Placing and Sizing
Distributed Photovoltaic Generators
for Optimal Reactive Power Compensation

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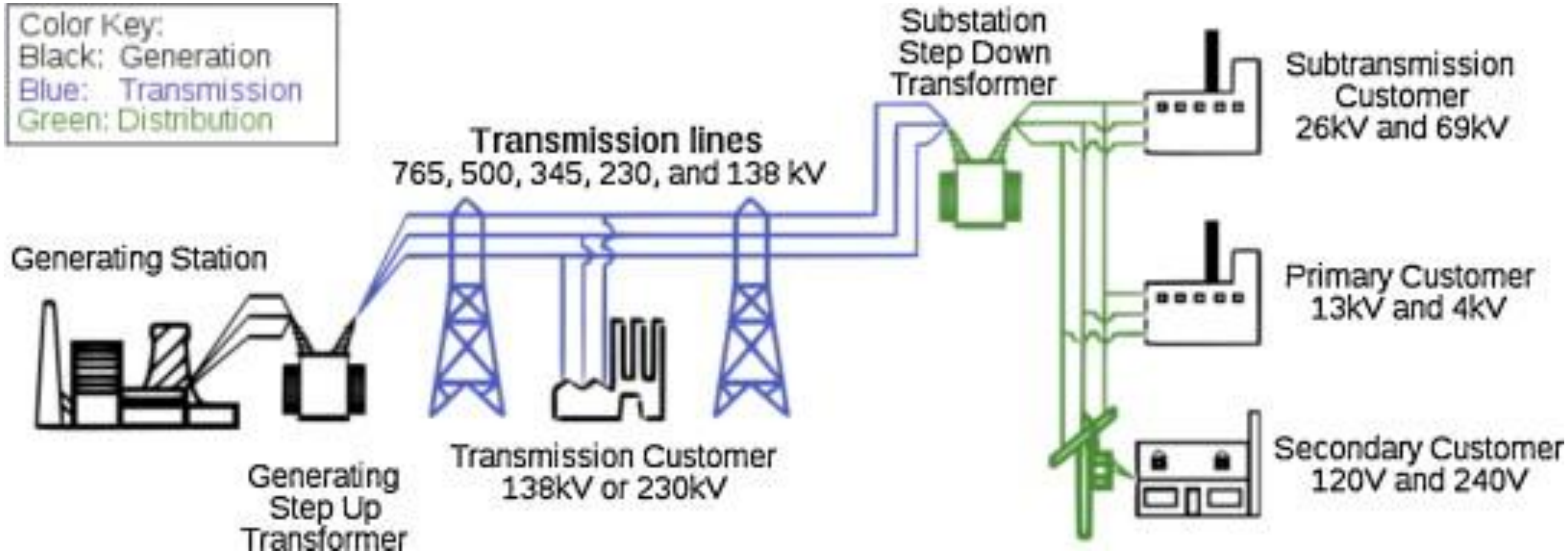
Electrical grid



Picture source: FERC, "Final Report on the August 14, 2003 Blackout in the United States and Canada," Apr. 2004. [Online]. http://www.ferc.gov/industries/electric/indus-act/reliability/blackout/ch1-3.pdf

Introduction

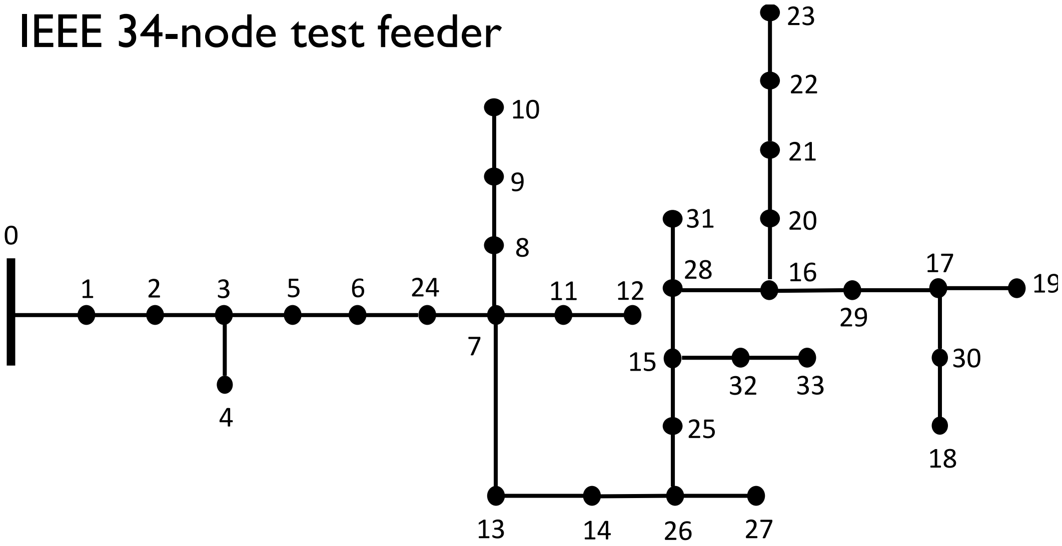
- Pressing issues
    - Demand is rising
    - Fossil fuel resources are limited
    - There is risk of regional blackouts

- Advantages of distributed renewable generation
    - Decentralized production-close to the point of consumption
    - Bypass transmission network congestion

- Example: Photovoltaic (PV) units
- Challenge: Renewable generation has uncertainty

Optimal PV placement and sizing

IEEE 34-node test feeder

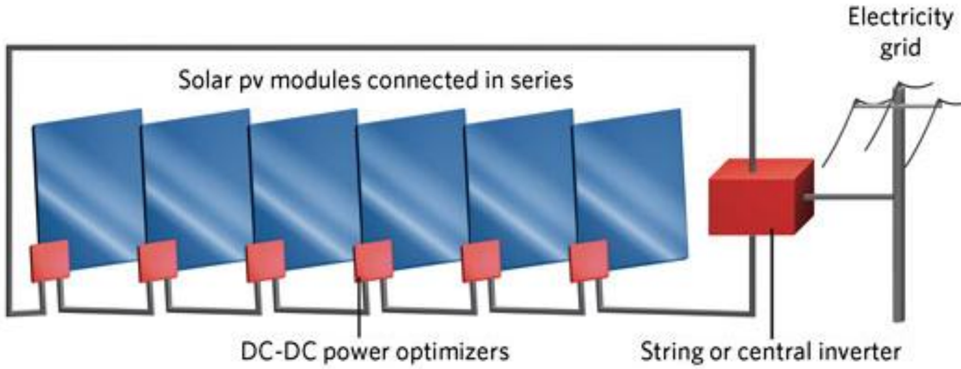


- Where to place PV units?
- Sizing: Area of panel and inverter capability
- Economic operation and voltage regulation

CPS Energy Proposes Leased Solar Option

IRIS DIMMICK on 10 February, 2015 at 01:03

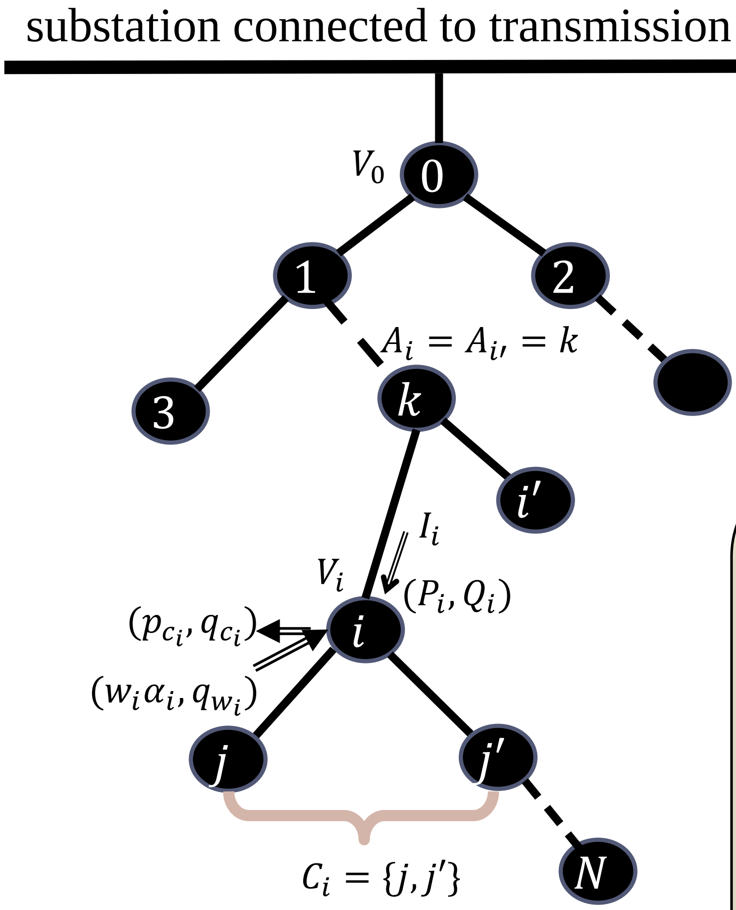
http://therivardreport.com/cps-energy-proposes-leased-solar-option/



Prior art and contributions

- Placement of distributed generation: Survey [Georgilakis-Hatziargyriou '13]
- Placement & sizing of DG units without reactive power compensation
  - Loss minimization [Atwa et al '10]
  - Conservation voltage reduction [Wang et al '15]
  - Unbalanced distribution systems [Dall'Anese-Giannakis '13]
- With reactive power compensation
  - Other DG: Independent real & reactive power limits, genetic alg. [Liu et al '11]
  - PVs: Linearized capability curve [Nick et al '14]
- This work: Two-stage stochastic programming approach
  - Joint optimization of real and apparent power capability
  - Scenario-adaptive reactive power compensation for voltage regulation
  - Minimization of installation and operation costs

SOCP power flow equations



[Jabr '06] [Low '12]

Change of variables

$$v_i = V_i^2,\ l_i = I_i^2$$

$$P_i = \sum_{j \in C_i} (P_j + r_j l_j) + p_{c_i} - w_i \alpha_i$$

$$Q_i = \sum_{j \in C_i} (Q_j + x_j l_j) + q_{c_i} - q_{w_i}$$

$$v_{A_i} = v_i + 2(r_i P_i + x_i Q_i) + (r_i^2 + x_i^2) l_i$$

$$P_i^2 + Q_i^2 \le v_i l_i$$ ⇨ SOCP relaxation

PV injection model: Real power

- Real power output is $\alpha_i w_i$
- α_i is normalized irradiance

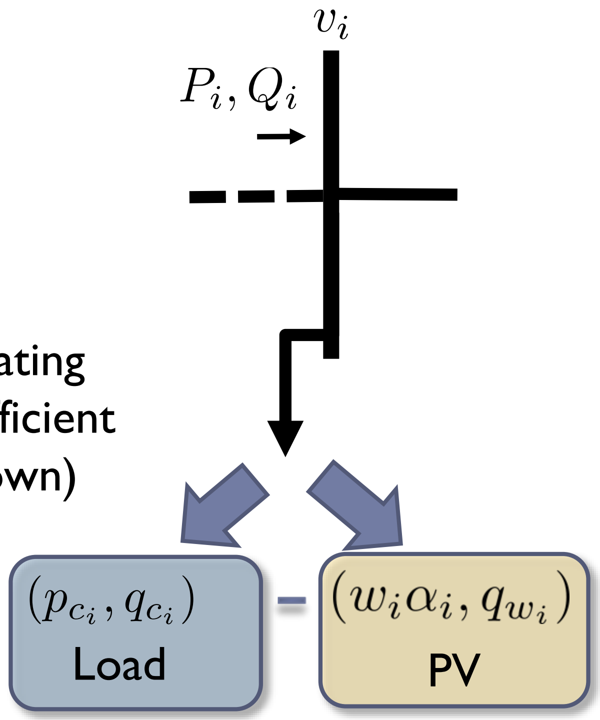
Maximum AC power output (Watts) →

$$w_i = a_i h_i^{\max} \eta_i$$

Area (sq. meters)

Maximum irradiance Watts/(sq. meters) (known)

Derating coefficient (known)



PV injection model: Reactive power

- To allow for reactive power control, size the inverter so that

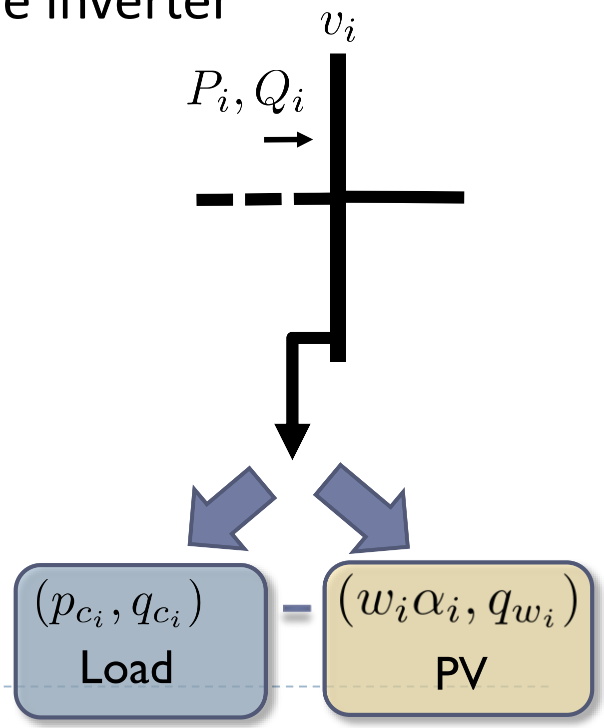
$$s_{w_i} > w_i$$

where s_{w_i} is the apparent power capability of the inverter

[Turitsyn, Šulc, Backhaus, Chertkov ’10-’11]

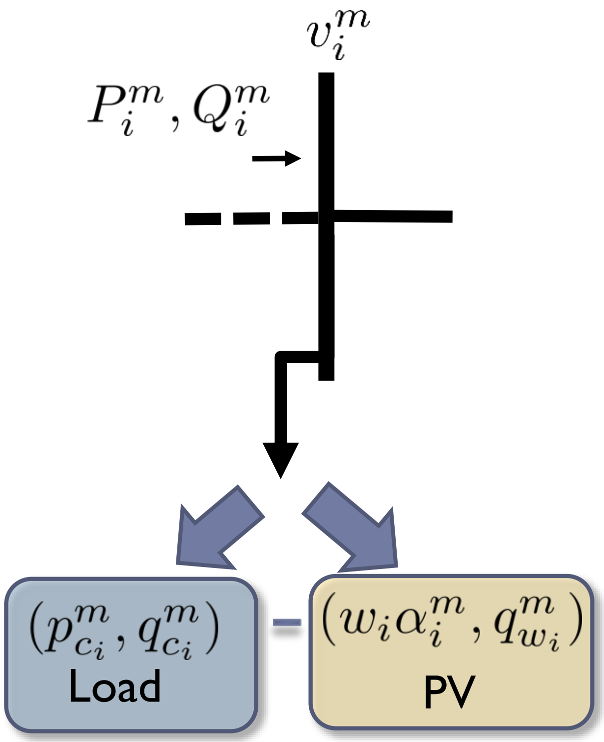
- Reactive power q_{w_i} *generated or consumed*

$$\sqrt{q_{w_i}^2 + (w_i\alpha_i)^2} \le s_{w_i}$$



Uncertainty model

- M plausible scenarios for irradiance and load
- Scenario m: irradiance α_i^m, load $p_{c_i}^m, q_{c_i}^m$
- Scenario m happens with probability π^m



- Scenario-dependent decision variables
  - Power flows, voltages, currents $P_i^m, Q_i^m, v_i^m, l_i^m$
  - Reactive power from the PVs $q_{w_i}^m$
- Scenario-independent decision variables
  - Placement binary decisions b_i
  - Sizing decision w_i and s_{w_i}

Scenario-dependent constraints

- Power flow equations

$$P_i^m = \sum_{j \in \mathcal{C}_i} (P_j^m + r_j l_j^m) + p_{c_i}^m - w_i \alpha_i^m$$

$$Q_i^m = \sum_{j \in \mathcal{C}_i} (Q_j^m + x_j l_j^m) + q_{c_i}^m - q_{w_i}^m - q_{s_i} v_i^m$$

$$v_{A_i}^m = v_i^m + 2(r_i P_i^m + x_i Q_i^m) + (r_i^2 + x_i^2) l_i^m$$

$$(P_i^m)^2 + (Q_i^m)^2 \leq v_i^m l_i^m$$

- Reactive power

$$\sqrt{(q_{w_i}^m)^2 + (w_i \alpha_i^m)^2} \leq s_{w_i}$$

Sizing and adaptive reactive power compensation cast as SOCP

- Penetration

$$\sum_{i=1}^{N} w_i \alpha_i^m \leq \theta \sum_{i=1}^{N} p_{c_i}^m$$

Scenario-independent constraints

- Binary decisions $b_i \in \{0, 1\}$
- Minimum and maximum area $b_i w_i^{\min} \le w_i \le b_i w_i^{\max}$
- Bounds on PV power factor $w_i \le s_{w_i} \le \frac{1}{\mathrm{PF}_{\min}} w_i$
- Budget on number of installations $\sum_{i=1}^{N} b_i \le B$

Objectives

- Objective comprises first-stage and the expected second-stage costs
- Examples follow
- First-stage cost
  - Installation

$$\sum_{i=1}^{N} C_i(s_{w_i}) = \sum_{i=1}^{N} \left(c_i s_{w_i}^2 + d_i s_{w_i}\right)$$

- Second-stage costs
  - Expected thermal loss

$$\sum_{m=1}^{M} \pi^m \sum_{i=1}^{N} r_i l_i^m$$

Losses in scenario m

  - Expected import cost $\sum_{m=1}^{M} \pi^m C(P_0^m)$ where

$$C_0(P_0^m) = \begin{cases} K_{0+} P_0^m & \text{if } P_0^m \geq 0 \\ K_{0-} P_0^m & \text{if } P_0^m < 0. \end{cases}$$

Placement and sizing (MISOCP)

First-stage cost

Second-stage cost

$$\min_{\substack{\mathbf{s}_w,\mathbf{w},\mathbf{b},\\ \mathbf{q}_w,\mathbf{P},\\ \mathbf{Q},\mathbf{v},\mathbf{l}}} \boxed{\sum_{i=1}^{N} C_i(s_{w_i})} + \boxed{\sum_{m=1}^{M} \pi^m \Big[\sum_{i=1}^{N} r_i l_i^m + C_0(P_0^m)\Big]}$$

subject to

Placement and Sizing

$$b_i \in \{0,1\}$$

$$b_i w_i^{\min} \le w_i \le b_i w_i^{\max}$$

$$w_i \le s_{w_i} \le \frac{1}{\mathrm{PF}_{\min}} w_i$$

$$\sum_{i=1}^{N} b_i \le B$$

Penetration

$$\sum_{i=1}^{N} w_i \alpha_i^m \le \theta \sum_{i=1}^{N} p_{c_i}^m$$

Reactive power

$$\sqrt{(q_{w_i}^m)^2 + (w_i \alpha_i^m)^2} \le s_{w_i}$$

Power flows and voltage regulation

$$P_i^m = \sum_{j \in \mathcal{C}_i} (P_j^m + r_j l_j^m) + p_{c_i}^m - w_i \alpha_i^m$$

$$Q_i^m = \sum_{j \in \mathcal{C}_i} (Q_j^m + x_j l_j^m) + q_{c_i}^m - q_{w_i}^m - q_{s_i} v_i^m$$

$$v_{A_i}^m = v_i^m + 2(r_i P_i^m + x_i Q_i^m) + (r_i^2 + x_i^2) l_i^m$$

$$(P_i^m)^2 + (Q_i^m)^2 \le v_i^m l_i^m$$

$$v_0 - \epsilon \le v_i^m \le v_0 + \epsilon$$

Constraints independent from scenarios

Constraints per scenario

Numerical test setup

IEEE 34-node feeder converted to single-phase system $V_{\text{base}} = 24.9 \text{ kV}, \quad S_{\text{base}} = 500 \text{ kVA}$

Shunt capacitors are included

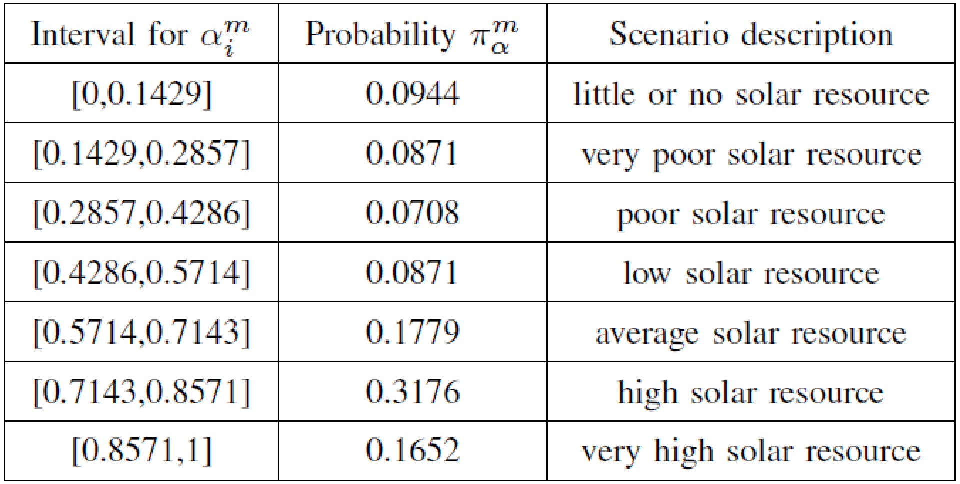
3 load scenarios: 50%, 100%, 150% of peak load with probabilities $\pi_c^m = \{0.3, 0.4, 0.3\}$

Solar Direct Normal Irradiance Data from NREL for a station near San Antonio, June-Aug, 11:00 am – 4:00 pm ➞ Create a histogram

Simplified scenario generation model using irradiance normalized to 1

$$C(s_{w_i}) = 0.01 s_{w_i}^2 + 0.03 s_{w_i}$$

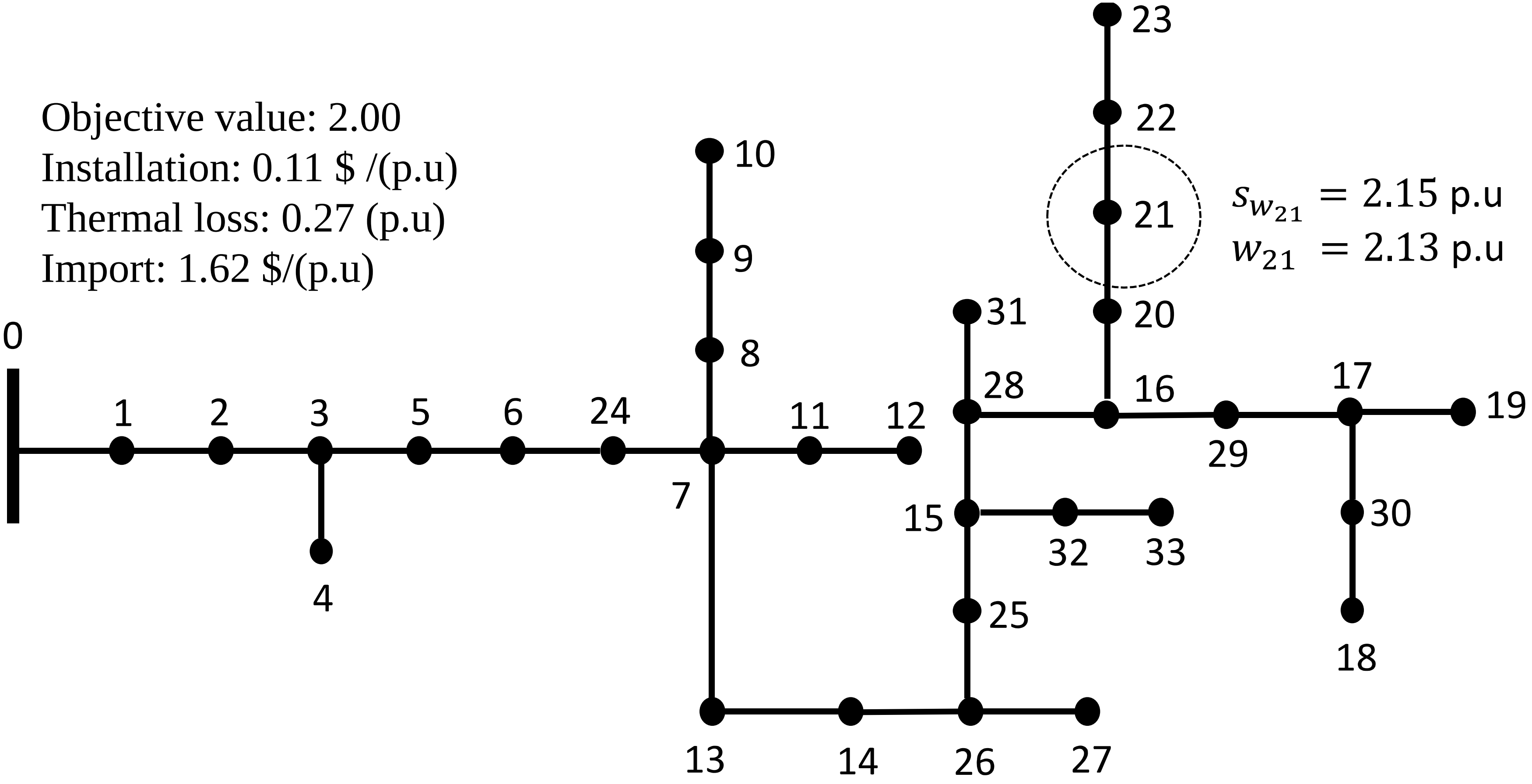
$$C(P_0^m) = \begin{cases} P_0^m & \text{if } P_0^m \geq 0 \\ 0.8 P_0^m & \text{if } P_0^m \leq 0 \end{cases}$$



Interval for α_i^m	Probability π_α^m	Scenario description
[0,0.1429]	0.0944	little or no solar resource
[0.1429,0.2857]	0.0871	very poor solar resource
[0.2857,0.4286]	0.0708	poor solar resource
[0.4286,0.5714]	0.0871	low solar resource
[0.5714,0.7143]	0.1779	average solar resource
[0.7143,0.8571]	0.3176	high solar resource
[0.8571,1]	0.1652	very high solar resource

Numerical test : $B = 1$

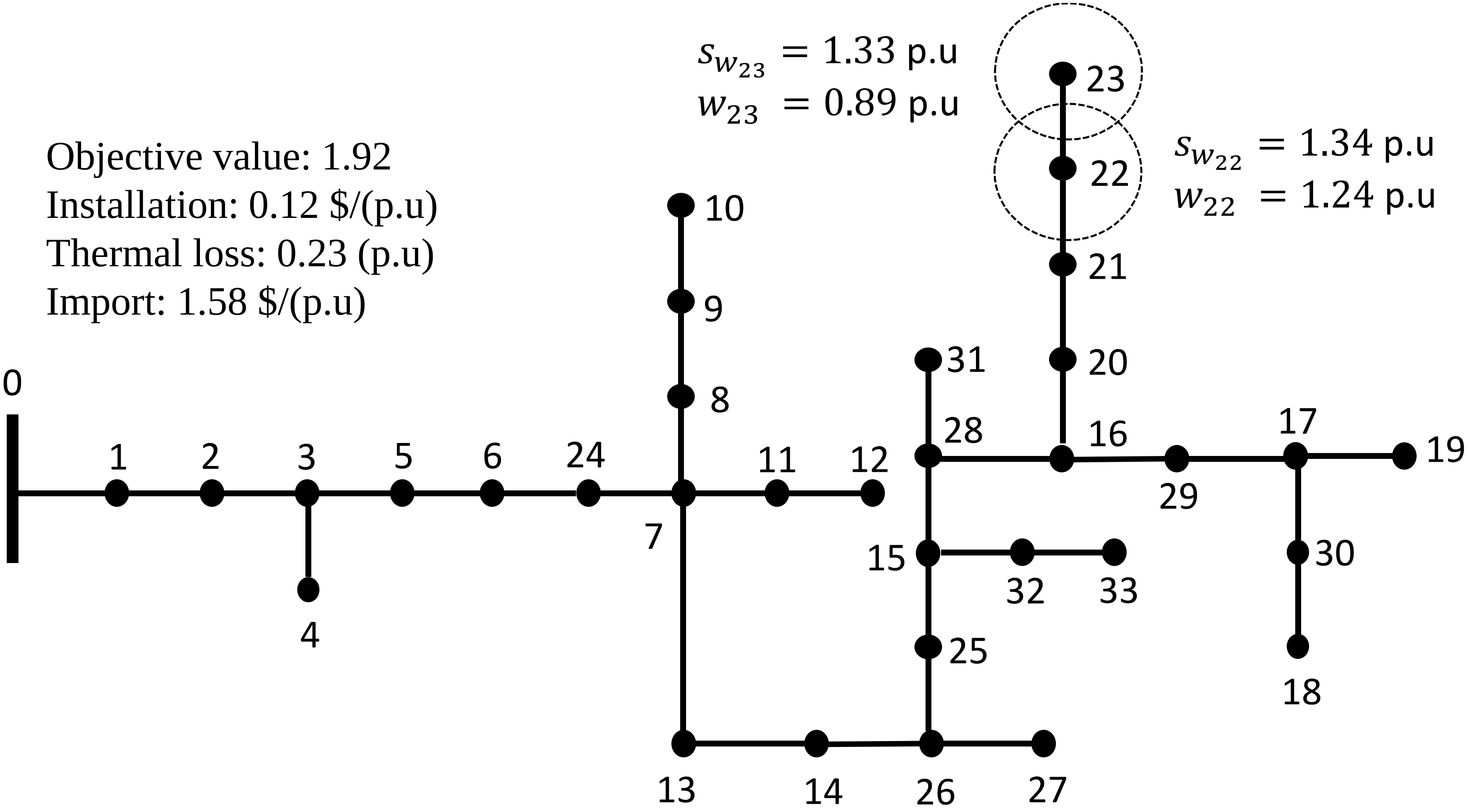
Objective value: 2.00
Installation: 0.11 $ /(p.u)
Thermal loss: 0.27 (p.u)
Import: 1.62 $/(p.u)



$s_{w_{21}} = 2.15$ p.u
$w_{21} = 2.13$ p.u

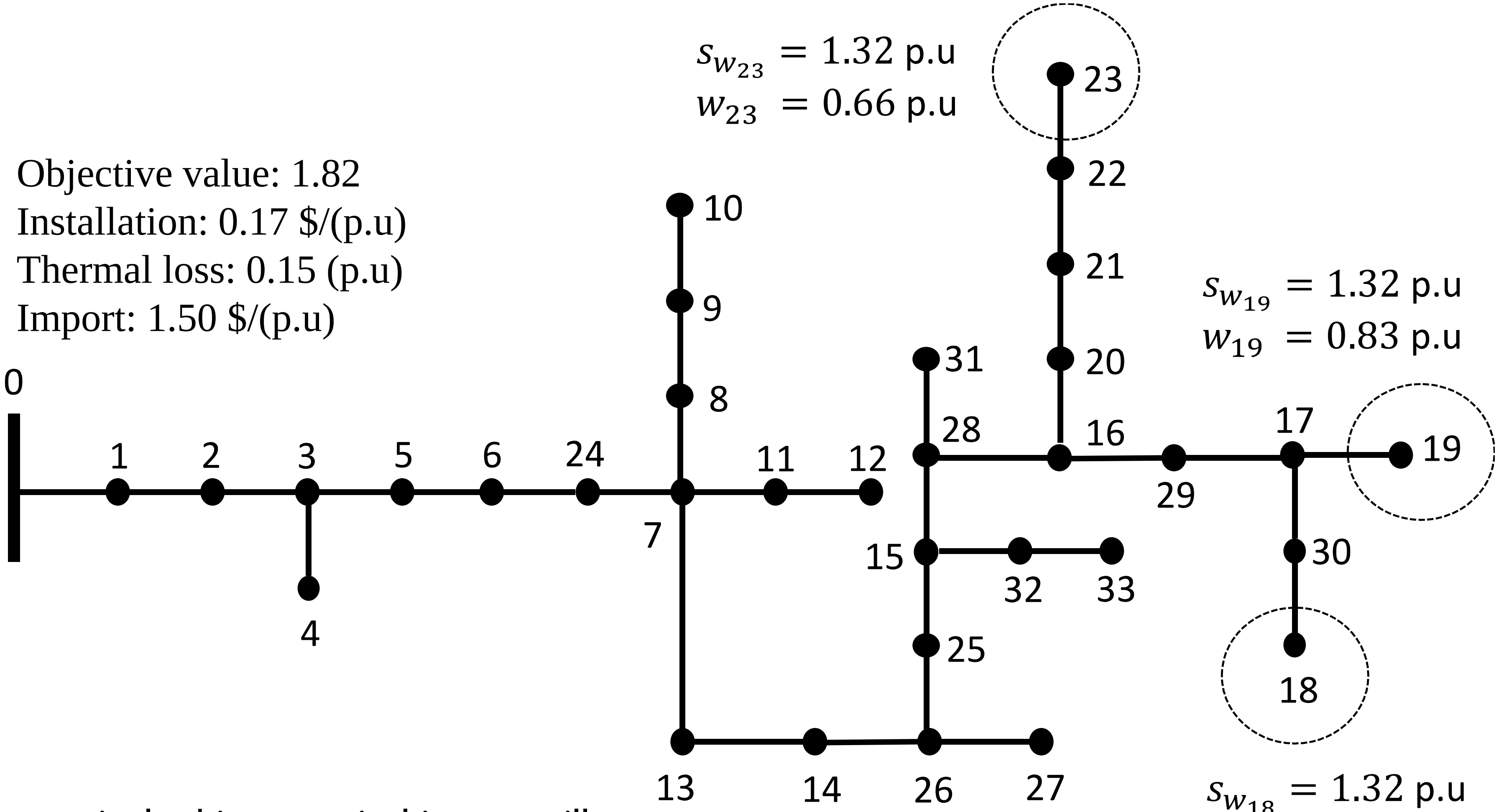
Numerical test : $B = 2$

Objective value: 1.92
Installation: 0.12 \$/(p.u)
Thermal loss: 0.23 (p.u)
Import: 1.58 \$/(p.u)



Numerical test : $B = 3$

Objective value: 1.82
Installation: 0.17 \$/(p.u)
Thermal loss: 0.15 (p.u)
Import: 1.50 \$/(p.u)



$s_{w_{23}} = 1.32$ p.u
$w_{23} = 0.66$ p.u

$s_{w_{19}} = 1.32$ p.u
$w_{19} = 0.83$ p.u

$s_{w_{18}} = 1.32$ p.u
$w_{18} = 0.64$ p.u

Interestingly, this numerical instance illustrates that a greedy placement algorithm is not optimal

Summary and future work

- Optimal placement and sizing
  - Real & reactive power capability optimization cast as a SOC constraint
  - Load and solar generation uncertainty
  - Scenario-adaptive power flow
  - Minimization of installation and operation cost

- Future work
  - Placement of distributed storage (batteries) with reactive power support
  - Three-phase distribution networks

Thank you!