Rate Maximization for Partially Connected Hybrid Beamforming in Single-User MIMO Systems

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Abstract

- Key idea:
  - Development of rate maximizing hybrid beamforming (HBF) algorithms for partially connected RF architecture.
- System:
  - Downlink single-user MIMO system with perfect CSIT.
- Problem:
  - Rate maximization with Tx power constraint.
- Solution:
  - Equivalent weighted minimum mean square error (WMMSE) problem solved by using alternating optimization between receive combiner, digital precoder and analog beamformer.
- Numerical results:
  - Partially connected HBF provides good balance between hardware complexity and system performance.

Introduction

- Massive MIMO technology can efficiently exploit the vast spectral resources available at millimeter waves.
- Digital beamforming is impractical for massive MIMO implementation due to demanding hardware requirements (one RF chain per antenna).
- Hybrid beamforming (HBF) is a promising approach to implement massive MIMO since it supports multi-stream transmission with affordable hardware complexity (low number of RF chains).
- We study partially connected HBF (with phase and amplitude control) against digital and analog beamforming.

System model

- Partially connected Hybrid BF at the BS side.
- Digital BF at the users side.
- $N_s = N_a \le N_r$

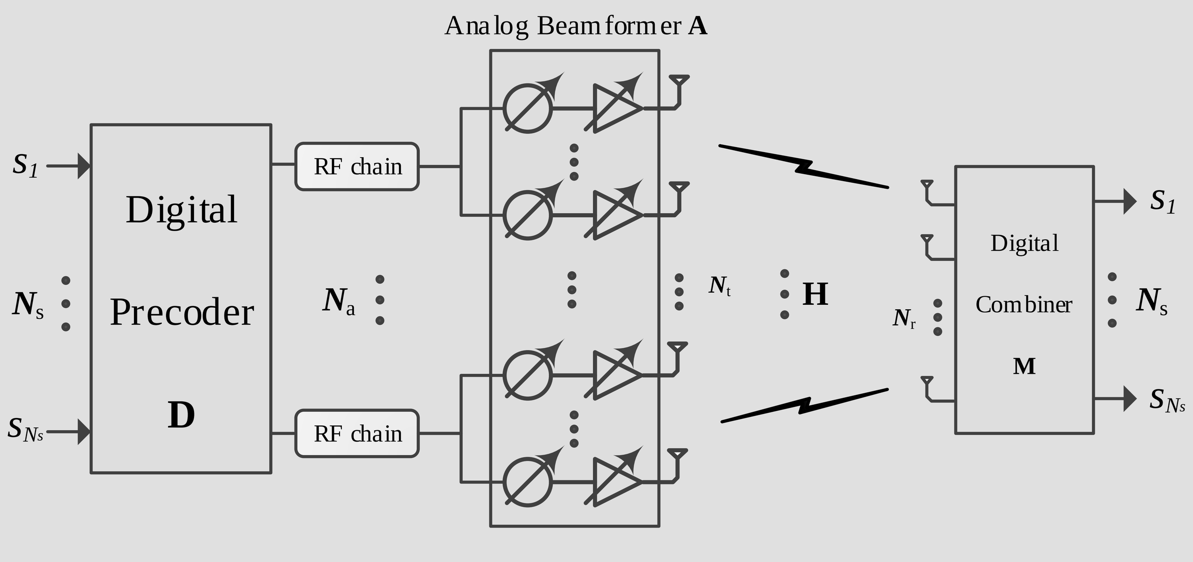
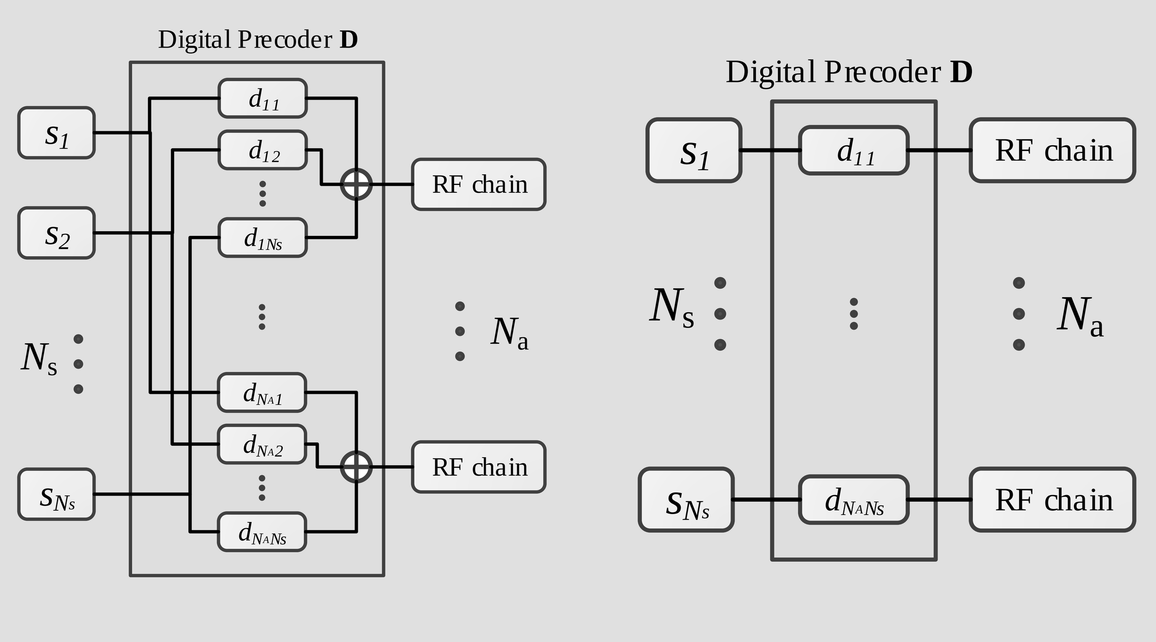


Figure: MIMO system with transmitter side HBF

- Full and subarray based processing strategies:



Problem Formulation

- The received signal vector at the user:

$$\mathbf{y} = \mathbf{HADs} + \mathbf{z} = \sum_{j=1}^{N_a} \mathbf{H}_j \mathbf{a}_j \mathbf{d}_j \mathbf{s} + \mathbf{z} \quad (1)$$

$\mathbf{s}$ is the vector of data streams with $\mathbb{E}[\mathbf{ss}^H] = \mathbf{I}_{N_s}$.

- Digital Precoder:

$$\mathbf{D} = \begin{pmatrix} d_{11} & d_{12} & \dots & d_{1N_s} \\ d_{21} & d_{22} & \dots & d_{2N_s} \\ \vdots & \vdots & \ddots & \vdots \\ d_{N_a1} & d_{N_a2} & \dots & d_{N_aN_s} \end{pmatrix} = \begin{pmatrix} \mathbf{d}_1 \\ \mathbf{d}_2 \\ \vdots \\ \mathbf{d}_{N_a} \end{pmatrix} \quad (2)$$

- Analog beamformer:

$$\mathbf{A} = \begin{pmatrix} \mathbf{a}_1 & \mathbf{0} & \dots & \mathbf{0} \\ \mathbf{0} & \mathbf{a}_2 & \dots & \mathbf{0} \\ \vdots & \vdots & \ddots & \vdots \\ \mathbf{0} & \mathbf{0} & \dots & \mathbf{a}_{N_a} \end{pmatrix} \quad (3)$$

Rate maximization problem with Tx power constraint:

$$\underset{\mathbf{A,D,M}}{\text{maximize}} \sum_{i=1}^{N_s} R_i \quad \text{s.t. } \operatorname{tr}(\mathbf{ADD}^H\mathbf{A}^H) \le P. \quad (4)$$

Equivalent rate formulations:

$$R = \sum_{i=1}^{N_s} R_i = \log_2 |\mathbf{E}^{-1}| = \sum_{i=1}^{N_s} \log_2 |e_i^{-1}|$$

Equivalent WMMSE problem:

$$\underset{\mathbf{A,D,M}}{\text{minimize}} \operatorname{tr}(\mathbf{WE}) \quad \text{s.t. } \operatorname{tr}(\mathbf{ADD}^H\mathbf{A}^H) \le P \quad (5)$$

or

$$\underset{\{\mathbf{a}_i\},\{\mathbf{d}_i\},\{\mathbf{m}_i\}}{\text{minimize}} \sum_{i=1}^{N_s} w_i e_i \quad \text{s.t. } \sum_{j=1}^{N_a} \operatorname{tr}(\mathbf{a}_j \mathbf{d}_j \mathbf{d}_j^H \mathbf{a}_j^H) \le P \quad (6)$$

Weight matrix:

$$\mathbf{W} = \operatorname{diag}(w_1\, w_2 \dots w_{N_s}) \quad (7)$$

Weight of stream i:

$$w_i = e_i^{-1} \quad (8)$$

Error term of stream i:

$$e_i = 1 - \mathbf{m}_i^H \sum_{j=1}^{N_a} \mathbf{H}_j \mathbf{a}_j d_{ji} - \sum_{j=1}^{N_a} d_{ji}^* \mathbf{a}_j^H \mathbf{H}_j^H \mathbf{m}_i + \mathbf{m}_i^H \sum_{j=1}^{N_a} \mathbf{H}_j \mathbf{a}_j \mathbf{d}_j \sum_{k=1}^{N_a} \mathbf{d}_k^H \mathbf{a}_k^H \mathbf{H}_k^H \mathbf{m}_i + N_0 \mathbf{m}_i^H \mathbf{m}_i. \quad (9)$$

Solution

Problem (5) is convex with respect to $\mathbf{M}$. The Lagrangian expression of (5):

$$\mathcal{L} = \operatorname{tr}(\mathbf{WE}) + \alpha(\operatorname{tr}(\mathbf{ADD}^H\mathbf{A}^H) - P) \quad (10)$$

Receive combiner optimization (MMSE receiver):

$$\mathbf{M} = (\mathbf{HADD}^H\mathbf{A}^H\mathbf{H}^H + N_0 \mathbf{I}_{N_r})^{-1}\mathbf{HAD}. \quad (11)$$

Digital precoder optimization (1st order optimality condition of Lagrangian of (10)):

$$\mathbf{D} = (\mathbf{A}^H\mathbf{H}^H\mathbf{MWM}^H\mathbf{HA} + \alpha \mathbf{A}^H\mathbf{A})^{-1}\mathbf{A}^H\mathbf{H}^H\mathbf{MW} \quad (12)$$

Analog beamformer optimization: 1st order optimality condition of Lagrangian of (6) yields (13). $\alpha \ge 0$ is chosen such that the Tx power constraint is satisfied.

$$\mathbf{a}_j = (\mathbf{H}_j^H \mathbf{MWM}^H \mathbf{H}_j + \alpha \mathbf{I}_n)^{-1} \mathbf{H}_j^H \left(\sum_{i=1}^{N_s} \mathbf{m}_i w_i d_{ji}^* - \mathbf{MWM}^H \sum_{k=1, k\neq j}^{N_a} \mathbf{H}_k \mathbf{a}_k \mathbf{d}_k \mathbf{d}_j^H \right) (\mathbf{d}_j \mathbf{d}_j^H)^{-1} \quad (13)$$

Proposed Algorithms

- Full array based WMMSE algorithm:
- Set iteration number $n = 0$.
- initialize $\mathbf{D}^n$ and $\mathbf{A}^n$.
- **repeat**
  - Solve (11) for $\mathbf{M}^n$ while $\mathbf{D}^{n-1}$ and $\mathbf{A}^{n-1}$ are fixed.
  - Compute $\mathbf{W}^n$ from (8), (9) given $\mathbf{D}^{n-1}$, $\mathbf{A}^{n-1}$, and $\mathbf{M}^n$.
  - Solve (12) for $\mathbf{D}^n$ while $\mathbf{M}^n$ and $\mathbf{A}^{n-1}$ are fixed.
  - Solve (13) for $\{\mathbf{a}_j^n\}$ while $\mathbf{M}^n$ and $\mathbf{D}^n$ are fixed.
- **until** convergence

- Subarray based WMMSE algorithm: Digital precoder is considered to be idendity, ($\mathbf{D} = \mathbf{I}$). Only optimization of $\mathbf{M}$, $\mathbf{W}$, and $\mathbf{A}$ is needed.

Convergence Results

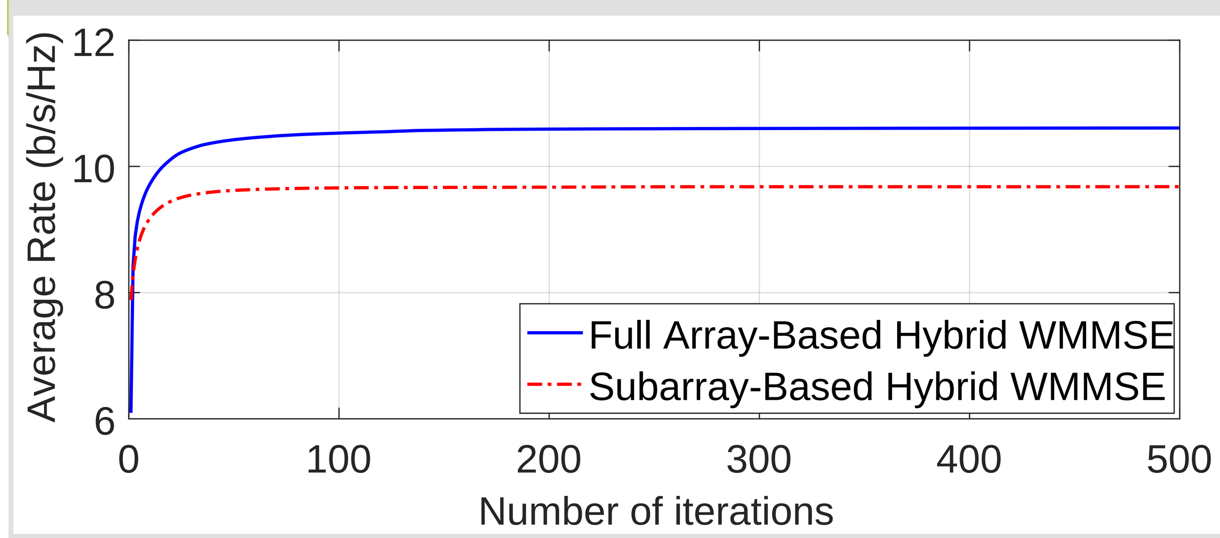


Figure: SNR = 0 dB.

Rate vs. SNR

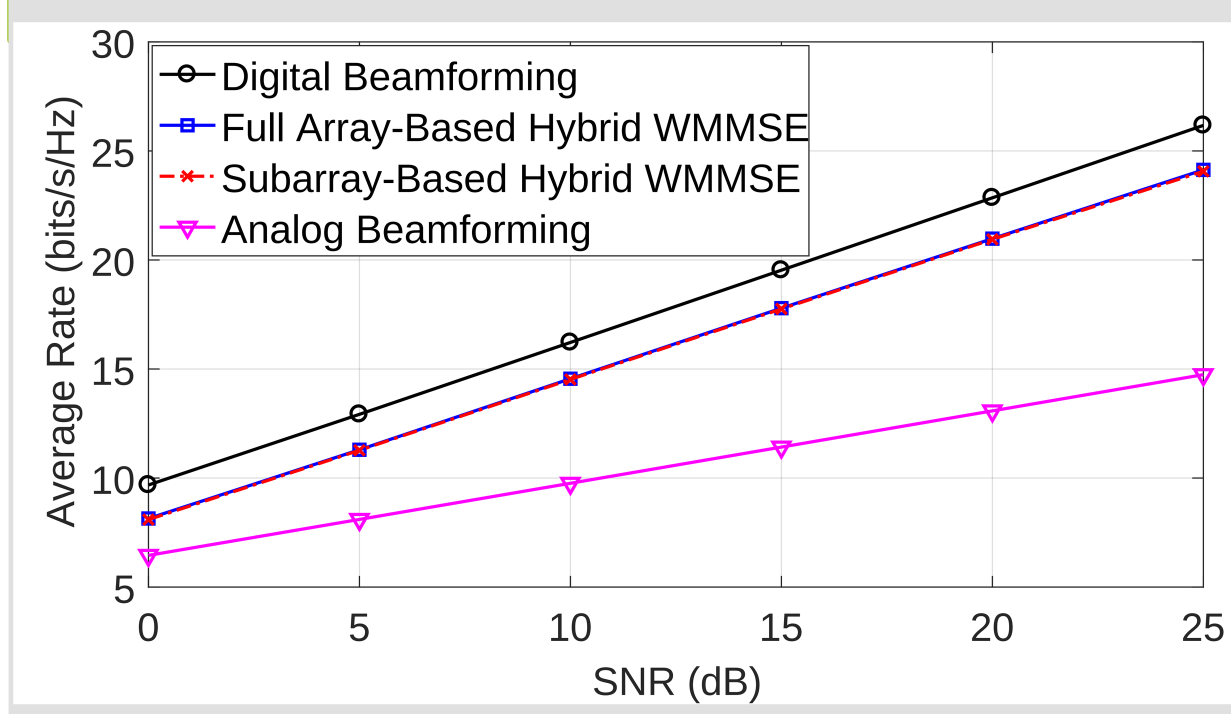


Figure: $N_t = 64$, $N_r = 2$, $N_s = 2$.

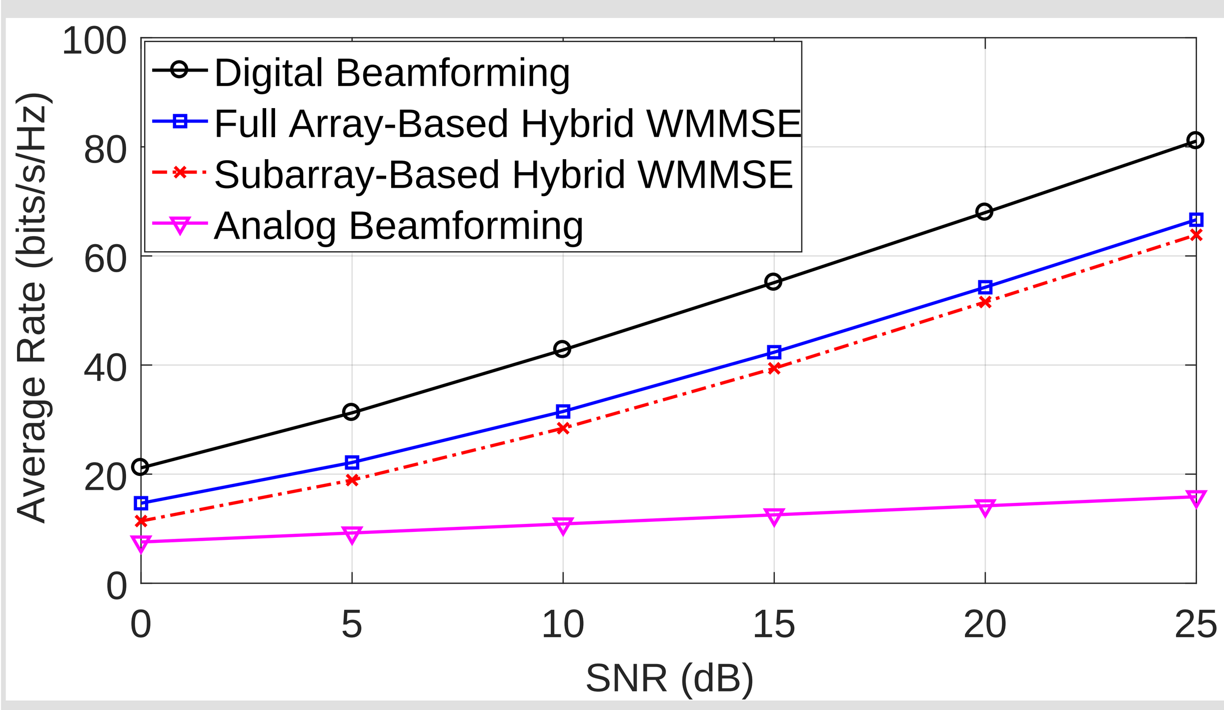


Figure: $N_t = 64$, $N_r = 8$, $N_s = 8$.

Conclusion

- Partially connected HBF obtains a good compromise between achievable rate and hardware complexity in comparison to digital and analog beamforming.
- Performance of full and subarray-based WMMSE algorithms are comparable for $N_s \le 4$ at medium/high SNRs.
- Rate maximizing results serve as upper bounds for lower complexity HBF algorithms.