Locally Linear Low-rank Tensor Approximation

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Outline

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  - Multidimensional Scaling (MDS): Embed the data into a graph to construct d-dimensional manifold (Tenenbaum et al. 2000).
  - Locally linear embedding (LLE)(Roweis and Saul 2000).
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  - Geometric Multi-resolution Analysis (GMRA): Data dependent multi-scale dictionaries (Allard et al. 2012).
- However, these approaches are not directly applicable to high order data, e.g. hyperspectral imaging, social and biological networks.

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- These methods are mostly limited to learning the optimal linear transformation for supervised classification of high-order data.

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- Two step approach:
  - Decompose the tensor into subtensors.
  - Apply higher order singular value decomposition (HOSVD) to these subtensors.

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Tensor Algebra

- A multidimensional array with N modes $\mathcal{X} \in \mathbb{R}^{I_1 \times I_2 \times \ldots \times I_N}$ is called a tensor, where $x_{i_1, i_2, ..i_N}$ denotes the $(i_1, i_2, ..i_N)^{th}$ element of the tensor $\mathcal{X}$.

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- Tensor *n-rank* of $\mathcal{X}$ is the collection of ranks of mode matrices $\mathbf{X}_{(n)}$:
  $n\text{-rank}(\mathcal{X}) = \left(\text{rank}(\mathbf{X}_{(1)}),\ \text{rank}(\mathbf{X}_{(2)}), ...,\ \text{rank}(\mathbf{X}_{(N)})\right)$.

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Any tensor $\mathcal{X} \in \mathbb{R}^{I_1 \times I_2 \times \ldots \times I_N}$ can be decomposed as:

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where $\mathbf{U}^{(n)} \in \mathbb{R}^{I_n \times I_n}$s are the left singular vectors of $\mathbf{X}_{(n)}$ and $\mathcal{S} \in \mathbb{R}^{I_1 \times I_2 \times \ldots \times I_N}$ is the core tensor computed as:

$$\mathcal{S} = \mathcal{X} \times_1 (\mathbf{U}^{(1)})^\top \times_2 (\mathbf{U}^{(2)})^\top \ldots \times_N (\mathbf{U}^{(N)})^\top. \tag{2}$$

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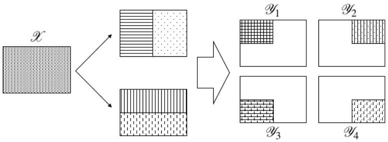

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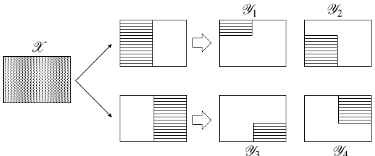

Locally Linear High Order Singular Value Decomposition

Goal: Low *n*-rank approximation to subtensors of an *N*th order tensor $\mathcal{X} \in \mathbb{R}^{I_1 \times I_2 \times \ldots \times I_N}$ to better capture local nonlinearities.

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- Decompose tensor $\mathcal{X}$ into K subtensors $\mathcal{Y}_k \in \mathbb{R}^{I_{1,k} \times I_{2,k} \times \ldots \times I_{N,k}}$ with $k \in \{1,\ 2, \ldots\ K\}$ by direct division or sequential division approaches.

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- Mapping functions f_ks are defined on the index sets from $\mathcal{X}$ to $\mathcal{Y}_k$ as:

$$f_k : J_1 \times J_2 \times \ \ldots \ \times J_N \mapsto J_{1,k} \times J_{2,k} \times \ \ldots \ \times J_{N,k}, \qquad (3)$$

where $J_n = \{1, 2, \ldots, I_n\}$, $J_{n,k} \subset \{1, 2, \ldots, I_{n,k}\}$ with $n \in \{1, 2, \ldots N\}$.

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- f_ks satisfy $\cup_{k=1}^{K} J_{n,k} = J_n$ and $J_{n,k} \cap J_{n,l} = \emptyset$ when $k \neq l$ for all $k, l \in \{1, 2, \ldots, K\}$.

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- HOSVD is used to obtain the low *n*-rank approximation for each $\mathcal{Y}_k$.

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- HOSVD is used to obtain the low n-rank approximation for each $\mathcal{Y}_k$.
- Let $\hat{\mathcal{Y}}_k$ be a low n-rank approximation of $\mathcal{Y}_k$ computed as:

$$\hat{\mathcal{Y}}_k = \hat{\mathcal{S}}_k \times_1 \hat{\mathbf{U}}^{(1,k)} \times_2 \hat{\mathbf{U}}^{(2,k)} \ldots \times_N \hat{\mathbf{U}}^{(N,k)}, \tag{4}$$

where $\hat{\mathbf{U}}^{(n,k)}$s are the truncated projection matrices of $\mathcal{Y}_k$ obtained by keeping the first r_n columns of $\mathbf{U}^{(n,k)}$ for $n \in \{1, 2, \ldots N\}$ and $\hat{\mathcal{S}}_k$ is the core tensor

$$\hat{\mathcal{S}}_k = \hat{\mathcal{Y}}_k \times_1 (\hat{\mathbf{U}}^{(1,k)})^\top \times_2 (\hat{\mathbf{U}}^{(2,k)})^\top \ldots \times_N (\hat{\mathbf{U}}^{(N,k)})^\top. \tag{5}$$

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- Combining all of the subtensors $\hat{\mathcal{Y}}_k$s by using the inverse mapping functions f_k^{-1} provides piecewise-linear approximation of $\mathcal{X}$:

$$\hat{\mathcal{X}} = \sum_{k=1}^{K} \hat{\mathcal{Y}}_{k,(f_k^{-1}(J_{1,k} \times J_{2,k} \times \ldots \times J_{N,k}))}. \tag{7}$$

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- Translating Subspaces

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  - Two point clouds with 100 Gaussian random variables in $\mathbb{R}^{20}$ were generated.
  - The two subspaces in which the point clouds live are orthogonal to each other in $\mathbb{R}^{100}$.
  - The first point cloud is static whereas the second one is translating in time $t \in \{1, 2, \ldots 60\}$.
  - A 3-mode tensor $\mathcal{X} \in \mathbb{R}^{100 \times 200 \times 60}$ is created.

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- Rotating Subspaces
  - Two point clouds with 100 Gaussian random variables in $\mathbb{R}^{20}$ were generated.
  - The first point cloud is static whereas the second one is rotating in time $t \in \{1, 2, ...60\}$ with the rotation matrix
    $\mathbf{A} = \mathbf{I}_{10\times 10} \otimes \begin{bmatrix} cos(\theta t) & sin(\theta t) \\ -sin(\theta t) & cos(\theta t) \end{bmatrix}$ and $\theta = \begin{cases} \frac{\pi}{120}, & t \leq 30 \\ \frac{\pi}{60}, & t > 30 \end{cases}$.
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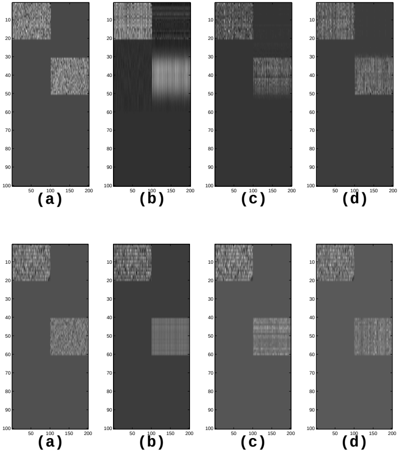

- Figure 1: Low n-rank approximations of $\mathcal{X}$ are computed by HOSVD, LL-HOSVD(DD) and LL-HOSVD(SD) with various n-rank and the cluster number along each mode $C = (4, 4, 4)$. Sample outputs for translating (left) and rotating (right) subspaces: (a) original slice, (b) HOSVD, (c)LL-HOSVD(DD), (d) LL-HOSVD(SD).

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TABLE I
AVERAGE MSE FOR THE RECONSTRUCTED 3-WAY TENSOR $\mathscr{X} \in \mathbb{R}^{100\times200\times60}$ FOR MOVING SUBSPACES BY HOSVD, LL-HOSVD(DD) AND LL-HOSVD(SD) APPROACHES AT VARYING n-RANK OVER 25 TRIALS.

	$R=(3,3,3)$	$R=(5,5,5)$	$R=(7,7,7)$	$R=(9,9,9)$
HOSVD	0.1131	0.1006	0.0943	0.0885
LL-HOSVD(DD)	0.1029	0.0926	0.0792	0.0662
LL-HOSVD(SD)	0.0838	0.0584	0.0407	0.0285

TABLE II
AVERAGE MSE FOR THE RECONSTRUCTED 3-WAY TENSOR $\mathscr{X} \in \mathbb{R}^{100\times200\times60}$ FOR ROTATING SUBSPACES BY HOSVD, LL-HOSVD(DD) AND LL-HOSVD(SD) APPROACHES AT VARYING n-RANK OVER 25 TRIALS.

	$R=(3,3,3)$	$R=(5,5,5)$	$R=(7,7,7)$	$R=(9,9,9)$
HOSVD	0.1016	0.0896	0.0786	0.0685
LL-HOSVD(DD)	0.685	0.0540	0.0474	0.0432
LL-HOSVD(SD)	0.0493	0.0231	0.0137	0.0084

PIE Dataset

- A 3-mode tensor $\mathcal{X} \in \mathbb{R}^{122 \times 160 \times 138}$ is created from PIE dataset (Sim et al. 2003).
- The tensor contains 138 images from 6 different yaw angles and varying illumination conditions collected from a subject.
- Each image is converted to gray scale and downsampled to 122×160.
- n-rank$(\hat{\mathcal{Y}}_k) = (20, 25, 15)$ and the cluster number along each mode is chosen as $C = (4, 4, 4)$.

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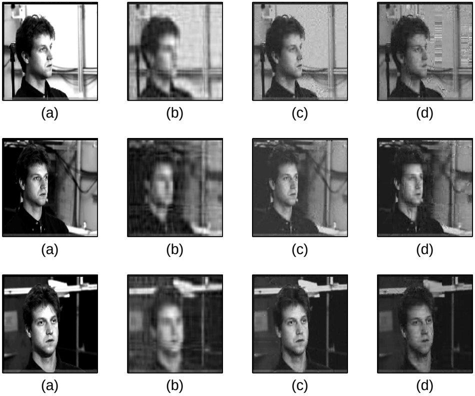

- Figure 2: Frames corresponding to 3 different yaw angles obtained from approximated low n-rank tensor: (a) original image, (b) HOSVD, MSE $=$ 439.0140, (c) LL-HOSVD(DD), MSE $=$ 140.6469, (d) LL-HOSVD(SD), MSE $=$ 378.3899

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- Learning multiresolution tree structure provides better compression rate than HOSVD.

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Acknowledgements

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