Low-Complexity MIMO Detector with 1024-QAM

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Outline:

Introduction
- Motivation
- System Model
- Popular Detectors

Proposed Work
- Low-Complexity LORD
- Optimizing Search Region
- Optimizing LLR Saturation

Results
- Complexity Study
- Simulation Scenario
- Simulation Results

Summary & future work

Motivation

- Quadrature Amplitude Modulation (QAM) is rising
  - 1024QAM and beyond
  - Mainly in microwave backhaul but also in WiFi
- Broadcom announced new 5G WiFi chips
  - NitroQAM™ (1024-QAM) technology
  - 8x8 MU-MIMO
- Detection with 1024QAM
  - Near-optimal detectors are complex
  - Their low complexity versions degrade performance
- Low complexity LORD detector has limitations
  - Optimize search region
  - Optimize LLR saturation

System Model

MIMO system combined with OFDM

Received signal at resource element is given by: $\mathbf{y} = \mathbf{H}\mathbf{x} + \mathbf{n}$

$\mathbf{H} = N_r \times N_t$ channel matrix

$\mathbf{x}$ transmitted QAM symbols

$\mathbf{n}$ complex additive white Gaussian noise with zero mean and variance $\sigma^2 = \dfrac{N_t}{\mathrm{SNR}}$

We consider the case $N_r = N_t = 2$

$$\mathbf{y} = \mathbf{h}_1 x_1 + \mathbf{h}_2 x_2 + \mathbf{n}$$

$\mathbf{h}_1$: channel coefficients of user of interest
$\mathbf{h}_2$: channel coefficients of interferer

$$\mathrm{E}[x_1 . x_1^*] = \mathrm{E}[x_1 . x_1^*] = 1$$

x_1 and x_2 are drawn from a 1024QAM constellation $\mathcal{M}$

Maximum Likelihood (ML) Detection

A hard-output ML detector solves:

$$\min_{\mathbf{x}\in S} \|\mathbf{y} - \mathbf{H}\mathbf{x}\|^2$$

where S is the lattice of symbol vectors ($|S| = |\mathcal{M}|^2$)

Let $\mathbf{b}_{\mathbf{x}} = [x_b]_{b=1}^{K}$ be the bit vector of $\mathbf{x}$, $x_b \in \{0,1\}$ and $K = \log_2(|S|)$

A soft-output ML detector calculates the log-likelihood ratio (LLR) of bit b as:

$$\lambda_b = \min_{\mathbf{x}\in S_{b,1}} \frac{\|\mathbf{y} - \mathbf{H}\mathbf{x}\|^2}{\boldsymbol{\sigma}^2} - \min_{\mathbf{x}\in S_{b,0}} \frac{\|\mathbf{y} - \mathbf{H}\mathbf{x}\|^2}{\boldsymbol{\sigma}^2}$$

$S_{b,1}$ corresponds to points in S having in the bit position b a value of 1
$S_{b,0}$ corresponds to points in S having in the bit position b a value of 0

Minimum Mean Square Error (MMSE) Detector

The MMSE detector solves for an equalized output $\hat{\mathbf{y}}$:

$$\hat{\mathbf{y}} = (\mathbf{H}^*\mathbf{H} + (1/\mathrm{SNR})\mathbf{I}_2)^{-1}\mathbf{H}^*\mathbf{y}$$

And the LLRs can be computed as:

$$\lambda_{\acute{b}}^{t} = \min_{\mathbf{x}(t)\in\mathcal{M}_{\acute{b},t,1}} \frac{|\hat{\mathbf{y}}(t) - \mathbf{x}(t)|^2}{\sigma_{\mathrm{MMSE}}^2} - \min_{\mathbf{x}(t)\in\mathcal{M}_{\acute{b},t,0}} \frac{|\hat{\mathbf{y}}(t) - \mathbf{x}(t)|^2}{\sigma_{\mathrm{MMSE}}^2}$$

where $t \in \{1,2\}$ is the symbol index
$\sigma_{\mathrm{MMSE}}^2 = \sigma^{\mathbf{2}}\mathbf{W}(t,t)$ is a scaled variance, where $\mathbf{W} = (\mathbf{H}^*\mathbf{H} + (1/\mathrm{SNR})\mathbf{I}_2)^{-1}$

$\mathcal{M}_{\acute{b},t,1}$ corresponds to points in $\mathcal{M}$ having in the bit position $\acute{b}$ of symbol t a 1
$\mathcal{M}_{\acute{b},t,0}$ corresponds to points in $\mathcal{M}$ having in the bit position $\acute{b}$ of symbol t a 0

Layered Orthogonal Lattice Detector (LORD)

QR decomposition in the preprocessing step:

$$\tilde{\mathbf{y}} = \mathbf{Q}^*\mathbf{y} = \mathbf{R}\mathbf{x} + \mathbf{Q}^*\mathbf{n} = \mathbf{R}\mathbf{x} + \tilde{\mathbf{n}}$$

$$\begin{bmatrix} \tilde{y}_1 \\ \tilde{y}_2 \end{bmatrix} = \begin{bmatrix} r_{1,1} & r_{1,2} \\ 0 & r_{2,2} \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} + \begin{bmatrix} \tilde{n}_1 \\ \tilde{n}_2 \end{bmatrix}$$

Exhaustively search layer 2. For each possibility $\bar{x}_2$, $\bar{x}_1 = slice\left(\left(\tilde{y}_1 - r_{1,2}\bar{x}_2\right)/r_{1,1}\right)$

The searched lattice of vectors $\bar{\mathbf{x}} = [\bar{x}_1, \bar{x}_2]$ is $\acute{S}$ ($|\acute{S}| = |\mathcal{M}|$)

Searching layer 2, LLRs of x_2 can be computed

$$\lambda_{\acute{b}}^2 = \min_{\mathbf{x} \in \acute{S}_{\acute{b},2,1}} \frac{\|\mathbf{y} - \mathbf{H}\mathbf{x}\|^2}{\sigma^2} - \min_{\mathbf{x} \in \acute{S}_{\acute{b},2,0}} \frac{\|\mathbf{y} - \mathbf{H}\mathbf{x}\|^2}{\sigma^2}$$

$\acute{S}_{\acute{b},2,1}$ corresponds to points in $\acute{S}$ having in the bit position $\acute{b}$ of symbol 2 a value 1

$\acute{S}_{\acute{b},2,0}$ corresponds to points in $\acute{S}$ having in the bit position $\acute{b}$ of symbol 2 a value 0

To compute LLRs of x_1 the layers are swapped and the same operation is repeated

Output identical to ML detector with 2x2 MIMO

Turbo LORD (T-LORD)

T-LORD is a generalization of LORD

It builds on the maximum-a-posteriori (MAP) detector instead of the ML detector

Used with iterative detection and decoding (T iterations)

MAP detector accepts a-priori LLRs ξ from the decoder

The modified distance metric is:

$$\varphi(\mathbf{x}) = -\frac{\|\mathbf{y} - \mathbf{H}\mathbf{x}\|^2}{\sigma^2} + \sum_{k=1}^{K} \mathbf{b}_{\mathbf{x}}(k)\xi(k)$$

The a-posteriori LLRs can then be calculated as:

$$\lambda_{\acute{b}}^{t} = \max_{\mathbf{x} \in \acute{S}_{\acute{b},t,1}} \varphi(\mathbf{x}) - \max_{\mathbf{x} \in \acute{S}_{\acute{b},t,0}} \varphi(\mathbf{x})$$

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Low Complexity LORD (LC-LORD)

Searching $|\mathcal{M}| = 1024$ lattice points is still computationally demanding

LC-LORD only explores a subset of the constellation at the root layer
- A reduced QAM θ
- A square subset centered on equalized output $\tilde{y}_2/r_{2,2}$

LLRs can not be computed when all points in θ have the same bit value at a specific bit
- An LLR saturation mechanism is required
- Especially for high order bits when Gray mapping is employed

LC-LORD need not be applied on all carriers
- Worst carriers can be isolated an treated with regular LORD
- This depends on the implementation constraints
- Criteria for sorting worst carriers is:

$$\min_{l=1,2} r^l(2,2)$$

where l denotes the antenna index at the root layer

Optimizing Search Region Location

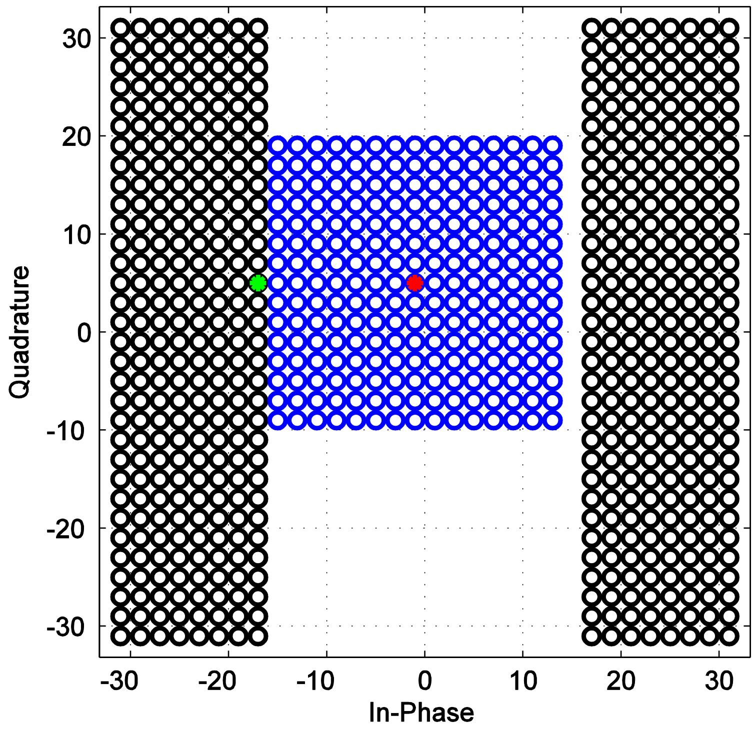
LC-LORD fails when the actual transmitted symbol lies outside θ

This is worse with correlated channels
- $\mathbf{H}$ ill-conditioned
- $r_{2,2}$ tends to zero

One solution uses the hard-output of MMSE detection as a center of search on both layers

This is called MMSE-LC-LORD

Note that operations on both layers are now dependent
- Can not be fully parallelized



Constellation Schematic - Black Circles Indicate that Third MSB is 1

Proposed Layer-Ordered LC-LORD (LO-LC-LORD)

This proposed solution is based on:
- Layer ordering
- Zero-forcing with decision feedback (ZFDF)

Find equalized output on layer 2

$$\bar{x}_2^1 = slice\left(\tilde{y}_2 / r_{2,2}\right)$$

Get its corresponding projection on layer 1

$$\bar{x}_1^1 = slice\left(\left(\tilde{y}_1 - r_{1,2}\bar{x}_2^1\right)/r_{1,1}\right)$$

We obtain $\bar{\mathbf{x}}^1 = [\bar{x}_1^1, \bar{x}_2^1]$

Permute layers and apply same procedure to obtain $\bar{\mathbf{x}}^2 = [\bar{x}_1^2, \bar{x}_2^2]$

The centers of reduced search on both layers are the components of $\mathbf{x}_{\text{center}}$

$$\mathbf{x}_{\text{center}} = \min_{\mathbf{x}\in\{\bar{\mathbf{x}}^1,\bar{\mathbf{x}}^2\}} \|\mathbf{y} - \mathbf{H}\mathbf{x}\|^2$$

Proposed Iterative LC-LORD (Iter-LC-LORD)

This solution adds an iterative behavior

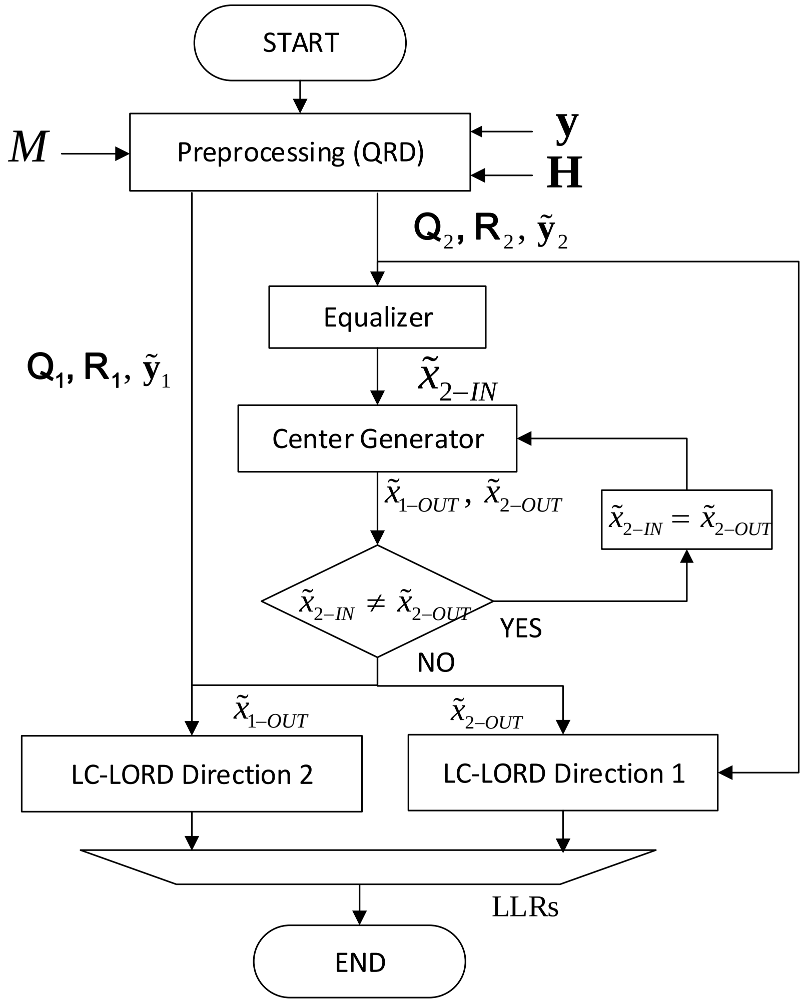
The Center Generator is
a hard-output LC-LORD
- single layer operation
- hard output constitutes
  updated centers of search

Updated search centers are
closer to ML hard output

Might get stuck in local minima

Algorithm halts after a maximum
of J iterations

With T-LORD center updates can take place
on every detection/decoding iteration



Proposed Region-Thresholding LC-LORD (RegTh-LC-LORD)

With LC-LORD one of the two terms below can go missing

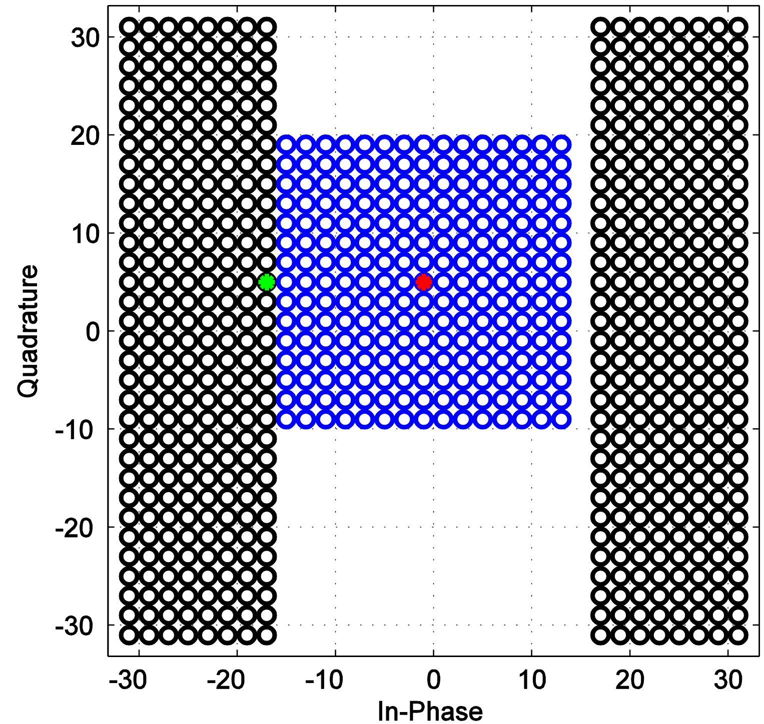
$$\lambda_{\acute{b}}^{t} = \max_{\mathbf{x}\in\acute{S}_{\acute{b},t,1}} \varphi(\mathbf{x}) - \max_{\mathbf{x}\in\acute{S}_{\acute{b},t,0}} \varphi(\mathbf{x})$$

LLR saturation in Literature:

- Saturate LLR to a threshold value
- Substitute missing term by maximum Euclidean norm within θ

Proposed approach (RegTh-LC-LORD):

- Locate the closest point to the center of θ having opposite bit value (in green)
- Project on other layer + slice
- Substitute missing term by the distance from resultant vector to received vector



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Complexity Study

Preprocessing complexity

- QR decomposition (can be avoided in 2x2 MIMO)
- Handling search region boundaries

Search routine complexity

- Summarized in table in terms of Euclidean distance computations (visited nodes)
- Table shows the worst case
- When search center is close to boundaries of $\mathcal{M}$, θ gets clipped
- In Iter-LC-LORD θs of subsequent iterations partially overlap and computations can be saved

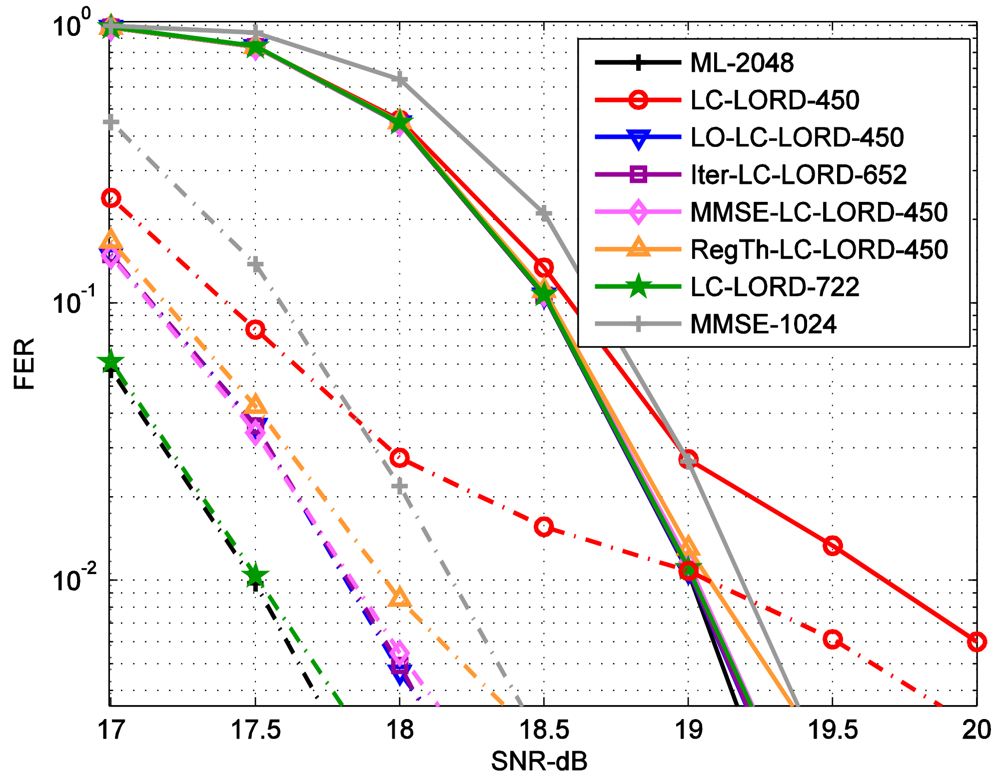
Approach	Description	Nodes Visited
ML	Full Complexity LORD	$2 \times \lvert\mathcal{M}\rvert$
LC-LORD	Low Complexity LORD	$2 \times \lvert\theta\rvert$
LO-LC-LORD	Layer Ordered LC-LORD + Region Thresholding	$2 \times \lvert\theta\rvert$
Iter-LC-LORD	Iterative LC-LORD + Region Thresholding	$(J + 2) \times \lvert\theta\rvert$
MMSE-LC-LORD	MMSE-based LC-LORD + Region Thresholding	$2 \times \lvert\theta\rvert$
RegTh-LC-LORD	LC-LORD + Region Thresholding	$2 \times \lvert\theta\rvert$
MMSE	Soft-output MMSE	$\lvert\mathcal{M}\rvert$

Simulation Scenario

- A 2x2 MIMO simulation chain was implemented
  - System model in introduction
  - All studied detectors were implemented
  - Iterative detection/decoding
- Turbo coding/decoding
  - Code rate 1/2
  - 8 iterations
- Two channel types
  - Uncorrelated (rich scattering)
  - Highly correlated ($\alpha = 0.9$)
- Performance measure
  - Frame Error Rate (FER)

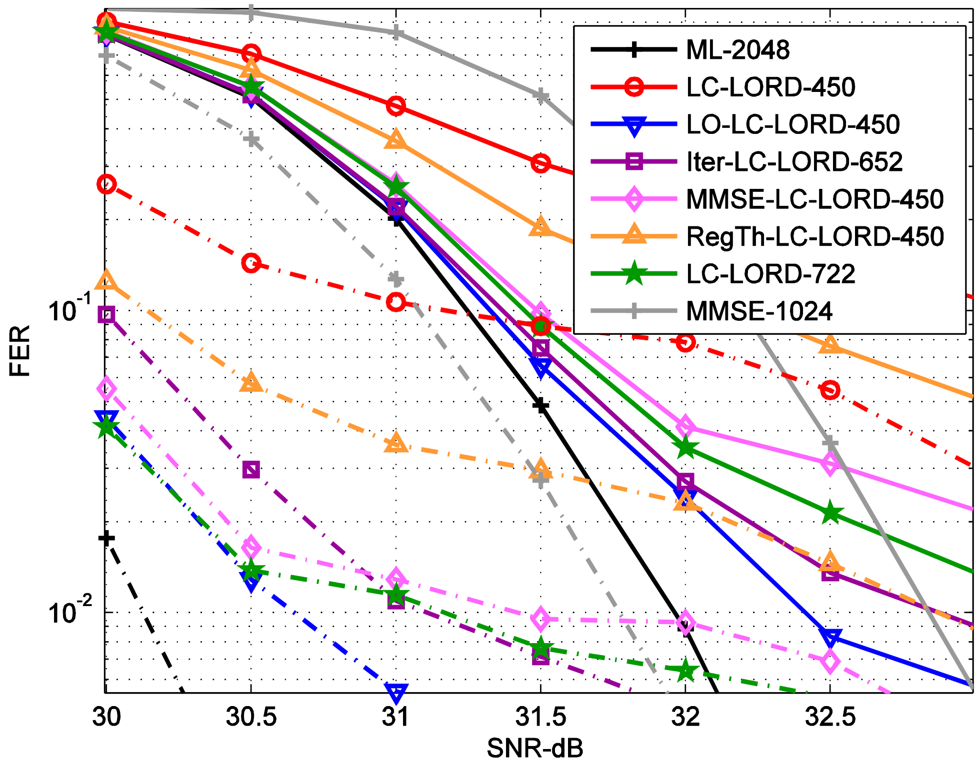
- Parameters
  - $|\theta| = 225$
  - $J = 8$ (1.8 on average)
  - $T = 4$

FER – Uncorrelated



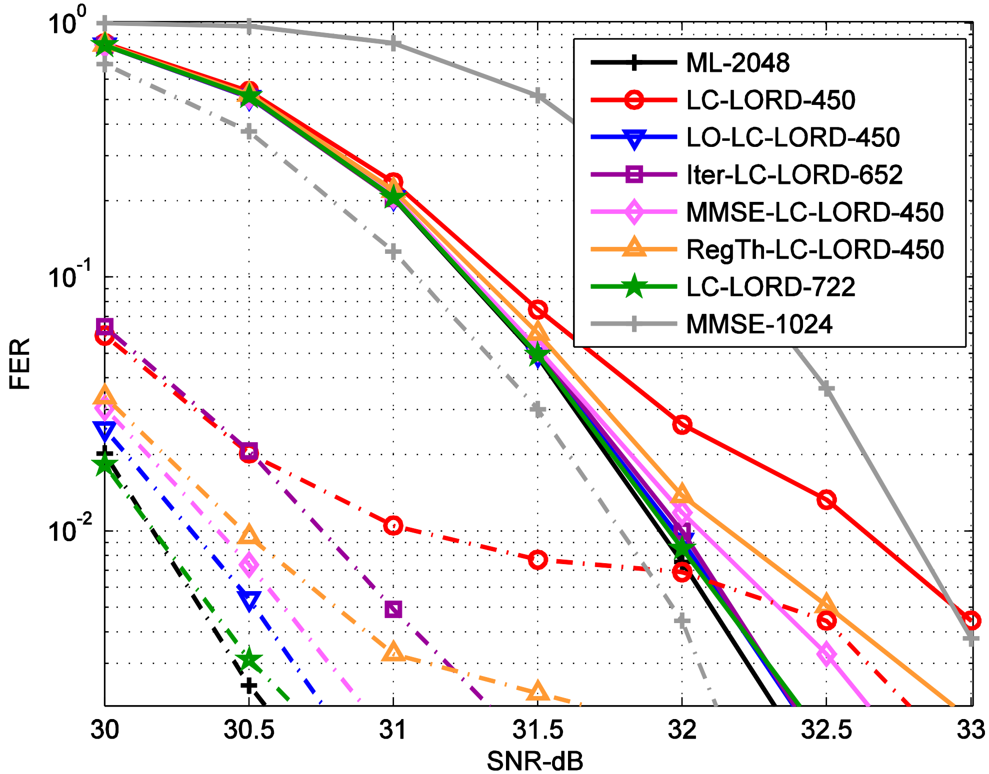
Detectors Performance with Uncorrelated Channels and 15% Full Complexity Carriers, for T = 1 (solid) and T = 4 (dotted)

FER – Correlated



Detectors Performance with Correlated Channels and 15% Full Complexity Carriers, for T = 1 (solid) and T = 4 (dotted)

FER – Correlated



Detectors Performance with Correlated Channels and 30% Full Complexity Carriers, for T = 1 (solid) and T = 4 (dotted)

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Summary and Future Work

- **2×2 MIMO** systems that use **1024-QAM** were studied.
- Building on the **LORD** detector, several algorithms were proposed.
- Optimizing the **location** of a reduced **region of search**.
- Optimizing **LLR saturation**.
- The optimizations resulted in an **enhanced performance**, at a **reduced complexity**.

- The proposed approaches are to be studied with **higher order MIMO**, where **LORD loses optimality**.

Thanks for listening

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