COMPRESSIVE PULSE-DOPPLER RADAR SENSING VIA ONE-BIT SAMPLING WITH TIME-VARYING THRESHOLD

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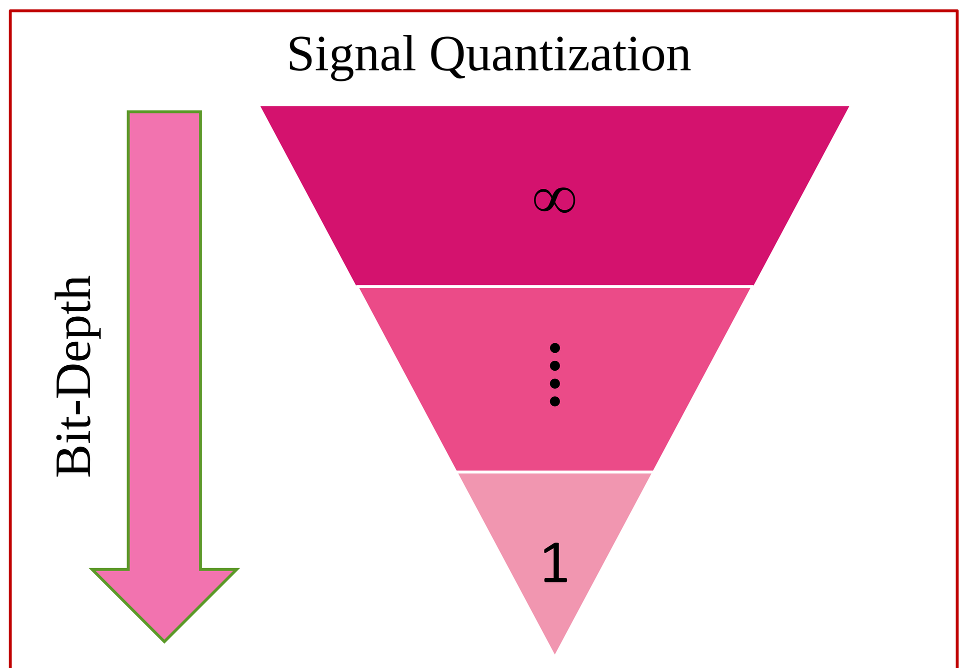
ABSTRACT

We propose a pulse-Doppler radar that works through 1-bit quantization of the received noisy signal. The 1-bit quantization is performed by comparing the signal with a time-varying threshold. Considering the sparsity of the targets in the range-Doppler domain, the problem is dealt with by a sparse recovery method. Numerical examples show that the proposed sensing method has a promising performance.

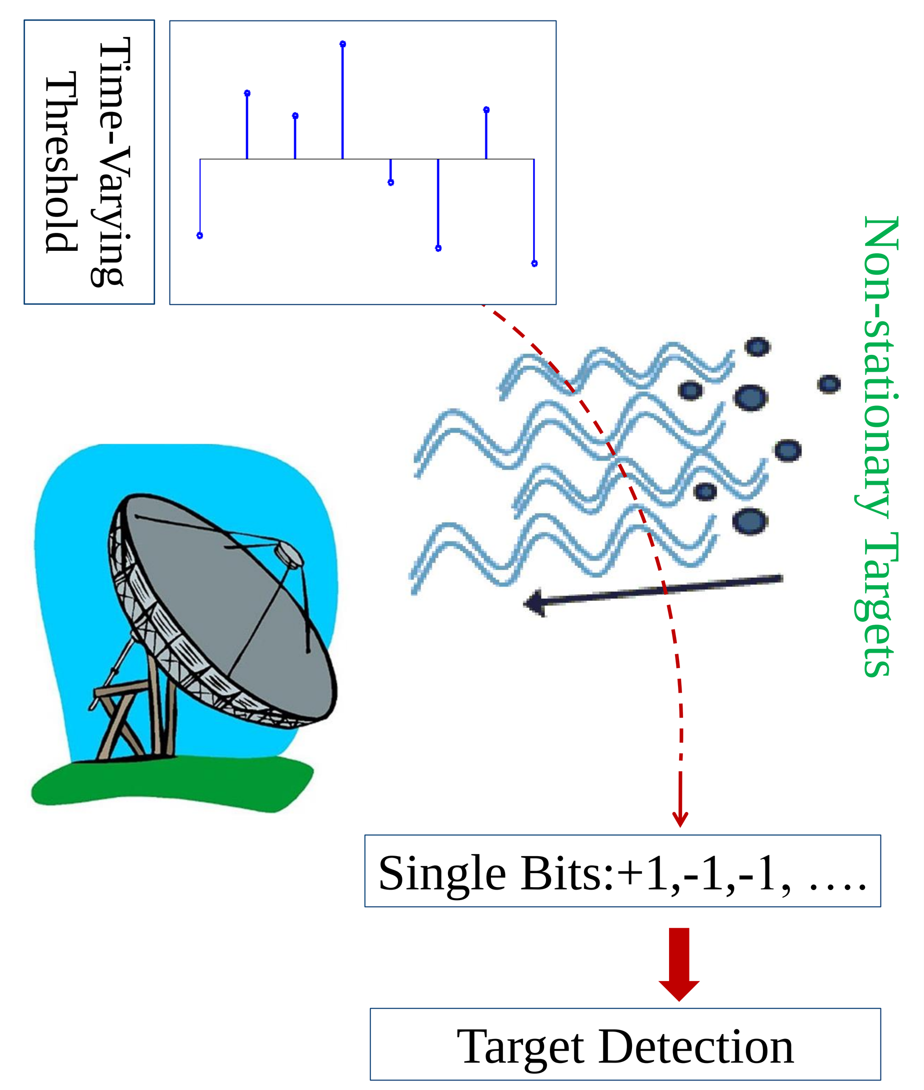
INTRODUCTION



One-Bit Sampling

Very High Sampling Rates	• Up to 240 Gigasamples per second
Low Power	• Only about 10 mW in a rate of 240 GSample/sec
Low Price	• Only a few dollars

OBJECTIVE



PROBLEM FORMULATION

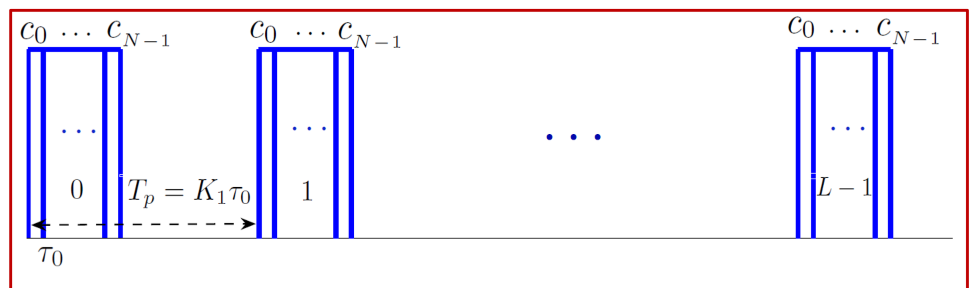


Fig. 1. The transmitted multi-pulse intra-coded signal.

Considering a pulse-Doppler radar under a single-input single-output setup:

$$s_T(t) = \sum_{\ell=0}^{L-1}\sum_{n=0}^{N-1} c_n p(t - n\tau_0 - \ell T_p)$$

$$s_{Rec}(t) = \sum_{k_r=1}^{K_r}\sum_{k_d=1}^{K_d} \alpha_{k_r k_d} s_T(t-\tau_{k_r}) e^{j\omega_{k_d} t} + \epsilon(t)$$

$$= \sum_{k_r,k_d}\sum_{\ell,n} \alpha_{k_r k_d} c_n p(t - n\tau_0 - \ell T_p - \tau_{k_r}) e^{j\omega_{k_d} t} + \epsilon(t)$$

$$\boldsymbol{\alpha} \triangleq \begin{bmatrix} \alpha_{11} & \cdots & \alpha_{1K_d} \\ \vdots & \ddots & \vdots \\ \alpha_{K_r 1} & \cdots & \alpha_{K_r K_d} \end{bmatrix}$$

$$\mathbf{f}(t) \triangleq \begin{bmatrix} \sum_{\ell=0}^{L-1}\sum_{n=0}^{N-1} c_n p(t - n\tau_0 - \ell T_p - \tau_1) \\ \vdots \\ \sum_{\ell=0}^{L-1}\sum_{n=0}^{N-1} c_n p(t - n\tau_0 - \ell T_p - \tau_{K_r}) \end{bmatrix}$$

$$\boldsymbol{\phi}(t) \triangleq [e^{j\omega_1 t}, \ \cdots, \ e^{j\omega_{K_d} t}]^T$$

$$\Rightarrow s_{Rec}(t) = \mathbf{f}^T(t)\boldsymbol{\alpha}\boldsymbol{\phi}(t) + \epsilon(t)$$

The time-varying thresholds:

$$h(t) \triangleq h_R(t) + i h_I(t)$$

The observed quantized data:

$$y(t) \triangleq y_R(t) + i y_I(t)$$

$$\begin{aligned} y_R(t) &= \mathrm{sgn}(\mathrm{Re}\,[s_{Rec}(t)] - h_R(t)) \\ &= \mathrm{sgn}\left(\mathrm{Re}\left[\mathbf{f}^T(t)\boldsymbol{\alpha}\boldsymbol{\phi}(t) + \epsilon(t)\right] - h_R(t)\right) \\ y_I(t) &= \mathrm{sgn}(\mathrm{Im}\,[s_{Rec}(t)] - h_I(t)) \\ &= \mathrm{sgn}\left(\mathrm{Im}\left[\mathbf{f}^T(t)\boldsymbol{\alpha}\boldsymbol{\phi}(t) + \epsilon(t)\right] - h_I(t)\right) \end{aligned}$$

Considering M samples at times $t_1, \ldots, t_M$

$$\begin{cases} \mathbf{h} \triangleq [h(t_1), \ldots, h(t_M)]^T, \\ \boldsymbol{\epsilon} \triangleq [\epsilon(t_1), \ldots, \epsilon(t_M)]^T, \\ \mathbf{y} \triangleq [y(t_1), \ldots, y(t_M)]^T. \end{cases}$$

Defining

$$\mathbf{F} \triangleq \left[\mathbf{f}(t_1)\middle|\mathbf{f}(t_2)\middle|\cdots\middle|\mathbf{f}(t_M)\right]^T$$

$$\boldsymbol{\Phi} \triangleq [\boldsymbol{\phi}(t_1)|\boldsymbol{\phi}(t_2)|\cdots|\boldsymbol{\phi}(t_M)]$$

$$\begin{cases} \mathbf{y}_R = \mathrm{sgn}(\mathrm{Re}\,[\mathrm{Diag}\{\mathbf{F}\boldsymbol{\alpha}\boldsymbol{\Phi}\} + \boldsymbol{\epsilon}] - \mathbf{h}_R) \\ \mathbf{y}_I = \mathrm{sgn}(\mathrm{Im}\,[\mathrm{Diag}\{\mathbf{F}\boldsymbol{\alpha}\boldsymbol{\Phi}\} + \boldsymbol{\epsilon}] - \mathbf{h}_I) \end{cases}$$

The problem is to estimate the vector $\boldsymbol{\alpha}$ from $\mathbf{y}$

THE PROPOSED SOLUTION

$$\boldsymbol{\alpha} \triangleq \begin{bmatrix} \alpha_{11} & \cdots & \alpha_{1K_d} \\ \vdots & \ddots & \vdots \\ \alpha_{K_r 1} & \cdots & \alpha_{K_r K_d} \end{bmatrix}$$

Vectorize (column by column):

$$\widetilde{\boldsymbol{\alpha}} \triangleq [\alpha_{11}, \cdots, \alpha_{K_r 1} \ \alpha_{12}, \cdots, \alpha_{K_r 2} \ \cdots \ \alpha_{1K_d} \cdots \alpha_{K_r K_d}]^T$$

$$\mathrm{Diag}\{\mathbf{F}\boldsymbol{\alpha}\boldsymbol{\Phi}\} = (\mathbf{1}_{1\times K_d} \otimes \mathbf{F}) \odot (\boldsymbol{\Phi}^T \otimes \mathbf{1}_{1\times K_r})\widetilde{\boldsymbol{\alpha}}$$

$$\begin{cases} \mathbf{y}_R = \mathrm{sgn}\left(\mathrm{Re}[\widetilde{\mathbf{F}}\widetilde{\boldsymbol{\alpha}} + \boldsymbol{\epsilon}] - \mathbf{h}_R\right), \\ \mathbf{y}_I = \mathrm{sgn}\left(\mathrm{Im}[\widetilde{\mathbf{F}}\widetilde{\boldsymbol{\alpha}} + \boldsymbol{\epsilon}] - \mathbf{h}_I\right), \end{cases}$$

$$\widetilde{\mathbf{F}} \triangleq (\mathbf{1}_{1\times K_d} \otimes \mathbf{F}) \odot (\boldsymbol{\Phi}^T \otimes \mathbf{1}_{1\times K_r})$$

Sparsity of the targets in the range-Doppler domain ⇒ Sparse $\widetilde{\boldsymbol{\alpha}}$

Adjusts the sparsity

$$\min_{\widetilde{\boldsymbol{\alpha}}, \mathbf{z}} \quad \|\mathbf{z}\|_2 + \lambda \|\widetilde{\boldsymbol{\alpha}}\|_0$$

$$\text{s.t.} \quad \begin{aligned} \mathbf{y}_R \odot (\mathrm{Re}[\widetilde{\mathbf{F}}\widetilde{\boldsymbol{\alpha}} + \mathbf{z}] - \mathbf{h}_R) \geq 0 \\ \mathbf{y}_I \odot (\mathrm{Im}[\widetilde{\mathbf{F}}\widetilde{\boldsymbol{\alpha}} + \mathbf{z}] - \mathbf{h}_I) \geq 0 \end{aligned}$$

✓ We must characterize $\mathbf{F}$ and $\boldsymbol{\Phi}$

$$t_m = m\frac{\tau_0}{r}, \qquad m = 1, \ldots, M.$$

$$\tau_{k_r} = k_r\frac{\tau_0}{r}, \qquad k_r = 1, \ldots, K_r$$

$$\omega_{k_d} = -\omega_D + (k_d - 1)\Delta, \quad \text{for } k_d = 1, \ldots, K_d$$

Maximum Doppler Frequency

$$\Rightarrow \boldsymbol{\Phi} = \left[\boldsymbol{\phi}\middle|\boldsymbol{\phi}^{(2)}\middle|\cdots\middle|\boldsymbol{\phi}^{(M)}\right]$$

$$\boldsymbol{\phi} = \left[e^{-j\omega_D\frac{\tau_0}{r}}, \ e^{-j(\omega_D - \Delta)\frac{\tau_0}{r}}, \cdots, \ e^{j\omega_D\frac{\tau_0}{r}}\right]^T$$

$\Rightarrow K_r > rN$

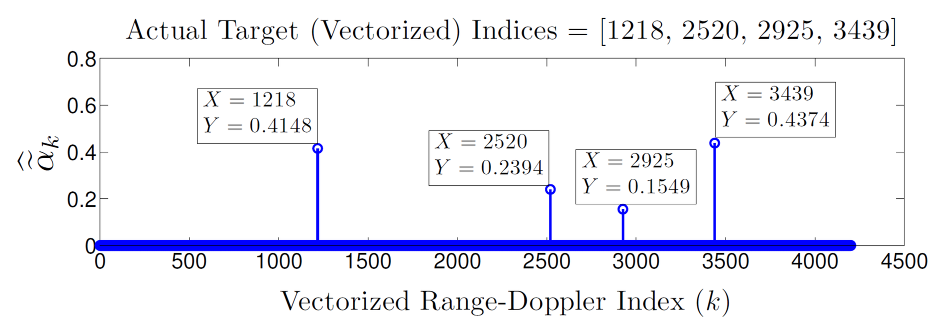
$$\mathbf{F} = \begin{array}{c} \begin{bmatrix} \mathbf{c}_0 & & \\ \vdots & \ddots & \\ \mathbf{c}_{N-1} & \ddots & \ddots \\ & \ddots & \ddots & \mathbf{c}_0 \\ & & \ddots & \vdots \\ & & & \mathbf{c}_{N-1} \\ \mathbf{c}_0 & & \\ \vdots & \ddots & \\ \mathbf{c}_{N-1} & \ddots & \ddots \\ & \ddots & \ddots & \mathbf{c}_0 \\ & & \ddots & \vdots \\ & & & \mathbf{c}_{N-1} \\ & \vdots & & \\ & & \vdots & \\ \mathbf{c}_0 & & \\ \vdots & \ddots & \\ \mathbf{c}_{N-1} & \ddots & \ddots \\ & \ddots & \ddots & \mathbf{c}_0 \\ & & \ddots & \vdots \\ & & & \mathbf{c}_{N-1} \end{bmatrix} \end{array} \begin{array}{l} 1 \\ \\ r(N-1)+1 \\ K_r \\ \\ K_r + rN - r \\ rK_r + 1 \\ \\ \\ \\ \\ \\ \\ \\ (L-1)rK_r + 1 \\ \\ \\ \\ \\ \\ \end{array}$$

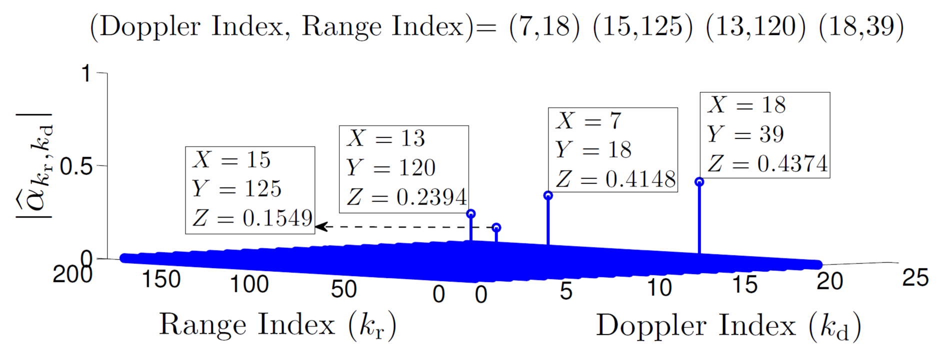
NUMERICAL EXAMPLES

✓Approximating the zero-norm with l_1-norm ⇒ Convex and Tractable

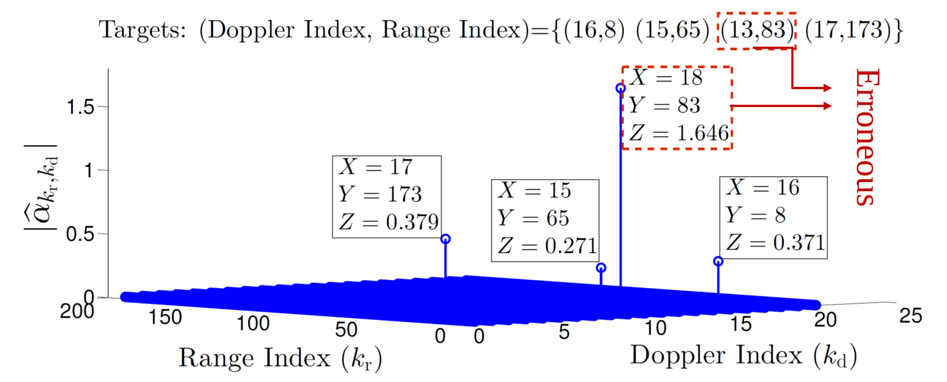
❖ Example 1:

N_t	SNR	N	r	K_r	K_d	L	τ_0	ω_D	λ
4	6 dB	16	2	200	21	5	0.01 ms	2000π rad/sec	1

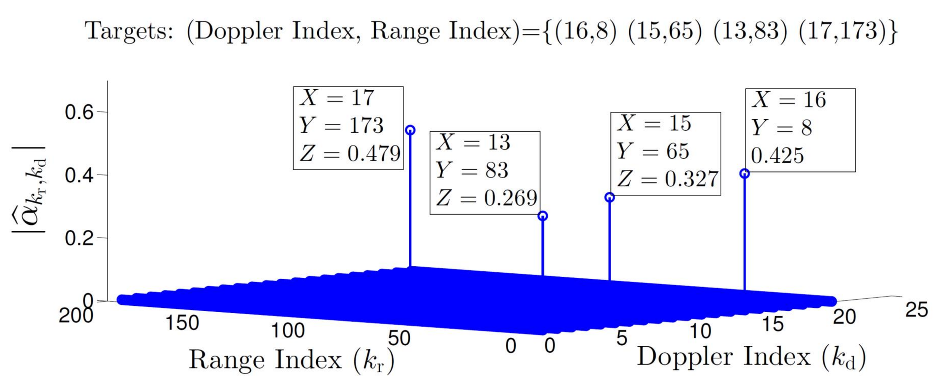




❖ Example 2: Reducing the SNR to 2 dB



❖ Example 3: Increasing the sampling rate with r =3 at SNR=2 dB.



CONCLUSION

❑ We presented a compressive pulse-Doppler radar based on one-bit quantization of the received noisy signal.

❑ The problem was approached by a sparse recovery method, which led to a norm-penalized optimization problem.

❑ Numerical examples showed that the proposed sensing method has a promising performance.

❑ Increasing the sampling rate compensates the performance loss of low SNR.

ACKNOWLEDGMENT

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