Kalman Filters with Bayesian Quadratic Game Fusion in Networks

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Abstract

Distributed filtering in network is a fundamental problem in the field of network signal processing. Each node estimates or tracks some unknown state relying on the private observation and the fusion information from the network. Network fusion is generally a way of interaction over network, by which nodes can learn from each other and make decision mutually. Unlike conventional methods, we construct a distributed filter using Bayesian network game as a fusion tool, where all the nodes exchange their best strategies instead of exchanging local estimators. The proposed algorithm is a coalition of signal processing and game theory in network, which can be extended to more general signal processing and decision making models.

Introduction

Distributed filtering in network has been an appealing research point in the field of network signal processing recently. The basic application deals with nodes in a network collaboratively tracking the real trajectory of an unknown state with limited information and interaction capability. A typical distributed network filter iterates with two steps, local innovation and network fusion [3]. For a specific node, the local innovation executes estimation update by private observations, which can be configured with traditional signal processing method such as RLS, RLMMSE and Kalman filter independent of network topology. The network fusion renewal the estimator using fused information from other nodes. Existing network fusion strategies include incremental [1], consensus [3], diffusion [4] and network game [2, 5]. All these strategies mingle and process the network information with commutative messages such as observations, estimations, auxiliary variables or actions.

The Bayesian network game is powerful to analyze the interaction of strategies among players in a network with incomplete information. Ceyhun Eksin *et al.* [2] proposed an analytical solution to a Bayesian network game of quadratic utilities (BQNG), which is a seminal work deploying network game into signal processing in network. The optimal actions, derived by best strategies, incorporate with available historical information and help nodes refine their estimations on a static unknown state. Exchanging strategies, rather than parameters, can be seen as a new type of *strategy-based* fusion way.

In [5], we proposed a recursive distributed filter for multiple observations based on BQNG fusion. However, it only deals with the estimation problem on a static state. In order to track a time-varying object, we construct a distributed KF extending our previous work, namely the BQNG-KF filter.

Main Objectives

- Design a more precise and convergent distributed network filter for adaptive target tracking which models Bayesian network game as a fusion tool.

Scenario

Consider a network $\mathcal{G} = (\mathcal{V}, \mathcal{E})$ distributedly tracks on an unknown state θ_t that transferred following a linear Gaussian form

$$\theta_t = f_t\theta_{t\text{-}1} + w_{\theta,t}, \qquad w_{\theta,t} \sim \mathcal{N}(0, \sigma^2_{\theta,t}),$$

and each node is supposed to receive a private observation at t

$$s_{i,t} = h_{i,t}\theta_t + n_{i,t}, \qquad \forall i \in \mathcal{V},$$

where $n_{i,t} \sim \mathcal{N}(0, \sigma^2_{i,t})$ is the measure noise. Every node maintains an estimator upon θ_t recursively and can exchange information with its neighbors. A distributed network filtering scheme is expected to regulate the nodes to implement objective tracking conformably.

Structure of the BQNG-KF filter

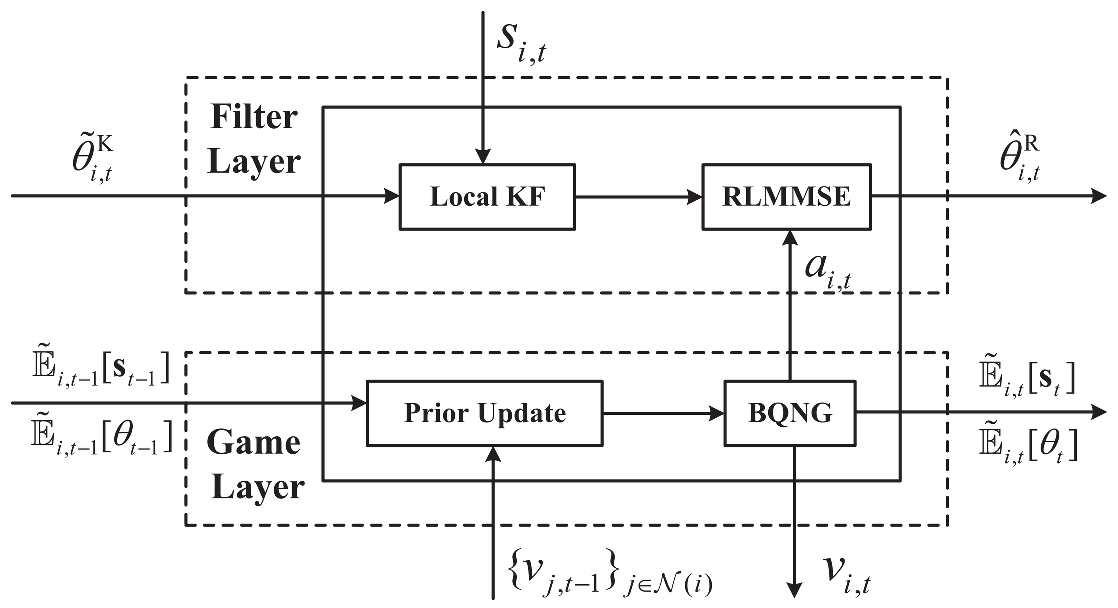

Figure 1: Structure of the BQNG-KF filter.

- Filter layer: iteratively update the estimator upon θ_t.
  - Local KF: local innovation process, use private observation $s_{i,t}$ to renew the estimator. The Kalman filter algorithm is configured.
  - RLMMSE: network fusion process, update the estimator by the linear Gaussian action $a_{i,t}$ from the game layer. The recursive least mean square estimating algorithm is configured.
- Game layer: imports the dynamic Bayesian quadratic network game for information fusion with its action $a_{i,t}$ feedback to the filter layer as the input of network fusion module.
  - Prior Update: renewals beliefs $\tilde{\mathbb{E}}_{i,t}[\mathbf{s}_t]$ and $\tilde{\mathbb{E}}_{i,t}[\theta_t]$ by reasoning from t-1 using $\{v_{j,t\text{-}1}\}_{j\in\mathcal{N}(i)}$. Based on RLMMSE and Bayesian reasoning.
  - BQNG: calculate the best action $a_{i,t}$ as a feedback to the filter layer and broadcast its best strategy message $v_{i,t}$ to its neighbors for Prior Update in the next iteration.

BQNG Structure

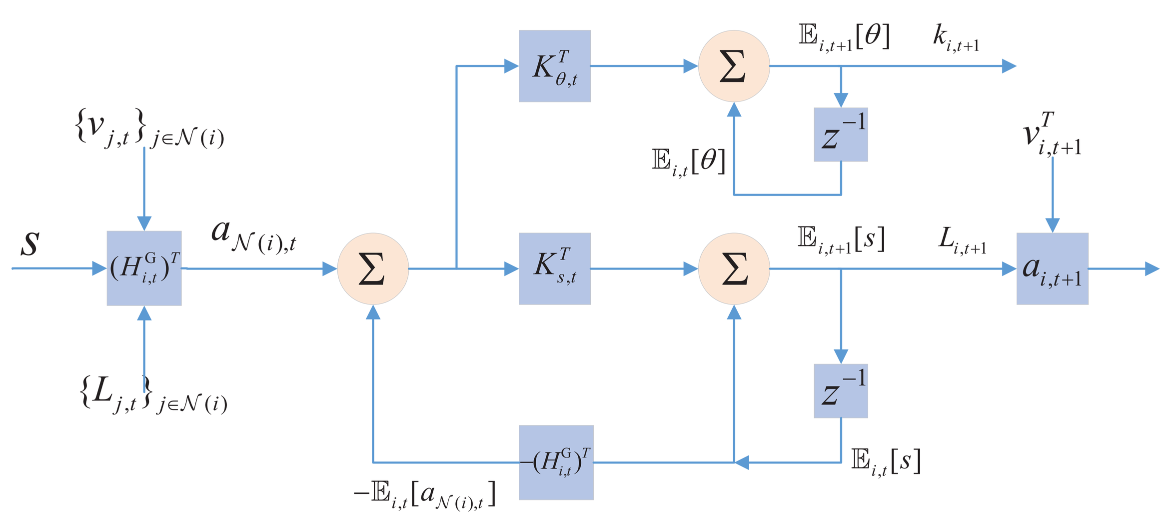

Figure 2: BQNG Structure.

Numerical Result

A network formulated by random topology with 25 nodes is considered. At initialization, suppose $\theta_0 = 5$, $f_t = 1.1, \sigma^2_{\theta,t} = 0.3$ and the observing function to be $h_{i,t} = 1, \sigma^2_{i,t} = 1, \forall i \in \mathcal{V}$.

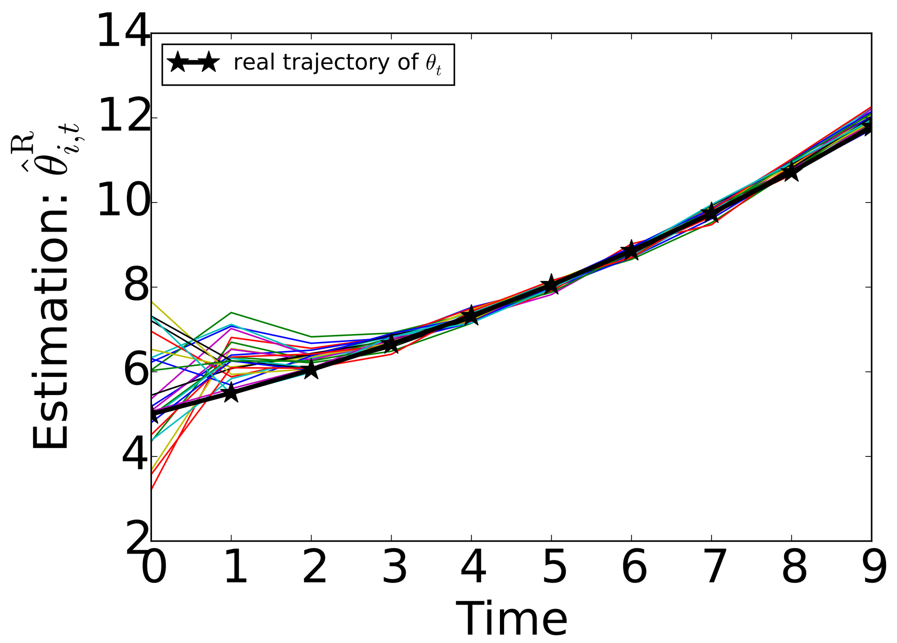

Figure 3: Tracking performance of the network.

Plots in Figure 3 exhibit the tracking performance of the BQNG-KF filter. The y-axis labels the tracking output $\hat{\theta}^{\mathrm{R}}_{i,t}$ while the x-axis labels the time slice. The Black-star-line marks the real transitions of θ_t while ordinary ones display the estimating trajectories of nodes in network. All nodes closely track on θ_t after few iterations.

Figure 4 shows the performance comparison among the BQNG-KF, the Consensus-KF and the Diffusion-KF filters. Define $\hat{\theta}^{\mathrm{R}}_t = [\hat{\theta}^{\mathrm{R}}_{1,t}, ..., \hat{\theta}^{\mathrm{R}}_{N,t}]^{\mathrm{T}}$ as the estimation vector at t. Denote the modulus of the estimation error vector as a measure of global performance, expressed as $\| \hat{\theta}^{\mathrm{R}}_t - \mathbf{1}\theta_t \|$. The BQNG-KF filter proves its tracking ability and shows strong robustness against diversiform topologies.

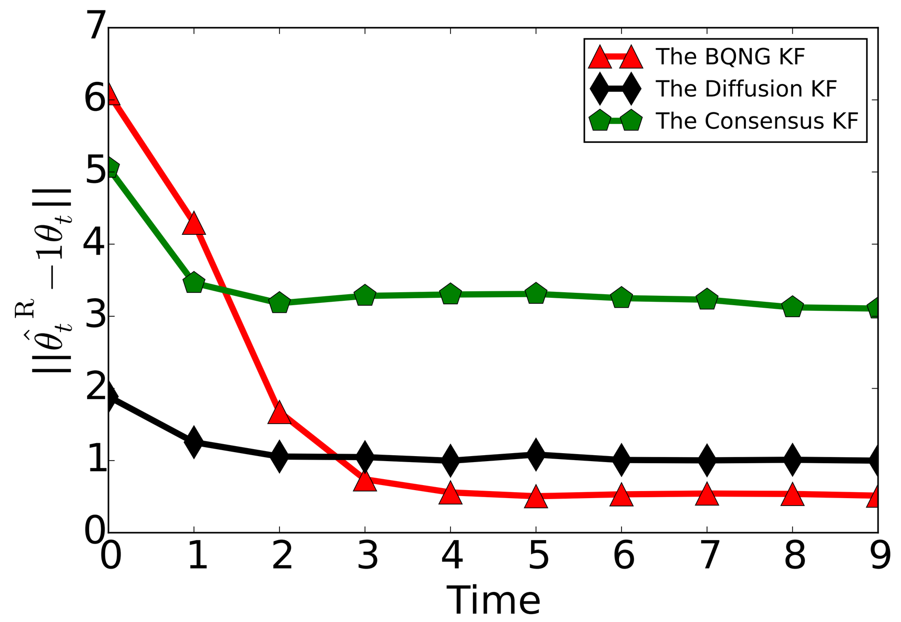

Figure 4: Comparison of BQNG-KF, Consensus-KF and Diffusion-KF filters.

Forthcoming Research

- Asynchronous algorithms would be introduced for more complex scenarios.
- More learning schemes for dynamic game would be considered for prior update.
- Bayesian games with other utility forms would be studied for convergence to more efficient equilibrium.

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Acknowledgements

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