# Power Scaling Laws for Massive MIMO Relay Systems with Linear Transceivers

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# Outline

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# Background

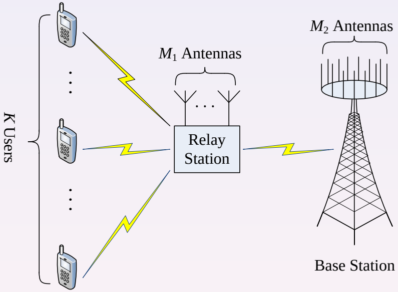
- Massive MIMO systems are capable of providing significant gains in both spectral efficiency and energy efficiency [Marzetta '10] [Ngo '13].
- Relaying technique can enhance coverage and improve throughput, etc.
- [Suraweera '13] investigated the one-way amplify-and-forward relay system, and the conclusion showed that the transmit power of each source or relay station can be made inversely proportional to the number of relay antennas while maintaining a given quality-of-service.
- As discussed by [Ngo '13] and [Jin '14], massive MIMO can be wellapplied in two-way AF relaying with distributed and centralized antennas.

# Our work

- In this paper, we investigate the uplink of a multiuser massive MIMO amplify-and-forward relay system, where K single-antenna users simultaneously communicate with the base station through a fixed relay station.
- Contribution in this paper:
  - ◇ We derive closed-form approximations of the ergodic achievable rate of the aforementioned system while MRC or ZF is performed at the relay station and the base station.
  - ◇ We study the power scaling laws, the conclusion shows that the transmit power of users or relay station can be cut down inverse proportionally to the number of the relay station antennas or the number of the base station antennas.

# Outline

# The model of relay system in cellular with large antenna arrays




- Scenario I: MRC is performed at both relay station and base station.
- Scenario II: ZF is performed at both relay station and base station.

- $\mathbf{G}_1 = \mathbf{H}_1 \mathbf{D}_1^{1/2}$ denotes the channel matrix between users and relay station, where $\mathbf{H}_1 \in \mathbb{C}^{M_1 \times K}$ and the diagonal matrix $\mathbf{D}_1 \in \mathbb{R}^{K \times K}$ represent fast fading and slow fading coefficients, respectively.
- $\mathbf{G}_2 = \eta_2^{1/2} \mathbf{H}_2 \in \mathbb{C}^{M_2 \times K}$ denotes the channel matrix between relay station and base station.
- For Scenario I, relay station and base station use $a_{\mathrm{mrc}} \mathbf{G}_1^{\dagger}$ and $\mathbf{G}_2^{\dagger}$ to process the received signal, respectively.
- For Scenario II, relay station and base station use $a_{\mathrm{zf}} \left( \mathbf{G}_1^{\dagger} \mathbf{G}_1 \right)^{-1} \mathbf{G}_1^{\dagger}$ and $\left( \mathbf{G}_2^{\dagger} \mathbf{G}_2 \right)^{-1} \mathbf{G}_2^{\dagger}$ to process the received signal, respectively.
- $a_{\mathrm{mrc}}$ and $a_{\mathrm{zf}}$ are instantaneous power coefficients which make relay station satisfies the power constraint.

## the received signal for MRC

- The received signal at the base station for MRC:

$$\begin{aligned}\tilde{\mathbf{y}}_{BS} =& a_{\mathrm{mrc}}\eta_2\Theta\mathbf{D}_1^{1/2}\Phi\mathbf{D}_1^{1/2}\mathbf{x} \\ &+ a_{\mathrm{mrc}}\eta_2\Theta\mathbf{D}_1^{1/2}\mathbf{H}_1^{\dagger}\mathbf{n}_R + \eta_2^{1/2}\mathbf{H}_2^{\dagger}\mathbf{n}_{BS}.\end{aligned} \tag{1}$$

- The instantaneous power coefficient for MRC:

$$a_{\mathrm{mrc}} = \sqrt{\frac{P_r}{\mathrm{Tr}\left(\mathbf{D}_1^{1/2}\Phi\mathbf{D}_1\mathbf{P}\Phi\mathbf{D}_1^{1/2}+\sigma_R^2\mathbf{D}_1^{1/2}\Phi\mathbf{D}_1^{1/2}\right)}}. \tag{2}$$

- $P_r$ denotes the transmit power of the relay station.
- $\mathbf{P}$ is a diagonal matrix, where $P_j$ ($j$th element of $\mathbf{P}$) denotes the transmit power of $j$th user.

# the received signal for ZF

- The received signal at the base station for ZF:

$$\begin{aligned}\tilde{\mathbf{y}}_{BS} =& a_{\text{zf}}\mathbf{x} + a_{\text{zf}}\mathbf{D}_1^{-1/2}\Phi^{-1}\mathbf{H}_1^{\dagger}\mathbf{n}_R \\ &+ \eta_2^{-1/2}\Theta^{-1}\mathbf{H}_2^{\dagger}\mathbf{n}_{BS}.\end{aligned} \tag{3}$$

- The instantaneous power coefficient for ZF:

$$a_{\text{zf}} = \sqrt{\frac{P_r}{\text{Tr}\left[\mathbf{P}+\sigma_R^2\mathbf{D}_1^{-1/2}\Phi^{-1}\mathbf{D}_1^{-1/2}\right]}}. \tag{4}$$

# Outline

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# MRC

- The ergodic rates of the $j$th user can be written as

$$R_j \approx \frac{1}{2}\log_2\left(1+\frac{M_1 M_2 P_j P_r \eta_{1j}\eta_2}{S_{\mathrm{N_{MRC}}}+S_{\mathrm{I}}}\right). \quad (5)$$

- $S_{\mathrm{N_{MRC}}}$ and $S_{\mathrm{I}}$ are defined as

$$\begin{aligned} S_{\mathrm{N_{MRC}}} =& \sigma_{BS}^2\left[M_1\frac{\mathrm{Tr}\left(\mathbf{D}_1^2\mathbf{P}\right)}{\eta_{1j}}+\frac{\mathrm{Tr}\left(\mathbf{D}_1\right)\mathrm{Tr}\left(\mathbf{D}_1\mathbf{P}\right)}{\eta_{1j}}\right] \\ &+P_r\sigma_R^2\eta_2\left[M_2+\frac{\mathrm{Tr}\left(\mathbf{D}_1\right)}{\eta_{1j}}\right]+\sigma_R^2\sigma_{BS}^2\frac{\mathrm{Tr}\left(\mathbf{D}_1\right)}{\eta_{1j}}, \quad (6) \end{aligned}$$

$$\begin{aligned} S_{\mathrm{I}} =& M_1 P_r\eta_2\frac{\sum_{i\neq j}P_i\eta_{1i}^2}{\eta_{1j}}+M_2P_r\eta_2\sum_{i\neq j}P_i\eta_{1i} \\ &+P_r\eta_2\mathrm{Tr}\left(\mathbf{D}_1\right)\frac{\sum_{i\neq j}P_i\eta_{1i}}{\eta_{1j}}. \quad (7) \end{aligned}$$

# Outline

# ZF

- The ergodic rates of the $j$th user can be written as

$$R_j \approx \frac{1}{2}\log_2\left[1+\frac{\left(M_1-K\right)\left(M_2-K\right)P_jP_r\eta_{1j}\eta_2}{S_{\mathrm{N_{ZF}}}}\right]. \quad (8)$$

- $S_{\mathrm{N_{ZF}}}$ is defined as

$$S_{\mathrm{N_{ZF}}} = \left(M_1-K\right)\mathrm{Tr}\left(\mathbf{P}\right)\sigma_{BS}^2\eta_{1j} + \left(M_2-K\right)P_r\sigma_R^2\eta_2 + \sigma_R^2\sigma_{BS}^2\eta_{1j}\mathrm{Tr}\left(\mathbf{D}_1^{-1}\right). \quad (9)$$

# Outline

- Three cases are considered in our work.
  - Case I: $P_j = E_j, P_r = E_r / M_2$.
  - Case II: $P_j = E_j / M_1, P_r = E_r$.
  - Case III: $P_j = E_j / M_1, P_r = E_r / M_2$.
- $E_r$ is fixed.
- $E_1, E_2, \cdots, E_K$ are fixed.

# Outline

# MRC

- The upper bound of case I:

$$R_j = \frac{1}{2}\log_2\left(1+\frac{E_j E_r \eta_{1j}^2 \eta_2}{\sigma_{BS}^2 \mathrm{Tr}\left(\mathbf{D}_1^2 \mathbf{E}\right)}\right). \tag{10}$$

- The upper bound of case II:

$$R_j = \frac{1}{2}\log_2\left(1+\frac{E_j \eta_{1j}}{\sigma_R^2}\right). \tag{11}$$

- The upper bound of case III:

$$R_j = \frac{1}{2}\log_2\left(1+\frac{E_j E_r \eta_{1j}^2 \eta_2}{\sigma_{BS}^2 \mathrm{Tr}\left(\mathbf{D}_1^2 \mathbf{E}\right) + E_r \sigma_R^2 \eta_{1j} \eta_2 + \sigma_R^2 \sigma_{BS}^2 \mathrm{Tr}\left(\mathbf{D}_1\right)}\right). \tag{12}$$

# Outline

# ZF

- The upper bound of case I:

$$R_j = \frac{1}{2}\log_2\left(1 + \frac{E_j E_r \eta_2}{\sigma_{BS}^2 \mathrm{Tr}\left(\mathbf{E}\right)}\right). \tag{13}$$

- The upper bound of case II:

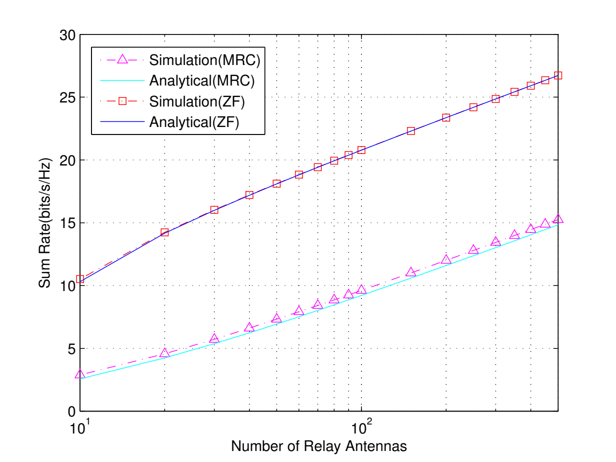
$$R_j = \frac{1}{2}\log_2\left(1 + \frac{E_j \eta_{1j}}{\sigma_R^2}\right). \tag{14}$$

- The upper bound of case III:

$$R_j = \frac{1}{2}\log_2\left(1 + \frac{E_j E_r \eta_{1j} \eta_2}{\sigma_{BS}^2 \mathrm{Tr}\left(\mathbf{E}\right)\eta_{1j} + E_r \sigma_R^2 \eta_2 + \sigma_R^2 \sigma_{BS}^2 \eta_{1j} \mathrm{Tr}\left(\mathbf{D}_1^{-1}\right)}\right) \tag{15}$$

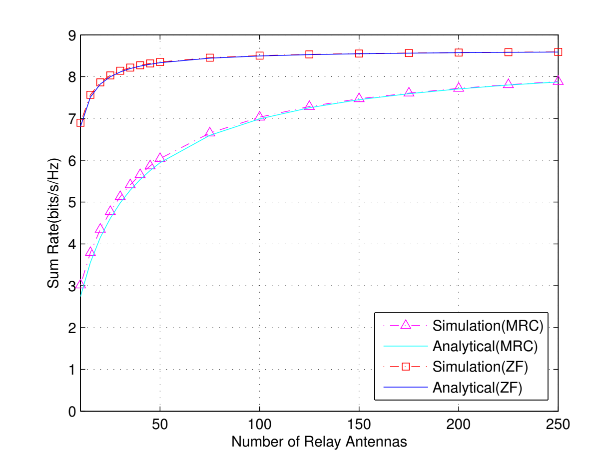
# Outline

# Sum rate of users for fixed transmit powers



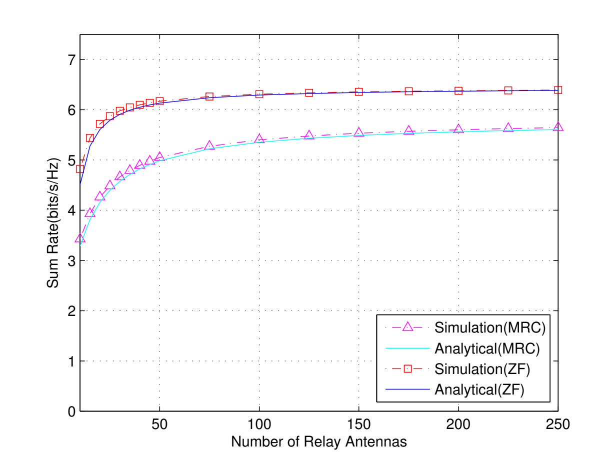

$K = 5$, $\mathbf{D}_1 = \mathbf{I}_5$, $\eta_2 = 1$, $\sigma_R^2 = 1$, $\sigma_{BS}^2 = 1$, $P_1 = P_2 = \cdots = P_5 = 5$, $P_r = 50$, $M_1 = M_2$.

# Sum rate of users for case I



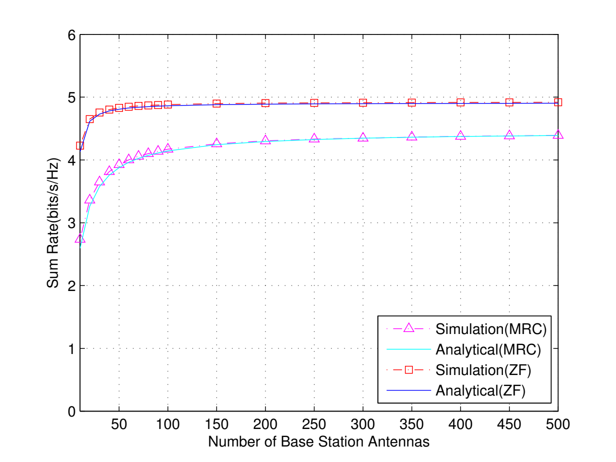

$K = 5$, $\mathbf{D}_1 = \mathbf{I}_5$, $\eta_2 = 1$, $\sigma_R^2 = 1$, $\sigma_{BS}^2 = 1$, $E_1 = E_2 = \cdots = E_5 = 5$, $E_r = 50$, $M_2 = 2M_1$.

# Sum rate of users for case II




$K = 5$, $\mathbf{D}_1 = \mathbf{I}_5$, $\eta_2 = 1$, $\sigma_R^2 = 1$, $\sigma_{BS}^2 = 1$, $E_1 = E_2 = \cdots = E_5 = 5$, $E_r = 50$, $M_2 = 100$.

# Sum rate of users for case III




$K = 5$, $\mathbf{D}_1 = \mathbf{I}_5$, $\eta_2 = 1$, $\sigma_R^2 = 1$, $\sigma_{BS}^2 = 1$, $E_1 = E_2 = \cdots = E_5 = 5$, $E_r = 50$, $M_1 = 50$.

# Outline

# Conclusion

- We derived the approximations of the ergodic rate lower bounds in closed-form while MRC or ZF is performed.
- To maintain a certain achievable rate, the average transmit powers of users can be cut down by increasing the number of relay antennas. Similarly, the transmit power of relay station can be cut down by increasing the number of base station antennas.

# THANK YOU!