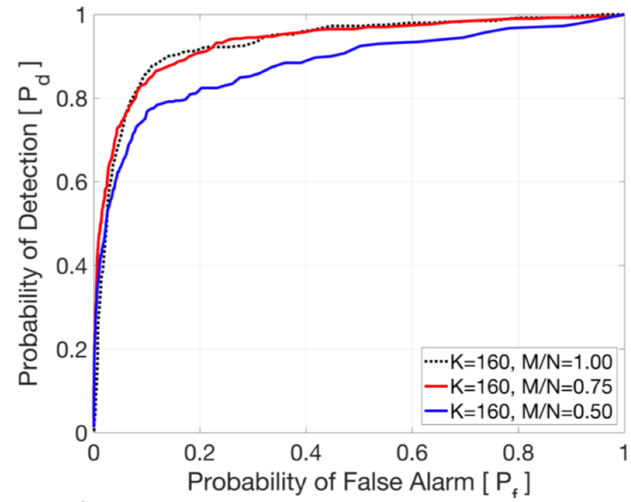
- The data is contaminated by RFI mainly generated by the Punta Borinquen Common Air Route Surveillance Radar (CARSR)

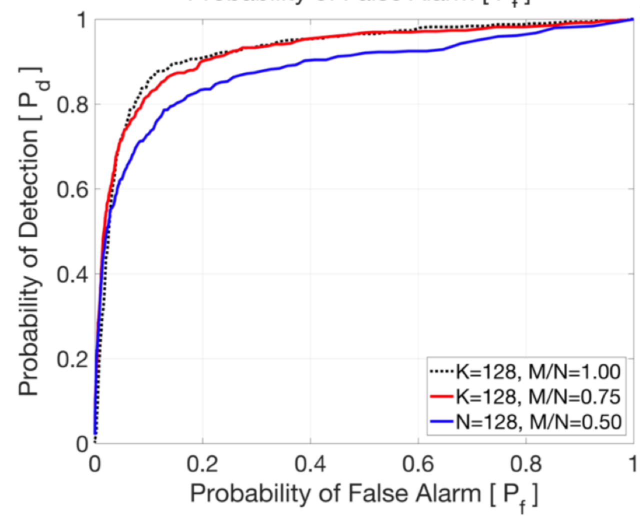
Application of CSS cyclic-stationary detection over RFI-contaminated L-band data reveals its effectiveness and feasibility for mitigation

Methodology for RFI detection:

- We use K-length time window shifting over the input data $\boldsymbol{x}(n)$. Here we use values $K = 160, 128$
- Each time window is reformed as $[N \times L]$ matrix with $K = N.L$. Here we use a fixed value of $N = 32$ and $K = 128, 160,$ therefore $L = 4, 5$, respectively
- A compression ratio M/N is defined. Here we use values: $M/N = 1, 0.75, 0.5$
- Next, CSS algorithm recovers the cyclic spectrum $S_x(\alpha, f)$.
- Band-by-band detection algorithm outputs a detected vector $\boldsymbol{f}^{(n)}$
- Ground truth vector $\boldsymbol{g}^{(n)}$ with AO and CARSR information is generated for each analyzed window.
- Receiver operating characteristic (ROC) curves are plotted to test the detection performance.

ROC curves for different window sizes and compression rates





Conclusion and Future Work

- Cyclic spectrum sensing is able to detect the presence and periodicity of quasi-periodic RFI in L-band spectra. Because both RFI sources operate at a low duty cycle, it is now possible to excise RFI features in the time domain rather than via frequency blanking or thresholding.

- The promising ROC performance achieved with a data compressive ratio of 0.5 together with short input time segments not only implies that short processing time allows for rapid estimation of frequency content but also that the burden of high sampling rates required by high-dimensional statistical analysis of wideband astronomy data can be relaxed to save acquisition costs.

- To increase the computational speed of the cyclic feature detection, future work will focus on analyze algorithms able to reduce the convergence speed in the $l_1 -$ regularization step.

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QUESTIONS/FEEDBACK