Decentralized Temperature Control via HVAC Systems in Energy Efficient Buildings: An Approximate Solution Procedure

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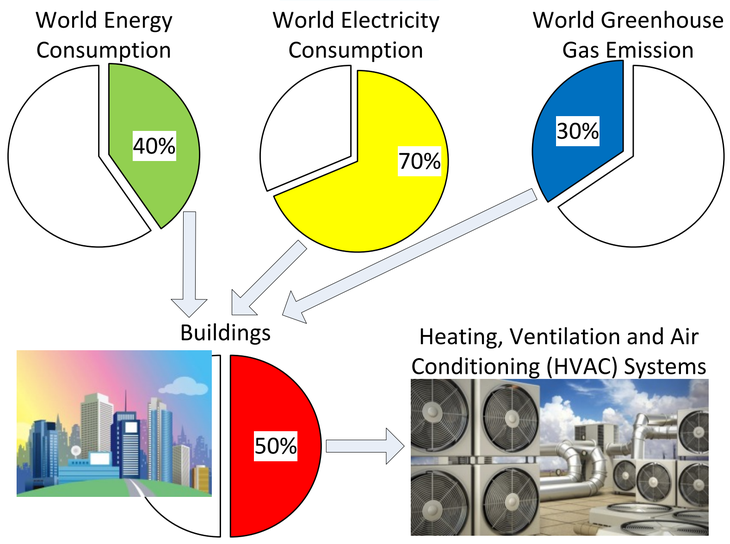
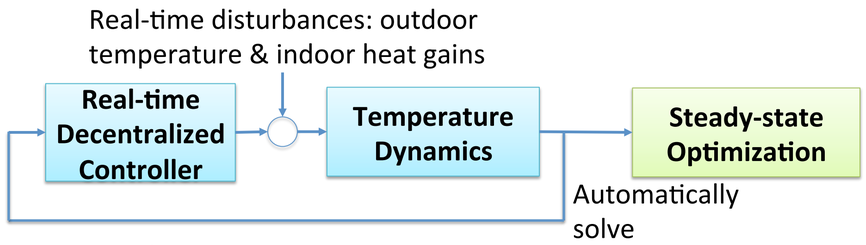
Motivation

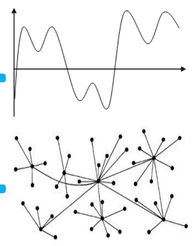


Challenge: **HVAC systems need to be more smart for both energy efficiency and user comfort.**

Desiderata: Design a real-time decentralized control scheme



Real-time Decentralized Controller



Continuously changing inputs to ensure optimality in the presence of real-time disturbances

Need only local measurement/information which respects user privacy, and is scalable with respect to building structures

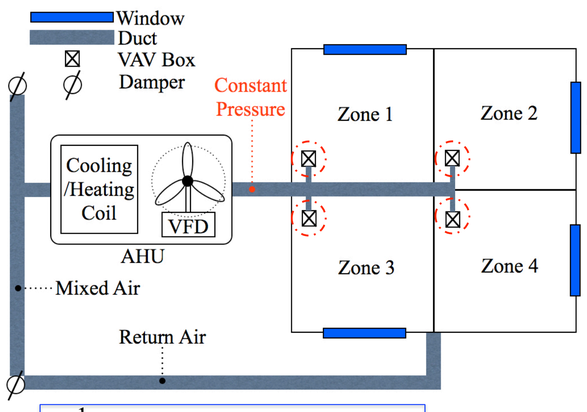
Steady-state Optimization

Balance user comfort and energy saving

Respect system operating constraints

Compared with convention controllers (PID, MPC, mean-field control, etc.), our controller can be either more efficient or be implemented in a more easier way.

Setup: Temperature dynamics and steady-state optimization



Reduced RC model:

$$C_i\dot{T}_i = \frac{T^o - T_i}{R_i} + \sum_{j\in N(i)} \frac{T_j - T_i}{R_{ij}} + c_a m_i (T^s - T_i) + Q_i$$

Approximation – ignore heat transfer between neighboring zones:

$$C_i\dot{T}_i = \frac{T^o - T_i}{R_i} + c_a m_i (T^s - T_i) + Q_i$$

$$\min_{Z_i, m_i} \sum_{i\in N} \frac{1}{2} r_i (Z_i - T_i^{set})^2 + \sum_{i\in N} \frac{1}{2} s_i m_i^2$$

$$\text{s.t. } \frac{T^o - Z_i}{R_i} + c_a m_i (T^s - Z_i) + Q_i = 0$$

NonConvexity

$$m_i^{\min} \le m_i \le m_i^{\max}$$

$$\sum_{i\in N} m_i \le \bar{m}$$

$\sum_{i\in N} \frac{1}{2} r_i (Z_i - T_i^{set})^2$: comfort indicator

$\sum_{i\in N} \frac{1}{2} s_i m_i^2$: consumption indicator

Z_i: desired steady-state temperature

m_i: zone flow rate, i.e., control input

Steady-state equation --

Zone flow rate bounds --

Total flow rate bound due to the fan --

Main Results

I. Convex relaxation

$$\min_{Z_i, m_i} \sum_{i\in N} \frac{1}{2} r_i (Z_i - T_i^{set})^2 + \sum_{i\in N} \frac{1}{2} s_i m_i^2$$

$$\text{s.t. } \frac{T^o - Z_i}{R_i} + c_a m_i (T^s - Z_i) + Q_i = 0$$

$$m_i^{\min} \le m_i \le m_i^{\max}$$

$$\sum_{i\in N} m_i \le \bar{m}$$

$f_i''(Z_i) > 0$ — Convex!

$$\min_{Z_i, m_i} \sum_{i\in N} \frac{1}{2} r_i (Z_i - T_i^{set})^2 + \sum_{i\in N} \frac{1}{2} s_i m_i^2$$

$$\text{s.t. } f_i(Z_i) \triangleq \frac{\frac{T^o - Z_i}{R_i} + Q_i}{c_a (Z_i - T^s)} \le m_i$$

$$m_i^{\min} \le m_i \le m_i^{\max}$$

$$\sum_{i\in N} m_i \le \bar{m}$$

Assume that in the cooling/heating mode, $T_i^{set} < T^o$ / $T_i^{set} > T^o$, and that the zone temperature set point satisfies $f_i(T_i^{set}) \ge m_i^{\min}, \forall i$. Then the above convex relaxation is always tight, i.e., the optimal solution of these two optimization problems are exactly the same.

II. Decentralized control design – motivated by a primal-dual gradient method

$$\dot{Z}_i = k_{Z_i} \left(r_i (T_i^{set} - Z_i) - \zeta_i \frac{\frac{T^s - T_i}{R_i} - C_i \dot{T}_i - c_a m_i (T_i - T^s)}{c_a (Z_i - T^s)^2} \right)$$

$$\dot{m}_i = k_{m_i} (-s_i m_i + \zeta_i - \mu_i^+ + \mu_i^- - \lambda^+)$$ System input

$$\dot{\zeta}_i = k_{\zeta_i} \left(\frac{\frac{T_i - Z_i}{R_i} + C_i \dot{T}_i + c_a m_i (T_i - T^s)}{c_a (Z_i - T^s)} - m_i \right)_{\zeta_i}^+$$ Positive projection

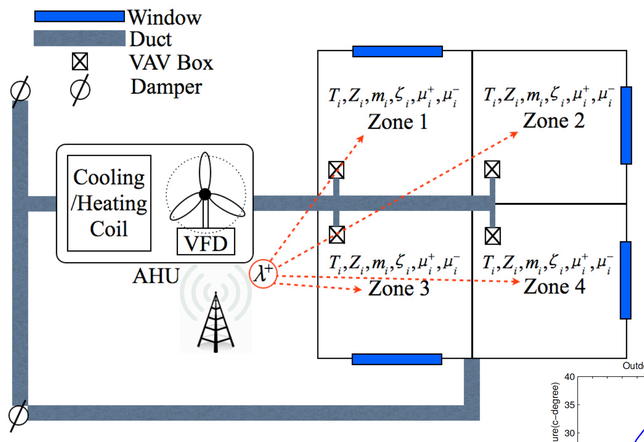
$$\dot{\mu}_i^+ = k_{\mu_i^+} (m_i - m_i^{\max})_{\mu_i^+}^+$$

$$\dot{\mu}_i^- = k_{\mu_i^-} (m_i^{\min} - m_i)_{\mu_i^-}^+$$

$$\dot{\lambda}^+ = k_{\lambda^+} \left(\sum_{i\in N} m_i - \bar{m} \right)_{\lambda^+}^+$$

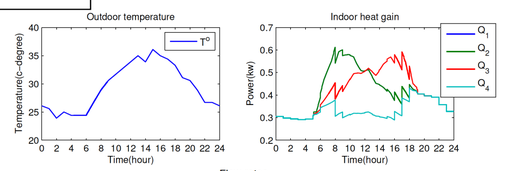
Under constant/slow-varying external disturbances, the above controller drives the approximate model to an equilibrium point which is the unique optimal solution of the steady-state optimization problem.

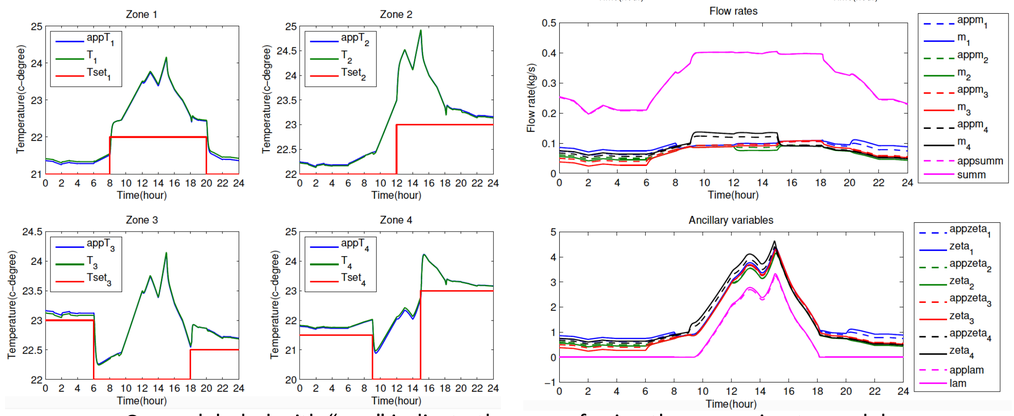
III. Decentralized implementation – a numerical example



Each zone collects its set point from users, locally measures its indoor temperature and receives the signal from the supply fan/duct.

The supply fan/duct locally measures the total flow rate, updates the signal and broadcasts it.





Curves labeled with "app" indicate the case of using the approximate model

IV. Extension – the community scenario

The proposed control framework can be applied to the following scenario – HVAC management in a community: each house is equipped with an HVAC system; the approximate model exactly describes the house temperature dynamics; the solution of the steady-state optimization problem determines an efficient resource allocation plan for the whole community.

References:

[1] X. Zhang, W. Shi, X. Li, B. Yan, A. Malkawi and N. Li, "Decentralized temperature control via HVAC systems in energy efficient buildings: An approximate solution procedure." *In Proc. of 2016 IEEE Global Conference on Signal and Information Processing*, 2016.

[2] X. Zhang, W. Shi, X. Li, B. Yan, A. Malkawi and N. Li, "Decentralized and distributed temperature control via HVAC systems in energy efficient buildings." *Submitted to journals*, 2016.