Performance Analysis of a DF based Dual Hop Mixed RF-FSO System with a Direct RF Link

Sanya Anees[\$], Priyanka Meena[*] and Manav R. Bhatnagar[#]

[\$]Bharti School of Telecommunication Technology & Management
&
[*#]Department of Electrical Engineering
Indian Institute of Technology Delhi, New Delhi, India

December 17, 2015

1. Introduction
2. Problem Approached
3. Statistical Characteristics
4. Performance Metrics
5. Numerical Results
6. References

Motivation

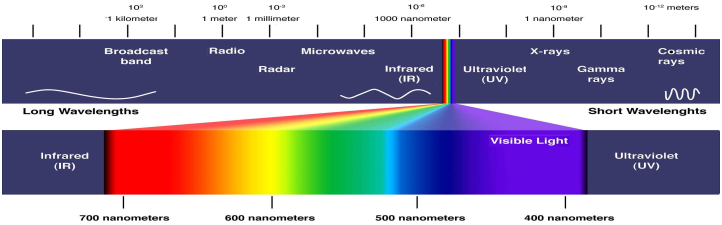

Figure: Electromagnetic spectrum

- "Wireless" synonymous to RF technologies
- RF band is limited, licensed, and costly
- Free Space Optical Communications (FSO) / Optical Wireless Communication (OWC) $\Longrightarrow$ transmission in unguided propagation media through use of optical carriers, i.e., visible, IR, and UV band.

FSO : Advantages

- High rate communication over distances up to several kilometers (10 Gbps)
- FSO systems use very narrow laser beams $\Longrightarrow$ inherent security and robustness to electromagnetic interference.
- Frequency used is above 300 GHz which is unlicensed worldwide.
- FSO systems are also easily deployable and can be reinstalled without the cost of dedicated fiber optic connections.
- Efficient solution for the "last mile" problem to bridge the gap between the end user and the backbone network.
- Enterprise/ campus connectivity
- Video surveillance and monitoring

FSO : Applications I

FSO communication can be potentially employed in a diverse range of communication applications. Based on the transmission range, OWC can be studied in five categories -

- Ultra-short range OWC (chip-to-chip communications in stacked and closely-packed multi-chip packages)
- Short range OWC (wireless body area network (WBAN) and wireless personal area network (WPAN) applications, underwater communications)
  Example : Disaster Recovery, e.g., 9/11 Terrorist Attacks in NY City when financial corporations were left out with no landlines.

FSO : Applications II

- Medium range OWC (indoor IR and VLC for wireless local area networks (WLANs), inter-vehicular and vehicle-to-infrastructure communications)
- Long range OWC (inter-building connections)
  Example : Broadcasting of live events, e.g., during 2010 FIFA World Cup, BBC deployed FSO links for Ethernet-based transport of HD video between studio locations setup in South Africa.
- Ultra-long range OWC (inter-satellite links and deep space links)

FSO : Limitations

The performance of FSO systems is strongly AFFECTED by

- Atmospheric Turbulence/ Scintillations : variations in temperature and pressure of atmosphere $\Longrightarrow$ variations in the refractive index along the transmission path $\Longrightarrow$ channel fading.
- ATMOSPHERIC LOSS : Rain, snow, fog, pollution, dust, smoke, etc absorb laser light energy attenuating optical power of the signal and cause light scattering.
- MISALIGNMENT LOSS or POINTING ERRORS $\Longrightarrow$ building sway phenomenon due to thermal expansion, earthquakes, etc.

FSO : Solutions

- Radio on FSO (RoFSO)
- MIMO-FSO
- Hybrid RF/FSO
- Asymmetric RF-FSO
- Serial FSO

Cooperation protocols

- Amplify-and-forward (AF)
- Decode-and-forward (DF)

Cooperative communication provides

- High reliability and fading mitigation
- Performance enhancement
- Broad and energy-efficient coverage area

Literature Review

- In [**S. Anees and M. R. Bhatnagar, IET Optoelectronics,2015**] Outage, BER, and capacity analysis for **DF** based asymmetric RF-FSO systems, where RF link $\Longrightarrow$ Nakagami distribution and FSO link $\Longrightarrow$ Gamma-Gamma turbulence & pointing errors.

- In [**I. S. Ansari, M. S. Alouini, and F. Yilmaz, IEEE VTC, 2013**] BER analysis of **fixed gain AF** based mixed RF-FSO system with direct RF link, where RF links $\Longrightarrow$ Rayleigh distribution and FSO link $\Longrightarrow$ Gamma-Gamma turbulence & pointing errors.

- In [**N. I. Miridakis, M. Matthaiou, and G. K. Karagiannindis, IEEE Trans. Commun., May 2014**]
Outage probability and ASEP analysis of DF based mutli-user mixed RF-FSO system, where simultaneous data is transmitted via RF links and the decoded information is sent to the destination via FSO link.

System model I

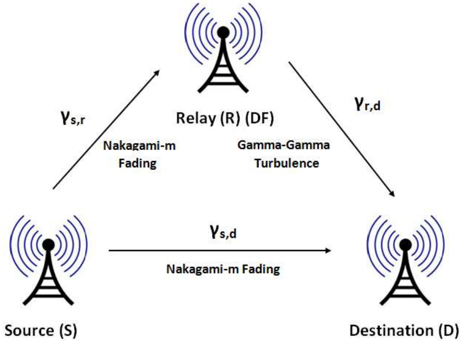

Figure: System Model of DF based dual-hop mixed RF-FSO system with a direct RF link.

System model II

Problem Statement

- Performance analysis of DF based dual hop mixed RF-FSO communication system with direct RF link, where
  - S-R and S-D links are characterized by Nakagami-*m* distributed fading
  - R-D link is characterized by Gamma-Gamma distributed turbulence and pointing error
- The system uses SC at the receiver; it selects the link with maximum SNR
- The system uses SIM scheme and direct mode of detection

System model III

- Signal received by R and D from S :

$$y_{s,q} = h_{s,q}x + e_{s,q}$$

  * $q \in \{r, d\}$
  * x denotes the signal transmitted by S
  * $h_{s,q}$ denotes the Nakagami-m distributed channel gain
  * $e_{s,q}$ denotes zero-mean AWGN noise with $\sigma^2_{s,q}$ variance

- Signal received by D after optical-to-electrical conversion from S over the FSO link :

$$y_{r,d} = \eta_{r,d} I_{r,d} \hat{x} + e_{r,d}$$

  * $I_{r,d}$ is the real-valued irradiance
  * $\eta_{r,d}$ is optical-to-electrical conversion coefficient
  * $e_{r,d}$ denotes zero-mean AWGN noise with $\sigma^2_{r,d}$ variance

System model IV

- For a DF based mixed RF/FSO system without a direct link, the end-to-end signal-to-noise (SNR) ($\gamma_{s,r,d}$)

$$\gamma_{s,r,d} \simeq \min\left(\gamma_{s,r}, \gamma_{r,d}\right)$$

- The instantaneous received SNR at D:

$$\gamma_z = \max(\gamma_{s,d}, \gamma_{s,r,d})$$

Channel Model I : RF Link

Assuming the fading of RF link to be Nakagami-m distributed, the PDF of $\gamma_{s,q}$ will be Gamma distributed

$$f_{\gamma_{s,q}}(\gamma) = \frac{m_{s,q}^{m_{s,q}} \gamma^{m_{s,q}-1}}{\Gamma(m_{s,q}) \bar{\gamma}^{m_{s,q}}} \exp\left(-\frac{m_{s,q}\gamma}{\bar{\gamma}_{s,q}}\right),$$

* $m \geq 1/2$ is the Nakagami parameter

Channel Model II : FSO Link

- PDF of $\gamma_{r,d}$ for direct detection

$$f_{\gamma_{r,d}}(\gamma) = \frac{\xi^2}{2\gamma\Gamma(a)\Gamma(b)} G_{1,3}^{3,0}\left(fab\sqrt{\frac{\gamma}{\bar{\gamma}_{r,d}}} \middle| \begin{matrix} \xi^2+1 \\ \xi^2, a, b \end{matrix} \right)$$

* $\gamma = \frac{\bar{\gamma}_{r,d}}{I_l A_0 p}$
* $f = \frac{\xi^2}{\xi^2+1}$
* $\xi = \frac{w_e}{2\sigma_s}$
* w_e is the equivalent beamwaist
* σ_s is the pointing error displacement standard deviation at the receiver
* $G(\cdot)$ is the Meijer-G function

Statistical Characteristics : Mixed RF-FSO Cooperative System without a Direct Link I

CDF :

$$F_{\gamma_{s,r,d}}(\gamma) = 1 - \left(1 - \mathcal{K}_1 \gamma\left(m_{s,r}, \frac{m_{s,r}\gamma}{\bar{\gamma}_{s,r}}\right)\right) \times \left(1 - \mathcal{K}_2 G_{3,7}^{6,1}\left(\mathcal{W}\gamma \middle| \begin{matrix} 1, \mathcal{P}_1 \\ \mathcal{P}_2, 0 \end{matrix}\right)\right)$$

* $\mathcal{K}_1 = \frac{1}{\Gamma(m_{s,r})}$
* $\mathcal{K}_2 = \frac{2^{z_1-2}\xi^2}{2\pi\Gamma(a)\Gamma(b)}$
* $\mathcal{W} = \frac{(fab)^2}{16\bar{\gamma}_{r,d}}$
* $\mathcal{P}_1 = \frac{\xi^2+1}{2}, \frac{\xi^2+2}{2}$
* $\mathcal{P}_2 = \frac{\xi^2}{2}, \frac{\xi^2+1}{2}, \frac{a}{2}, \frac{a+1}{2}, \frac{b}{2}, \frac{b+1}{2}$

Statistical Characteristics : Mixed RF-FSO Cooperative System without a Direct Link II

PDF :

$$f_{\gamma_{s,r,d}}(\gamma) = \left(\mathcal{K}_1 \left(\frac{m_{s,r}}{\bar{\gamma}_{s,r}}\right)^{m_{s,r}} \gamma^{m_{s,r}-1} \exp\left(\frac{-m_{s,r}\gamma}{\bar{\gamma}_{s,r}}\right)\right) \times \left(1 - \mathcal{K}_2 G_{3,7}^{6,1}\left(\mathcal{W}\gamma \middle| \begin{matrix} 1, \mathcal{P}_1 \\ \mathcal{P}_2, 0 \end{matrix}\right)\right) - \mathcal{K}_2 \gamma^{-1} \times \left(1 - \mathcal{K}_1 \gamma\left(m_{s,r}, \frac{m_{s,r}\gamma}{\bar{\gamma}_{s,r}}\right)\right) G_{2,6}^{6,0}\left(\mathcal{W}\gamma \middle| \begin{matrix} \mathcal{P}_1 \\ \mathcal{P}_2 \end{matrix}\right)$$

Statistical Characteristics : Mixed RF-FSO Cooperative System with a Direct Link

- CDF:

$$F_{\gamma_z}(\gamma)=\mathcal{K}_3\gamma\left(m_{s,d},\frac{m_{s,d}\gamma}{\bar{\gamma}_{s,d}}\right)\left[1-\left(1-\mathcal{K}_1\gamma\left(m_{s,r},\frac{m_{s,r}\gamma}{\bar{\gamma}_{s,r}}\right)\right)\right.$$
$$\left.\times\left(1-\mathcal{K}_2 G_{3,7}^{6,1}\left(\mathcal{W}\gamma\left|\begin{matrix}1,\mathcal{P}_1\\ \mathcal{P}_2,0\end{matrix}\right.\right)\right)\right]$$

where $\mathcal{K}_3=1/\Gamma(m_{s,d})$.

- PDF:

$$f_{\gamma_z}(\gamma)=F_{\gamma_{s,d}}(\gamma)f_{\gamma_{s,r,d}}(\gamma)+f_{\gamma_{s,d}}(\gamma)F_{\gamma_{s,r,d}}(\gamma).$$

Outage Probability

- Without Direct Link:

$$P_{out}(\gamma_{th}) = F_{\gamma_{s,r,d}}(\gamma_{th}) = 1 - \left(1 - \mathcal{K}_1 \gamma\left(m_{s,r}, \frac{m_{s,r}\gamma_{th}}{\bar{\gamma}_{s,r}}\right)\right) \times \left(1 - \mathcal{K}_2 G_{3,7}^{6,1}\left(\mathcal{W}\gamma_{th}\middle| \begin{matrix} 1, \mathcal{P}_1 \\ \mathcal{P}_2, 0 \end{matrix}\right)\right).$$

- With Direct RF Link:

$$P_{out}(\gamma_{th}) = \mathcal{K}_3 \gamma\left(m_{s,d}, \frac{m_{s,d}\gamma_{th}}{\bar{\gamma}_{s,d}}\right)\left[1 - \left(1 - \mathcal{K}_1 \times \gamma(m_{s,r}, \frac{m_{s,r}\gamma_{th}}{\bar{\gamma}_{s,r}})\right)\left(1 - \mathcal{K}_2 G_{3,7}^{6,1}\left(\mathcal{W}\gamma_{th}\middle| \begin{matrix} 1, \mathcal{P}_1 \\ \mathcal{P}_2, 0 \end{matrix}\right)\right)\right].$$

Bit Error Rate I

Table: BER parameters for Various Modulation Techniques

Modulation techniques	ϕ	ψ
Coherent Binary Frequency Shift Keying (CBFSK)	0.5	0.5
Coherent Binary Phase Shift Keying (CBPSK)	0.5	1
Non-Coherent Binary Frequency Shift Keying (NBFSK)	1	0.5
Differential Binary Phase Shift Keying (DBPSK)	1	1

Bit Error Rate II

Average BER Without Direct Link :

$$P_e = \frac{\mathcal{K}_1}{2\Gamma(\phi)} G_{2,2}^{1,2}\left(\frac{m_{s,r}}{\psi\bar{\gamma}_{s,r}}\middle| \begin{matrix} 1-\phi, 1 \\ m_{s,r}, 0 \end{matrix}\right) + \sum_{k=0}^{m_{s,r}-1} \frac{\psi^{\phi}\mathcal{K}_2}{2k!\Gamma(\phi)}\left(\frac{m_{s,r}}{\bar{\gamma}_{s,r}}\right)^k$$

$$\times\left(\psi+\frac{m_{s,r}}{\bar{\gamma}_{s,r}}\right)^{-\phi-k} G_{4,7}^{6,2}\left(\frac{\mathcal{W}\bar{\gamma}_{s,r}}{m_{s,r}+\psi\bar{\gamma}_{s,r}}\middle| \begin{matrix} 1-\phi-k, 1, \mathcal{P}_1 \\ \mathcal{P}_2, 0 \end{matrix}\right).$$

Bit Error Rate III

Average BER With Direct RF Link :

$$
\begin{aligned}
P_e &= \frac{\mathcal{K}_3}{2\Gamma(\phi)} G_{2,2}^{1,2}\left(\frac{m_{s,d}}{\psi\bar{\gamma}_{s,d}} \middle| \begin{matrix} 1-\phi, 1 \\ m_{s,d}, 0 \end{matrix} \right) - \frac{\psi^{\phi}\mathcal{K}_3}{2\Gamma(\phi)} \\
&\times \sum_{k=0}^{m_{s,r}-1} \left(\frac{m_{s,r}}{\bar{\gamma}_{s,r}}\right)^k \frac{1}{k!} \left(\psi + \frac{m_{s,r}}{\bar{\gamma}_{s,r}}\right)^{-\phi-k} G_{2,2}^{1,2}\left(\frac{m_{s,d}\bar{\gamma}_{s,r}}{\bar{\gamma}_{s,d}(\psi\bar{\gamma}_{s,r}+m_{s,r})} \middle| \begin{matrix} 1-\phi-k, 1 \\ m_{s,d}, 0 \end{matrix} \right) \\
&+ \frac{\psi^{\phi}\mathcal{K}_2}{2\Gamma(\phi)} \sum_{k=0}^{m_{s,r}-1} \left(\frac{m_{s,r}}{\bar{\gamma}_{s,r}}\right)^k \frac{1}{k!} \left(\psi + \frac{m_{s,r}}{\bar{\gamma}_{s,r}}\right)^{-\phi-k} G_{4,7}^{6,2}\left(\frac{\mathcal{W}\bar{\gamma}_{s,r}}{(\psi\bar{\gamma}_{s,r}+m_{s,r})} \middle| \begin{matrix} 1-k-\phi, 1, \mathcal{P}_1 \\ \mathcal{P}_2, 0 \end{matrix} \right) \\
&- \frac{\psi^{\phi}\mathcal{K}_2}{2\Gamma(\phi)} \sum_{k=0}^{m_{s,r}-1} \sum_{l=0}^{m_{s,d}-1} \left(\frac{m_{s,d}}{\bar{\gamma}_{s,d}}\right)^l \left(\frac{m_{s,r}}{\bar{\gamma}_{s,r}}\right)^k \frac{1}{l!k!} \left(\psi + \frac{m_{s,r}}{\bar{\gamma}_{s,r}} + \frac{m_{s,d}}{\bar{\gamma}_{s,d}}\right)^{-\phi-k-l} \\
&\times G_{4,7}^{6,2}\left(\frac{\mathcal{W}}{(\psi + \frac{m_{s,d}}{\bar{\gamma}_{s,d}} + \frac{m_{s,r}}{\bar{\gamma}_{s,r}})} \middle| \begin{matrix} 1-k-l-\phi, 1, \mathcal{P}_1 \\ \mathcal{P}_2, 0 \end{matrix} \right).
\end{aligned}
$$

Result I: Outage Probability

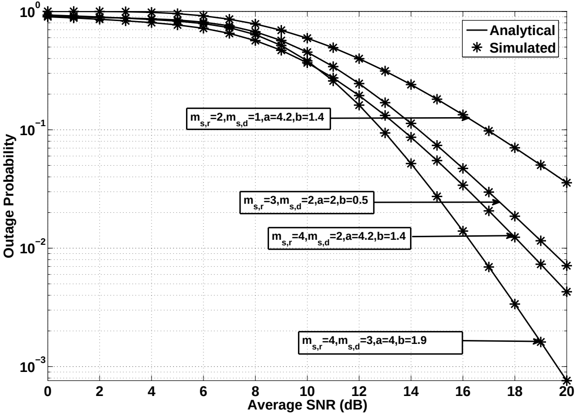

Figure: Outage Probability versus average SNR of the mixed RF-FSO system with direct link, for different values of fading parameters and ξ=1.2.

Result II: BER for Different Modulation Schemes

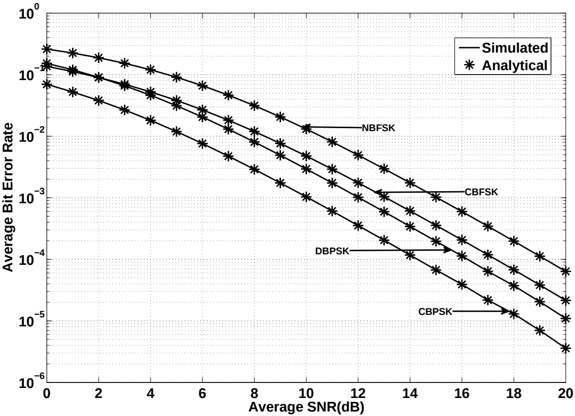

Figure: Average BER versus average SNR of the dual hop mixed RF-FSO system with direct link, for different modulation techniques and fading parameters, $m_{s,d}=2$, $m_{s,r}=4$, $a=4.2$, $b=1.4$, and $\xi=1.2$.

Result III: BER with and without Direct RF Link

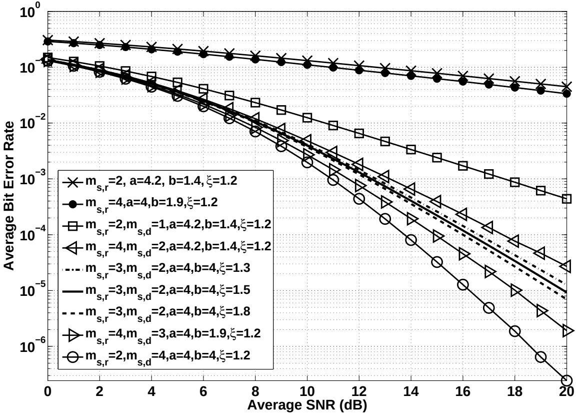

Figure: Average BER versus average SNR of dual hop mixed RF-FSO system with and without direct link for CBFSK modulation technique and different values of fading parameters and ξ.

M. A. Khalighi and M. Uysal
"Survey on Free Space Optical Communication: A Communication Theory Perspective," *IEEE Commun. Surveys & Tutorials*, vol. 16, no. 4, pp. 2231-2258, Nov. 2014.

W. Gappmair
"Further results on the capacity of free space optical channels in turbulent atmosphere," *IET Commun.*, vol. 5, no. 9, pp. 1262–1267, June 2011.

M. R. Bhatnagar
"Performance analysis of decode-and-forward relaying in Gamma-Gamma fading channels," *IEEE Photonics Tech. Lett.*, vol. 24, no. 7, pp. 545–547, 2012.

H. Samimi and M. Uysal
"End-to-End Performance of Mixed RF/ FSO Transmission Systems," *IEEE J. Opt. Commun. & Netw.*, vol. 5, no. 1, pp. 1139–1144, Nov. 2013.

I. S. Ansari, M. S. Alouini, and F. Yilmaz
"On the performance of hybrid RF and RF/FSO fixed gain dual-hop transmission systems," in *IEEE Vehicular Tech. Conf. (VTC)*, 2013.

S. Anees and M R. Bhatnagar
"Performance evaluation of an decode-and-Forward dual-hop asymmetric RFFSO System," *IET Optoelectronics*, June 2015.

N. I. Miridakis, M. Matthaiou, M., and G. K. Karagiannidis
"Multiuser Relaying over Mixed RF/FSO Links ," *IEEE Trans. Commun.*, vol. 62, no. 5, pp. 1634–1645, March 2014.

- Y. L. Luke
  The Special Functions and their Approximations, vol. 1, Academic Press, New York, USA, 1969.
- http://functions.wolfram.com/HypergeometricFunctions/MeijerG/.
- A. P. Prudnikov, Y. A. Brychkov and O. I. Marichev
  Integrals and Series, Ed 1, vol 3, Gordon and Breach Science Publishers, New York, USA, 2010.

Thank You